

CS301 :: Homework 4

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Problem 1. Context Free Grammars

Produce the CFG for the following language L . Let $\Sigma = \{a, b, c, d\}$

$$L = \{c^{w+x}a^{y+x}b^zc^zd^{w+x} : w, x \geq 0; y, z \geq 1\}$$

Solution ::

$$S \longrightarrow cSd \mid A \tag{1}$$

$$A \longrightarrow caA \mid B \tag{2}$$

$$B \longrightarrow aBd \mid aCd \tag{3}$$

$$C \longrightarrow bCc \mid bc \tag{4}$$

Problem 2. Chomsky Normal Form

Convert the following grammar into CNF, $\Sigma = \{0, 1\}$.

Show your work at each of the five steps:

START, BIN, DEL, UNIT, TERM.

$$S \longrightarrow 0XY \mid Y$$

$$X \longrightarrow 1X \mid Y$$

$$Y \longrightarrow S \mid 0 \mid \epsilon$$

Solution ::

START ::

$$S_0 \longrightarrow S \quad (5)$$

$$S \longrightarrow 0XY \mid Y \quad (6)$$

$$X \longrightarrow 1X \mid Y \quad (7)$$

$$Y \longrightarrow S \mid 0 \mid \epsilon \quad (8)$$

BIN ::

$$S_0 \longrightarrow S \quad (9)$$

$$S \longrightarrow 0S_1 \mid Y \quad (10)$$

$$S_1 \longrightarrow XY \quad (11)$$

$$X \longrightarrow 1X \mid Y \quad (12)$$

$$Y \longrightarrow S \mid 0 \mid \epsilon \quad (13)$$

DEL ::

$$S_0 \longrightarrow S \mid \epsilon \quad (14)$$

$$S \longrightarrow 0S_1 \mid Y \mid 0 \quad (15)$$

$$S_1 \longrightarrow XY \mid X \mid Y \quad (16)$$

$$X \longrightarrow 1X \mid Y \mid 1 \quad (17)$$

$$Y \longrightarrow S \mid 0 \quad (18)$$

UNIT ::

$$S_0 \longrightarrow \epsilon \mid 0S_1 \mid 0 \quad (19)$$

$$S \longrightarrow 0S_1 \mid 0 \quad (20)$$

$$S_1 \longrightarrow XY \mid 1X \mid 0S_1 \mid 1 \mid 0 \quad (21)$$

$$X \longrightarrow 1X \mid 0S_1 \mid 1 \mid 0 \quad (22)$$

$$Y \longrightarrow 0S_1 \mid 0 \quad (23)$$

TERM ::

$$S_0 \longrightarrow \epsilon \mid U_0S_1 \mid 0 \quad (24)$$

$$S \longrightarrow U_0S_1 \mid 0 \quad (25)$$

$$S_1 \longrightarrow XY \mid U_1X \mid U_0S \mid 1 \mid 0 \quad (26)$$

$$X \longrightarrow U_1X \mid U_0S_1 \mid 1 \mid 0 \quad (27)$$

$$Y \longrightarrow U_0S_1 \mid 0 \quad (28)$$

$$U_0 \longrightarrow 0 \quad (29)$$

$$U_1 \longrightarrow 1 \quad (30)$$

Problem 3. Non-context free Proof

Prove that the following language is not context free. Let $\Sigma = \{a, b, c\}$

$$L = \{a^n b^n c^i : n < i < 2n\}$$

Solution ::

I assume $n = 0$ isn't possible because $0 < 0 + 1 < 2(0)$ isn't true.

I assume $n = 1$ isn't possible because $0 < 1 + 1 < 2(1)$ isn't true.

Suppose, for the sake of contradiction, that L is context free. Then by definition, there must be a CFG G with pumping length p that generates it. Let $s = a^p b^p c^{p+1}$.

By the pumping lemma, s can be partitioned into u, v, x, y, z such that $|vxy| \leq p$ and $|vy| \geq 1$. We proceed on cases for v and y :

If v and y consist of the same characters:

- If v and y consist of only a 's or only b 's then vy consist of k a 's or k b 's, for some $k \geq 1$. Because $|vxy| \leq p$, vxy is contained within the p a 's or p b 's. Then uxz is equal to either $a^{p-k} b^p c^{p+1}$ or $a^p b^{p-k} c^{p+1}$, none of which are within L when v/y is pumped down, thus uv^0xy^0z is not within L , which is a contradiction.
- If v and y consist of only c 's then vy must consist of k c 's, for some $k \geq 1$. Because $|vxy| \leq p$, vxy is contained within the $p + 1$ c 's. Therefore uxz is equal to $a^p b^p c^{p-k+1}$. From this we can see that c^{p-k+1} will make it not be contained within L due to k needing to be ≥ 1 when we pump down, this we've reached a contradiction.

If v and y consist of different characters:

- If v consists of only a 's and y consists of only b 's, for simplicity sake let's say they contain the same number of their respective character, say k characters for each, for some $k \geq 1$. Because $|vxy| \leq p$, vxy should be contained within the last k characters of a , and the first k characters of b .

From this we can see that uxz is equal to $a^{p-k} b^{p-k} c^{p+1}$. From this now, we can see that this will not be within L if we pump down and are left with $a^{p-k} b^{p-k} c^{p+1}$, this is not within L because k must be ≥ 1 , causing a contradiction.

- If v consists of only b 's and y consists of only c 's, for simplicity sake let's say they contain the same number of their respective characters, say k characters for each, for some $k \geq 1$.
Because $|vxy| \leq p$, vxy should be contained within the last k characters of b , and the first k characters of c . From this we can see that uxz is equal to $a^p b^{p-k} c^{p-k+1}$, when we pump down we are left with that result, which falls out of L , thus causing a contradiction.

All cases result in a contradiction of the pumping lemma.

$\therefore L$ is not context-free.