MATH210 :: Homework 13

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Problem 1.

$$\varnothing(x, y) = \arctan\left(\frac{y}{3x}\right)$$

Solution ::

a)

$$\nabla\varnothing = \left\langle -\frac{3y}{y^2 + 9x^2}, \frac{3x}{y^2 + 9x^2} \right\rangle$$

b) Domain of definition is:

$$\{(x, y) : (x, y) \neq (0, 0)\}$$

c) $y = x$ parallel.

$\langle 1, 1\rangle$ is parallel to the line $y = x$, the gradient $\nabla\varnothing$ at point (x, y) is parallel to $\langle 1, 1\rangle$ if there is a λ such that:

$$\left\langle \frac{-3y}{y^2 + 9x^2}, \frac{3x}{y^2 + 9x^2} \right\rangle = \langle 1, 1\rangle \cdot \lambda$$

This happens when the x and y components of $\nabla\varnothing$ are equal thus showing that the points on $x = y$ eccept $(0, 0)$ are where F is parallel to $y = x$.

Problem 2. $0 \leq r \leq 2$, $\pi \leq t \leq \frac{3\pi}{2}$, $x \leq 0$, $y \leq 0$

$$\int_C xy\,ds$$

Solution ::

$$r(t) = \langle 2\cos(t), 2\sin(t)\rangle,\ \ \pi \leq t \leq \frac{3\pi}{2} \tag{1}$$

$$|r'(t)| = \sqrt{(-2\sin(t))^2 + (2\cos(t))^2} = 2 \tag{2}$$

$$f(r(t)) = 2\cos(t) \cdot 2\sin(t) \tag{3}$$

$$\int_{\pi}^{\frac{3\pi}{2}} 2\cos(t) \cdot 2\sin(t) \cdot 2\,dt = 8 \cdot \int_{\pi}^{\frac{3\pi}{2}} \cos(t)\sin(t)\,dt \tag{4}$$

$$u = \sin(t) \tag{5}$$

$$du = \cos(t) \tag{6}$$

$$\int u = \frac{u^2}{2} = \frac{\sin^2(t)}{2}\bigg|_{\pi}^{\frac{3\pi}{2}} = \frac{1}{2} \tag{7}$$

$$8 \cdot \frac{1}{2} = 4 \tag{8}$$

Problem 3.
C is line segment $(0,1,1)$ to $(1,0,1)$

$$\int_C y - xz\, ds$$

Solution ::

$$r(t) = \langle 0+t, 1-t, 1 \rangle,\ \ 0 \le t \le 1 \tag{9}$$
$$r'(t) = \langle 1, -1, 0 \rangle \tag{10}$$
$$|r'(t)| = \sqrt{1^2 + (-1)^2} = \sqrt{2} \tag{11}$$
$$f(r(t)) = (1-t) - t(1) = 1 - 2t \tag{12}$$
$$\int_0^1 (1-2t) \cdot \sqrt{2}\, dt = \sqrt{2}(1-1) \tag{13}$$
$$= 0 \tag{14}$$