CS301 :: Midterm 1

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71/80 Score

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Problem 1: Multiple Choice

Solution ::

a) Which of the following constraints exists for the transition function in an NFA 5-tuple.
$\delta : Q \times (\Sigma \cup \epsilon) \longrightarrow P(Q)$.

b) Given that A and B are regular languages, which of the following are not regular.
$A^n B^n$.

c) Which of the following languages is finite?
$L_b = \{w \in \{0,1\}^* : |w| \leq 20\}$.

d) From the pumping lemma, any string s with more than p characters can be partitioned into xyz. Identify the property of x, y, z below which is true.
z must be a suffix of s.

e) For a given DFA M, which of the following can be infinite?
The language it decides.

f) All finite languages are regular.
True.

g) Which of the following is a base case for the recursive structure of regular expressions?
$\emptyset :$ The empty set.

h) Which of the following languages are not regular?
$L_a = \{w \in \{0,1\}^* : w$ has at least three zeros for every one$\}$.

Problem 2: Short Answer

a) Give an example of an infinitely large binary language.
Solution ::

$$L = (01)^*$$

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b) Given an NFAs M_A, M_B which decide L_A, L_B respectively, describe how to generate an NFA which decides their concatenation, $L_A L_B$.
Solution ::

Given M_A and M_B we can make the concatenation $L_A L_B$ by adding epsilons from the final states of NFA M_A to the start state of NFA M_B. We must then make the start state of M_B. We must then make the start state of M_B become a normal state, the final states of M_A normal.

$$\begin{aligned} L(L_A) \circ L(L_B) &= \{xy \,|\, x \in L(L_A) \cap y \in L(L_B)\} \\ Q &= \{Q_A \cup Q_B\} \\ \Sigma &= \Sigma_A = \Sigma_B \\ q_0 &= q_{0_A} \\ F &= \{F_B\} \\ \delta &= \delta_A \cup \delta_B \cup (F_A, \epsilon \longrightarrow q_{0_B}) \end{aligned}$$

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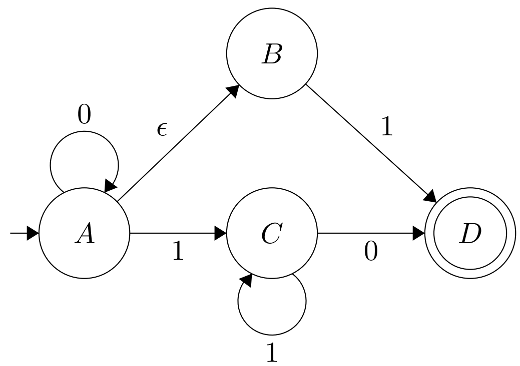
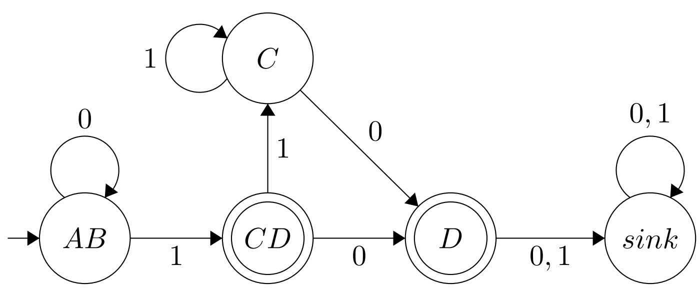
c) Give the 5-tuple for a DFA which accepts the empty language.
Let $\Sigma = \{0, 1\}$
Solution ::

$$\begin{aligned} Q &= \{q_0\} \\ \Sigma &= \{0, 1\} \\ q_0 &= q_0 \\ F &= \emptyset \\ \delta &= \{(q_0, 0) \longrightarrow q_0 \,|\, (q_0, 1) \longrightarrow q_0\} \end{aligned}$$

Problem 3: NFA to DFA
Consider the following NFA M:

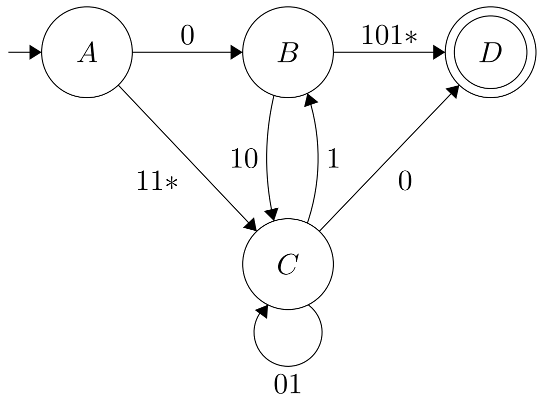


Give the state diagram for a DFA that is equivalent to M.
Solution ::

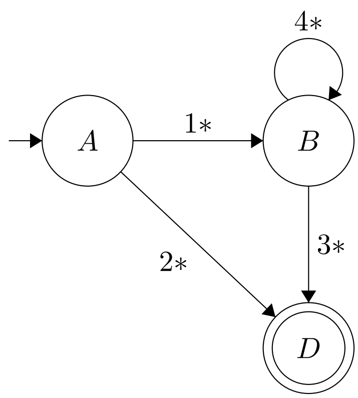


Problem 4: NFA to REGEX

Consider the following GNFA M. Give an equivalent GNFA w/out state C.



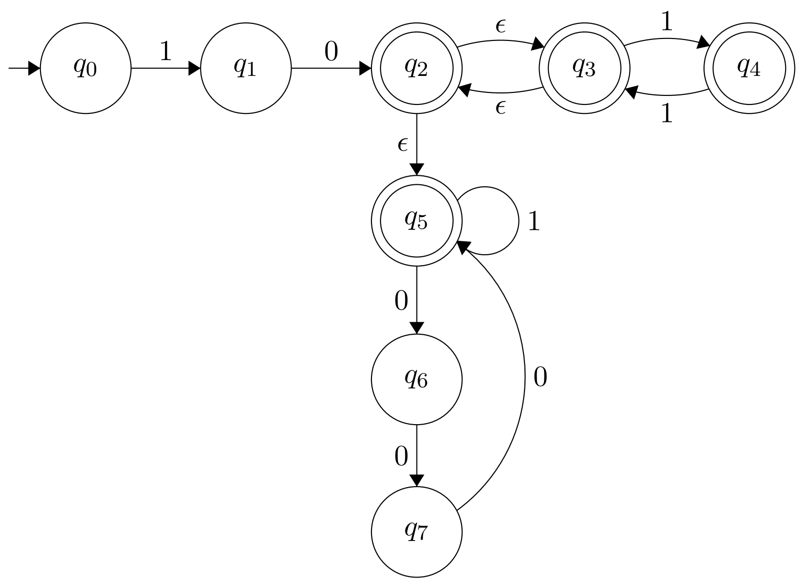
Solution ::



$$1* = 0 \cup 11^*(01)^*1$$
$$2* = 11^*(01)^*0$$
$$3* = 10(01)^*0 \cup 101^*$$
$$4* = 10(01)^*1$$

Problem 5: Regex to NFA
Give the state diagram for an NFA which decies the language L.
$\Sigma = \{0, 1\}$
$$L = 10(11)^*(000 \cup 1)^*$$

Solution ::



Problem 6: Non-Context-Free Proof
Prove that the following language L is irregular.

$$L = \{w \in \Sigma^* : w \text{ contains more 0's than 1's}\}$$

Solution ::

Let's assum L is regular for the sake of contradiction. There must be some DFA that decides L. Therefore there is a pumping length p.
Let $s = 1^p 0^{p+1}$, s can be partitioned into $xy^iz, i \geq 0$.
x must be 1^α, $0 \leq \alpha \leq p - \beta$.
y must be $1^{i\beta}$, $1 \leq \beta < p - \alpha$.
xy is within p. $\alpha + \beta \leq p$.
If we pick $i = 2$ we get the form $xyyz$ which is $\in L$. Therefore we get:

$$1^\alpha 1^\beta 1^{p-\beta-\alpha} 0^{p+1} = 1^{p+\beta} 0^{p+1}$$

However, we get the contradiction $p + \beta \geq p + 1$. The original regular language condition was that there were more 0s than 1s but because β must always be ≥ 1 they will always be at least equal.
$\therefore L$ is not regular.