MATH 210 :: Homework 14

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Problem 1.
$C =$ curve $y = x^2$, from $(0, 0)$ to $(1, 1)$.

$$\int_C x^2 dy + y^3 dx$$

Solution ::

$$r(t) = \left\langle t, t^2 \right\rangle,\ 0 \leq t \leq 1 \quad (1)$$
$$r'(t) = \langle 1, 2t \rangle \quad (2)$$
$$x^2 = t^2 \quad (3)$$
$$y^3 = t^6 \quad (4)$$
$$\int_0^1 2t^3 + t^6 dt = \frac{2t^4}{4} + \frac{t^7}{7}\bigg|_0^1 \quad (5)$$
$$= \frac{1}{2} + \frac{1}{7} \quad (6)$$
$$= \frac{9}{14} \quad (7)$$

Problem 2.
$C =$ circle, radius 3, centered at $(0, 0)$.

$$\oint_C \frac{-y}{(x-5)^2+y^2}dx + \frac{x-5}{(x-5)^2+y^2}dy$$

Solution ::

$$F(x,y) = \left\langle \frac{-y}{x^2}, \frac{x}{x^2+y^2} \right\rangle \tag{8}$$

$$r(t) = \langle 3\cos(t), 3\sin(t)\rangle,\ 0 \le t \le 2\pi \tag{9}$$

$$r'(t) = \langle -3\sin(t), 3\cos(t)\rangle,\ 0 \le t \le 2\pi \tag{10}$$

$$F(r(t)) = \left\langle \frac{-\sin(t)}{3}, \frac{\cos(t)}{3} \right\rangle \tag{11}$$

$$F(r(t)) \cdot r'(t) = \sin^2(x) + \cos^2(x) = 1 \tag{12}$$

$$\int_0^{2\pi} 1\,dt = 2\pi \tag{13}$$

Problem 3.
C on bi-line from $(0,0,0)$ to $(1,1,1)$, to $(2,1,1)$.

$$F(x,y,z) = \left\langle 2xy + 2x, x^2 - 2yz^3, -6z - 3z^2y^2 \right\rangle$$

$$f = x^2 + yx^2 + h(y,z)$$

Solution ::

$$\begin{aligned}
x^2 &= x^2 - 2yz^3 && (14) \\
\int -2yz^3 &= -z^3y^2 && (15) \\
\frac{\partial}{\partial z}(x^2 + yx^2) &= 0 && (16) \\
gz(y,z) &= -6z - 3z^2y^2 && (17) \\
&= -z^3y^2 + h(z) && (18) \\
\frac{\partial}{\partial z}(-z^3y^2 + h(z)) &= -6z - 3z^2y^2 && (19) \\
-3z^2y^2 + hz(z) &= -6z - 3z^2y^2 && (20) \\
hz(z) &= -6z && (21) \\
\int -6z\,dz &= -3z^2 && (22) \\
\emptyset(x,y,z) &= x^2 + yz^2 - z^3y^2 - 3z^2 && (23) \\
\emptyset(2,1,1) - \emptyset(0,0,0) &= 4 - 0 && (24) \\
&= 4 && (25)
\end{aligned}$$