CS301 :: Homework 3

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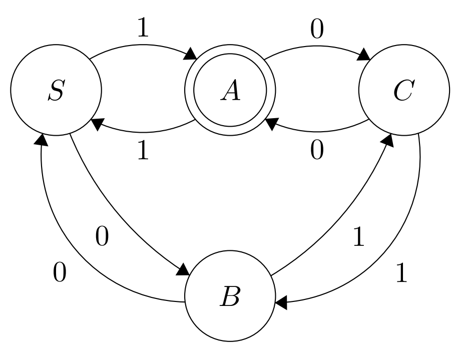
Problem 1. Regular Grammars
Consider the following language L. $\sum = \{0, 1\}$. **You do not need to produce any tuples.**

$$L = \{w : w \text{ has an even number of 0s and an odd number of 1s}\}$$

a) Give the NFA which decides L.

Solution ::



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b) Produce the right-linear, single-step CFG which is equivalent to your NFA in a).

Solution ::

$$S \longrightarrow 1A \,|\, 0B \tag{1}$$
$$A \longrightarrow 1S \,|\, OC \,|\, \epsilon \tag{2}$$
$$B \longrightarrow 0S \,|\, 1C \tag{3}$$
$$C \longrightarrow 0A \,|\, 1B \tag{4}$$

Problem 2. Context Free Grammars
Produce the CFG for the following languages. $\sum = \{a, b, c\}$. *You do not need to produce the 4-tuples.*

a) $L_a = \{w :$ the # of a's is equal to the # of b's and c's combined $\}$

Solution ::

$$S \longrightarrow aSx \,|\, xSa \,|\, SaX \,|\, XaS \,|\, SXa \,|\, XSa \tag{5}$$
$$S \longrightarrow \epsilon \tag{6}$$
$$X \longrightarrow b \,|\, c \tag{7}$$

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b) $L_b = \{a^i b^{2j} c^{i+j} : i, j \geq 1\}$

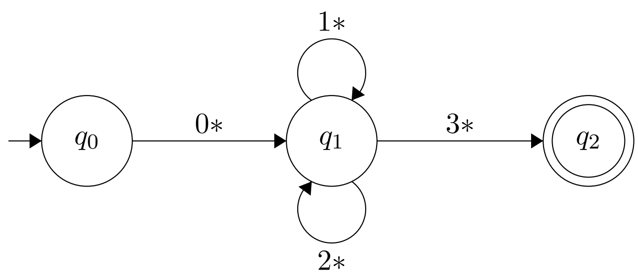
Solution ::

$$S \longrightarrow aSc \,|\, abbXcc \tag{8}$$
$$X \longrightarrow \epsilon \,|\, bbXc \tag{9}$$
$$\tag{10}$$

Problem 3. Pushdown Automata

a) Produce a PDA which decides the following language. $\sum = \{a, b, c, d\}$
$L = \{w :$ the # of a's is equal to the # of b's and c's combined $\}$
Note: unlike L_a from Q2, strings in this L may contain any # of d's.

Solution ::



$$0* = \epsilon, \epsilon \rightarrow \$ \tag{11}$$
$$1* = d, \epsilon \rightarrow \epsilon \,|\, c, x \rightarrow \epsilon \,|\, b, x \rightarrow \epsilon \,|\, a, \epsilon \rightarrow x \tag{12}$$
$$2* = b, \epsilon \rightarrow y \,|\, c, \epsilon \rightarrow y \,|\, a, y \rightarrow \epsilon \tag{13}$$
$$3* = \epsilon, \$ \rightarrow \epsilon \tag{14}$$

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b) Give the 6-tuple $(Q, \sum, \Gamma, \delta, q_0, F)$ for your PDA from a)
You do **not** need to provide the transition function δ.

Solution ::

$$Q = \{q_0, q_1, q_2\}$$
$$\sum = \{a, b, c, d\}$$
$$\Gamma = \{x, y\}$$
$$q_0 = q_0$$
$$F = \{q_2\}$$