CS301 :: Homework 5

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Problem 1. Turing Machines

a) Give a *high-level* description of a Turing Machine that decides the language:

$$L = \{0^n 1^m \mid n \geq 2m\}$$

Solution ::

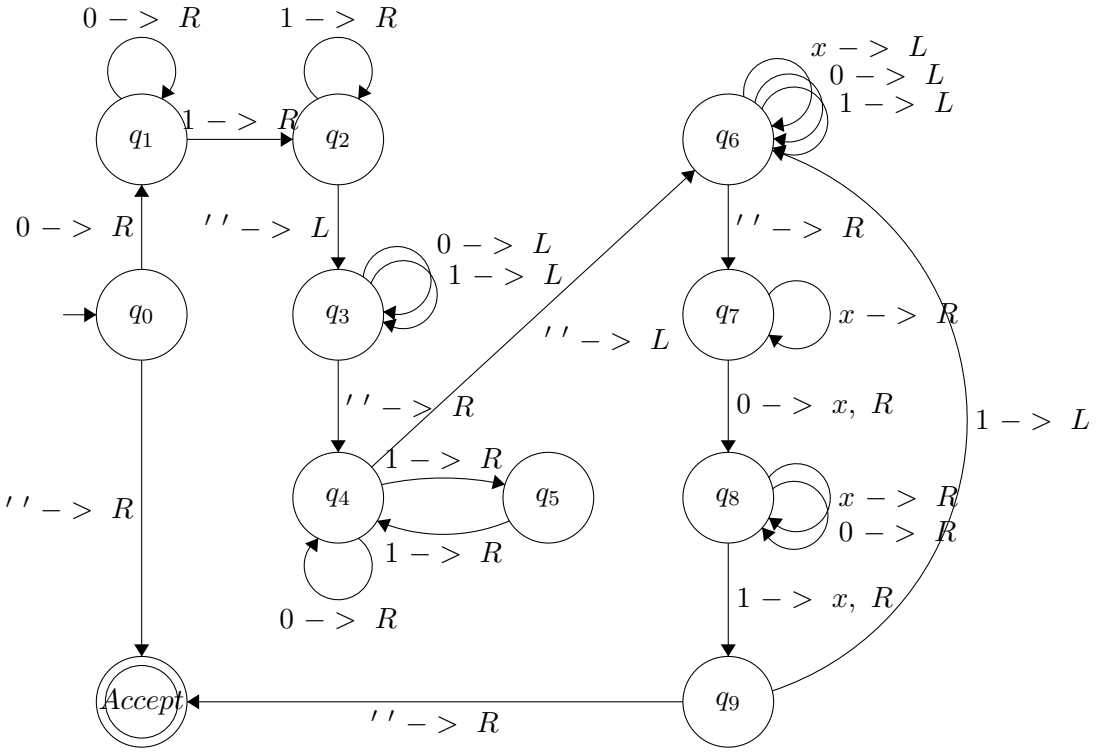
$M =$ "On input string w:

1. If the input string is empty (starts on ' ') then accept, else go to step 2.
2. Scan the input from left to right and make sure that the input is of the form "0*1*", and reject if it isn't.
3. Return tape head to the front of the input tape.
4. Move head to the first '1' character on tape.
5. Repeat the following until head is over a ' ', if so go to step 8.
6. Scan right once and check it is a '1', else reject.
7. Move head right once, go to step 5 loop condition check step.
8. Return tape head to the front of the input tape and scan right until first 0.
9. Repeat the following until no more 1s are on the tape, if no 1s go to step 14.
10. Replace the leftmost '0' with a 'x'.
11. Scan right until a '1' occurs. If there are no 1s, reject.
12. Replace the leftmost '1' with a 'x'.
13. Return tape head to front end of tape, and go to step 9.
14. Accept."

b) Give a state diagram for the Turing Machine you described in part (a), for the language:

$$L = \{0^n 1^m \mid n \geq 2m\}$$

Solution ::



Problem 2. Decidable Languages
Recall that to show a language is decidable, you must show either a halting algorithm which decides it, or a halting recution to a known decidable problem. In either case, **you must show that either the algorithm or the reduction halts.**

Show that the language $ALL_{DFA} = \{\langle D \rangle \mid D \text{ is a DFA and } L(D) = \sum^*\}$
You may use any problem we have shown to be decidable in lecture for a reduction.

Solution ::
Algorithm / machine using E_{DFA}:
$M =$ "On input $\langle D \rangle$:

1. Construct a new DFA A such that the language of A, $L(A)$ is the complement of $L(D)$.
2. Use a TM T that decides E_{DFA} on input $\langle A \rangle$.
3. Accept if T accepts, else reject.