Curve and Surfaces for Computer Graphics (SC303)

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Problem No -1

In computer graphics, moving an object along a parametrized curve with constant speed is known as the curve to be reparametrized by arc length. We have $C_1(s)$, s $\in$[0,L] where L is length of the curve, be an arc-length parametrized curve and $C_2(t)$, t $\in$ [t_{min} , t_{max}] , be another parametrization of the same curve. And that provides a relationship between t and s which is defined as

$$(t, s) \text{ for which } C_2(t) = C_1(s)$$

Applying chain rule results in,

$$\frac{dC_2}{dt} = \frac{dC_1}{ds} \frac{ds}{dt}$$

furthermore,

$$\left|\frac{dC_2}{dt}\right| = \left|\frac{dC_1}{ds}\right| \left|\frac{ds}{dt}\right| = \frac{ds}{dt}$$

And to obtain relationship between s and t, let us integrate the above equation, we obtain s as a function of t,

$$s = g(t) = \int_{t_{min}}^{t} \left|\frac{dC_{2(\tau)}}{dt}\right| d\tau$$

From above, given the time t, we can determine the corresponding **arc length s** from the integration.

Let say we have 6 points p0(0,0,0), p1(0,0,1), p2(0,1,0), p3(1,0,0), p4(1,1,1) and p5(0,1,1). from these points we divided the entire arc into three segments A1, A2 and A3. A1 segment is constructed by the points p0, p1, p2 and p3 similarly A2 is constructed by the points p1, p2, p3 and p4 and A3 is constructed by the points p2, p3, p4 and p5 . And we have the total length of curve is L .

Since, we have the cubic uniform B-Spline equation for 4 points (will use the equation for parametrization) :

$$P_i(t) = \frac{1}{6}(-t^3+3t^2-3t+1)P_{i-1} + \frac{1}{6}(3t^3-6t^2+4)P_i + \frac{1}{6}(-3t^3+3t^2+3t+1)P_{i+1} + \frac{1}{6}(t^3)P_{i+2} \quad ....(1)$$

(a) : In first case we have to determine t, for a given an arc length s, at which that arc length is achieved.

In this problem we have the function g(t):

$$s = g(t) = \int_{t_{min}}^{t} |\frac{dC_{2(\tau)}}{dt}| d\tau$$

Defining a new function F(t) = g(t) - s. It is just a root finding problem in which we have to find t for a given s such that F(t) = 0.

Using Newton's method :

$$t_{i+1} = t_i - \frac{F(t_i)}{F'(t_i)} \text{ for given } i \geq 0$$

We have the value of F(t) and we can find the value of F'(t) by:

$$F'(t) = \frac{dg}{dt} = |\frac{dY}{dt}|$$

Iterating for the value of t and t will approach to that value at which s is achieved. To start with, the initial value for t will be:

$$t_0 = t_{min} + \frac{s}{L}(t_{max} - t_{min})$$

These iterations are computed until either F'(t) is sufficiently close to zero or until maximum number of iterates.

(b) : In the second case begin with a parametrized curve C_3(u) for u $\in$[u_{min} ,u_{max}] , and determine a parametrization by time t, say, C_2(t) = C_3(u) for t $\in$ [t_{min} ,t_{max}] so that the speed at time t is a specified function σ(t) .

In this problem we have parameterized curve, Y(u) and determine a parametrization by time t say X(t) = Y(s), so that the speed at time t is a specified function $\sigma(t)$. In this problem curve parameter u is not about time. So we need to relate time variable and curve parameter.

Compute distance l traveled by particle for that time:

$$l = \int_{t_{min}}^{t} \sigma(\tau) d\tau \quad \in [0, \text{ L}]$$

Now we have to find a parameter u that results the same arc length l.

$$l = \int_{u_{min}}^{u} |\frac{dY_{(\mu)}}{du}| d\mu$$

So we can write as following:

$$l = \int_{u_{min}}^{u} |\frac{dY_{(\mu)}}{du}| d\mu \ = \int_{t_{min}}^{t} \sigma(\tau) d\tau$$

* The velocity function is user defined function.

Solving a Differential Equation

This is the differential Equation.

$$\frac{du}{dt} = \frac{\sigma(t)}{|\frac{dY(u)}{du}|}$$

We are using 4th order **Runge Kutta** Method to find the value of u for given value of t.

```
  h = ( t - t_min )/ n ;
  u = u_min ;
  t = t_min ;

while(i<=n)
  k1 = h * Sigma( t ) / F'( u ) ;
  k2 = h * Sigma ( t + h / 2 ) / F'( u + k1 / 2 ) ;
  k3 = h * Sigma ( t + h / 2 ) / F'( u + k2 / 2 ) ;
  k4 = h * Sigma ( t + h ) / F'( u + k3 ) ;
  t = t + h ;
  u = u + ( k1 + 2 ( k2 + k3 ) + k4 ) / 6 ;
endwhile
```

The final return value of this function will be u.