MATH210 :: Homework 1

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Problem 1.

Let $u = \langle 2, 1, -1 \rangle$ and $v = \langle 1, 2, 1 \rangle$.

Solution ::

a)

$$\text{proj}_v u = \left(\frac{u \cdot v}{|v|^2} \right) \cdot v \tag{1}$$

$$w = \left(\frac{2(1) + 1(2) - 1(1)}{\left(\sqrt{1^2 + 2^2 + 1^2} \right)^2} \right) \cdot \langle 1, 2, 1 \rangle \tag{2}$$

$$w = \frac{3}{6} \langle 1, 2, 1 \rangle = \frac{1}{2} \langle 1, 2, 1 \rangle \tag{3}$$

$$w = \left\langle \frac{1}{2}, 1, \frac{1}{2} \right\rangle \tag{4}$$

b) $u - w$ or $u + (-w)$:

$$\langle 2, 1, -1 \rangle - \left\langle \frac{1}{2}, 1, \frac{1}{2} \right\rangle \tag{5}$$

$$u - w = \langle 1.5, 0, -1.5 \rangle \tag{6}$$

c) Show that $u - w$ is orthagonal to v.

$$u - w \cdot v = 1.5(1) + 0(2) - 1.5(1) = 0$$

$\therefore u - w$ and v are orthagonal.

d) Write v as the sum of orthagonal perp. to v.

$$u_1 = u_1 + u_2 = \text{proj}_v u + (u - \text{proj}_v u) \tag{7}$$

$$u_1 = \left(\frac{u \cdot v}{v \cdot v}\right) \cdot v = \langle 0.5, 1, 0.5 \rangle \tag{8}$$

$$u_2 = \langle 2, 1, -1 \rangle - \langle 0.5, 1, 0.5 \rangle \tag{9}$$

$$u_2 = \langle 1.5, 0, -1.5 \rangle \tag{10}$$

$$u = \langle 0.5, 1, 0.5 \rangle + \langle 1.5, 0, -1.5 \rangle \tag{11}$$

$$u = \langle 2, 1, -1 \rangle \tag{12}$$

Problem 2.

$$|u| = 1, |v| = 3, u \cdot v = -2$$

Solution ::

a)

$$|u+v| = \sqrt{|u+v|^2} \tag{13}$$
$$= \sqrt{|u|^2 + u \cdot v + u \cdot v + |v|^2} \tag{14}$$
$$= \sqrt{1^2 + (-2) + (-2) + 3^2} \tag{15}$$
$$|u+v| = \sqrt{6} \tag{16}$$

b) $||(u+2v) \times v||$

$$|u+2v| = \sqrt{|u|^2 + u \cdot 2v + u \cdot 2v + 2|v|^2} \tag{17}$$
$$= \sqrt{1^2 + (-2) + (-2) + 18} \tag{18}$$
$$= \sqrt{15} \tag{19}$$