CS 362 :: Homework 2

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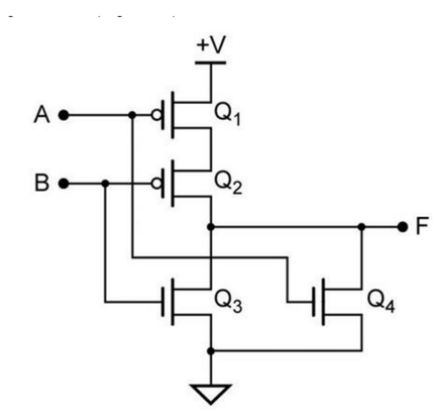
Problem 1. Write the truth table for the CMOS Circuit given below and specify which logic gate it represents.

Solution ::

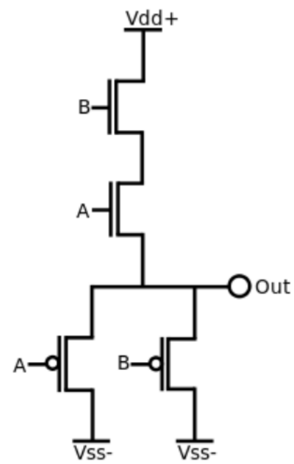
a) ::



A	B	Q_1	Q_2	Q_3	Q_4	F
0	0	1	1	0	0	1
0	1	1	0	1	0	0
1	0	0	1	0	1	0
1	1	0	0	1	1	0

This circuit represents a NOR gate.

b) ::

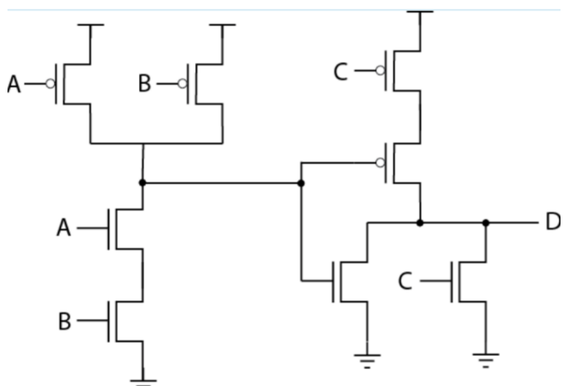


A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1

This circuit represents an AND gate.

Write the truth table for the CMOS Circuit given below and specify the equation it represents.

a) ::

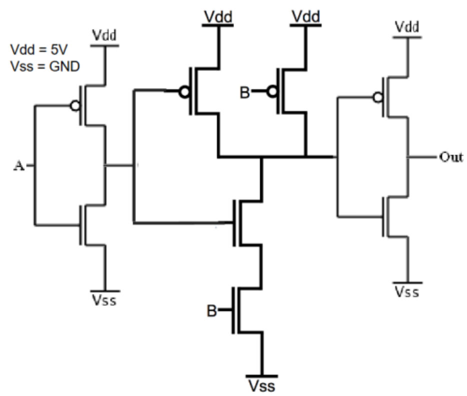


A	B	C	D
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

This circuit table can be represented like so:

$$D = ABC'$$

b) ::



A	B	Out
0	0	0
0	1	1
1	0	0
1	1	0

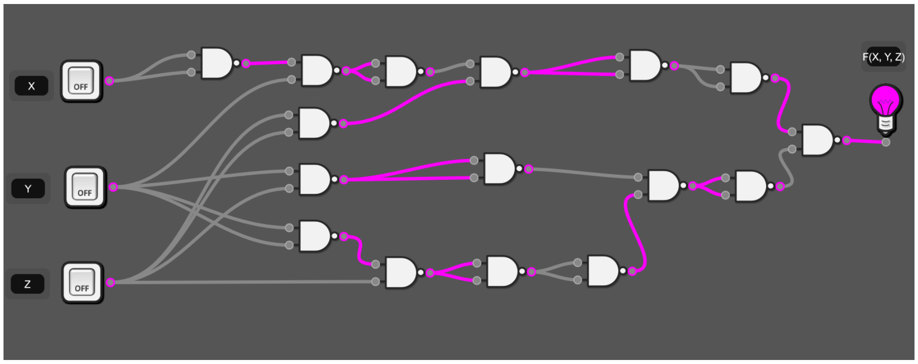
This circuit table can be represented like so:

$$Out = A'B$$

Problem 2. NAND gates and NOR gates are two types of Universal Gates. Represent each equation as a circuit using only **a single type** of a Universal Gate. I.E. Each answer must contain only NAND gates or must contain only NOR gates. Assume literals are not available in complemented form.

Solution ::

a) $F(x, y, z) = \overline{y}\,\overline{z} + \overline{y}z + \overline{x}y\overline{z}$



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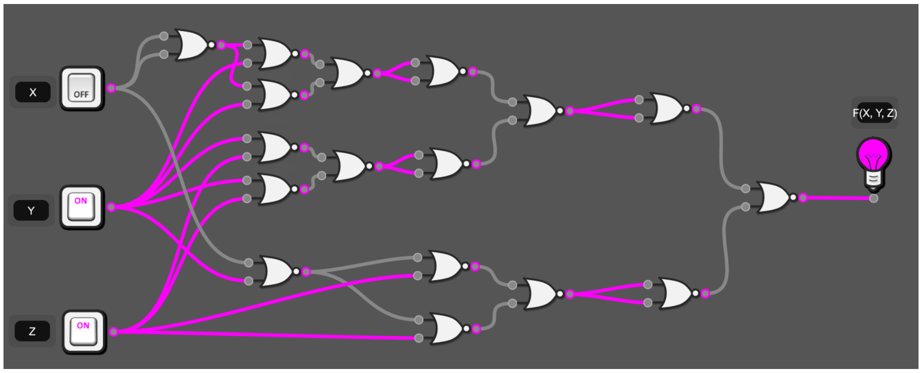
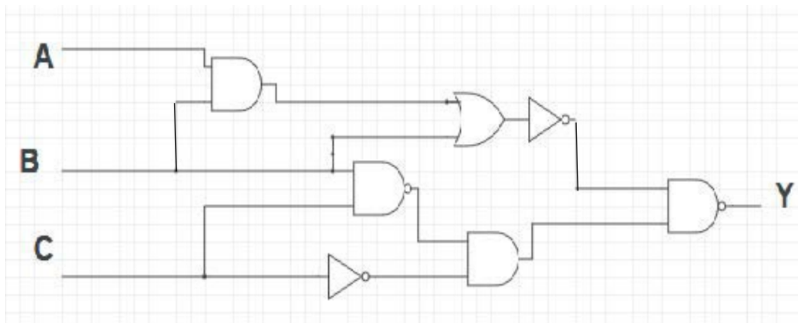
b) $F(x, y, z) = (x' + y) \cdot (y + z) \cdot (x' + y' + z)$



Problem 3. Write the truth table and equation for the following circuit diagram. Do not do any simplification on the equation when written for your answer.



Solution ::

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Equation:

$$Y = ABC + ABC' + AB'C + A'BC + A'BC' + A'B'C$$

The following Boolean Properties are to be used and referenced by name for each/every step to receive full credit when you "show your work" during the remaining problems in this homework.

$$
\begin{aligned}
a \cdot (b + c) = a \cdot + a \cdot c &= \text{Distributive (AND)} \\
a + (b \cdot c) = (a + b) \cdot (a + c) &= \text{Distributive (OR)} \\
a \cdot b = b \cdot a &= \text{Commutative} \\
a + b = b + a & \\
(a \cdot b) \cdot c = a \cdot (b \cdot c) &= \text{Associative} \\
(a + b) + c = a + (b + c) & \\
a \cdot a' = 0 &= \text{Complement (AND)} \\
a + a' = 1 &= \text{Complement (OR)} \\
a \cdot 1 = a &= \text{Identity (AND)} \\
a + 0 = a &= \text{Identity (OR)} \\
a \cdot 0 = 0 &= \text{Null elements} \\
a + 1 = 1 & \\
a \cdot a = a &= \text{Idempotent} \\
a + a = a & \\
(a')' = a &= \text{Involution} \\
(a \cdot b)' = (a' + b') &= \text{DeMorgan's (AND)} \\
(a + b)' = (a' \cdot b') &= \text{DeMorgan's (OR)}
\end{aligned}
$$

Problem 4. Expand the following equations to sum–of–minterm equations

Solution ::

a) $ab' + a'bc$ (assume literals are a, b, c)

$$\begin{aligned} ab'(1) + a'bc & \qquad \text{(Null elements)} \\ ab'(c' + c) + a'bc & \qquad \text{(Complement (OR))} \\ ab'c + ab'c + a'bc & \qquad \text{(Distributive (AND))} \end{aligned}$$

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b) $ac + bc + a'b$ (assume literals are a, b, c)

$$\begin{aligned} ac(1) + bc(1) + a'b(1) & \qquad \text{(Null elements))} \\ ac(b + b') + bc(a + a') + a'b(c' + c) & \qquad \text{(Complement (OR))} \\ ab'c + abc + abc + a'bc + a'bc' + a'bc & \qquad \text{(Distributive (AND))} \end{aligned}$$

---

c) $ad' + b'c$ (determine literals based on original equation)

$$\begin{aligned} & ad'(1)(1) + b'c(1)(1) \\ & \text{(Null elements)} \\ & ad'(b + b')(c + c') + b'c(a + a')(d + d') \\ & \text{(Complement (OR))} \\ & ad'(bc + bc' + b'c' + b'c) + b'c(a'd' + a'd + ad' + ad) \\ & \text{(Distributive (AND))} \\ & abcd' + abc'd' + abc'd' + ab'c'd' + ab'cd + ab'cd' + a'b'cd + a'b'cd' \\ & \text{(Distributive (AND))} \end{aligned}$$

Problem 5. Determine if the two equations (y & z) are equivalent by expanding each to a sum-of-minterm equation. Show your work.

Solution ::

a) $y = ab' + a'b + c$
$z = ab'c' + ac + a'bc + a'c$

$$y = ab'c + ab'c' + a'bc + a'bc' + a'b'c + a'bc' + ab'c + abc$$
$$z = a'b'c + a'bc + abc + ab'c + a'bc' + ab'c'$$

y and z are equivalent equations.

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b) $y = a'b' + bc'$
$z = a'c' + b'c$

$y = a'bc'd' + a'bc'd + abc'd' + abc'd + a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd$

$z = a'b'cd' + a'b'cd + ab'cd' + ab'cd + a'b'c'd' + a'b'c'd + a'bc'd' + a'bc'd$

$y = abc'd + abc'd'$

$z = ab'cd + ab'cd'$

y and z are not equivalent equations.

Problem 6a. Convert the truth table to sum-of-minterms expression. Show your work.

a	b	c	F	minterm
0	0	0	1	
0	0	1	1	
0	1	0	0	
0	1	1	0	
1	0	0	1	
1	0	1	0	
1	1	0	1	
1	1	1	1	

Solution ::

$$
\begin{aligned}
&= a'b'c' + a'b'c + ab'c' + abc' + abc \\
&= (m_0 + m_1 + m_4 + m_6 + m_7) \\
F(a,b,c) &= \sum(0,1,4,6,7)
\end{aligned}
$$

Problem 6b. Convert the truth table to product-of-maxterms expression. Show your work.

a	b	c	F	maxterm
0	0	0	0	
0	0	1	1	
0	1	0	0	
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	0	
1	1	1	1	

Solution ::

$$\begin{aligned} &= (a' + b' + c) \cdot (a + b' + c) \cdot (a + b + c) \\ &= (M_0 \cdot M_2 \cdot M_6) \\ F(a, b, c) &= \Pi M(0, 2, 6) \end{aligned}$$

Problem 6c. Express the truth table to **either a sum-of-minterms expression or a product-of-maxterms expression.**
Choose the one with the simplest expression (i.e. fewest number of total literals). Show your work

a	b	c	F	minterm
0	0	0	1	
0	0	1	1	
0	1	0	0	
0	1	1	0	
1	0	0	1	
1	0	1	0	
1	1	0	1	
1	1	1	1	

Solution ::

$$
\begin{aligned}
&= (a' + b + c' + (a + b + c)) \\
&= (M_0 \cdot M_5), \\
F(a, b, c) &= \Pi M(0, 5)
\end{aligned}
$$

Problem 6d. Express the truth table to either a sum-of-minterms expression or a product-of-maxterms expression. Choose the one with the simplest expression (i.e. fewest number of total literals). Show your work.

a	b	c	F	minterm	maxterm
0	0	0	0		
0	0	1	0		
0	1	0	1		
0	1	1	0		
1	0	0	0		
1	0	1	0		
1	1	0	1		
1	1	1	0		

Solution ::

$$= abc' + ab'c$$
$$= (m_2, m_6)$$
$$F(a, b, c) = \sum m(2, 6)$$

Problem 7. Simplify the sum-of-minterms expression below to sum-of-products form using Boolean Algebra properties. The "simplest" expression has the fewest literals. Show your work. (I'm super lost on this one)

Solution ::

a) $F(x, y, z) = x'y'z' + x'yz + xy'z' + xyz' + xyz$

$$= x'yz + y'z'(x' + x) + xy(x' + z) \quad \text{Null elements and comp. (OR)}$$
$$= xy + y'z' + x'yz$$
$$= y(x'y + x) + y'z' \quad \text{Dist. (AND)}$$

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b) $F(x, y, z) = xyz' + x'y'z + x'y'z' + x'yz + x'yz'$

$$xyz' + x'y'(x + z') + z'y(x + x')$$
$$xyz' + x'y'(1) + x'y(1) \quad \text{Null elements}$$
$$xyz' + x'y' + x'y \quad \text{complement (OR)}$$
$$x'y' + y(x' + z'x) \quad \text{Distributive (AND)}$$
$$x'y' + y((x + 1) \cdot z') \quad \text{Null elements}$$
$$y'x + z'y$$

: (Ran out of time,