CS301 :: Homework 4

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Problem 1. Context Free Grammars
Produce the CFG for the following language L. Let $\sum = \{a, b, c, d\}$

$$L = \left\{c^{w+x}a^{y+x}b^z c^z d^{w+x} : w, x \geq 0; y, z \geq 1\right\}$$

Solution ::

$$\begin{align} S &\longrightarrow cSd \mid A \tag{1} \\ A &\longrightarrow caA \mid B \tag{2} \\ B &\longrightarrow aBd \mid aCd \tag{3} \\ C &\longrightarrow bCc \mid bc \tag{4} \end{align}$$

Problem 2. Chomsky Normal Form
Convert the following grammar into CNF, $\sum = \{0, 1\}$.
Show your work at each of the five steps:
START, BIN, DEL, UNIT, TERM.
$S \longrightarrow 0XY \mid Y$
$X \longrightarrow 1X \mid Y$
$Y \longrightarrow S \mid 0 \mid \epsilon$

Solution ::
START ::

$$S_0 \longrightarrow S \tag{5}$$
$$S \longrightarrow 0XY \mid Y \tag{6}$$
$$X \longrightarrow 1X \mid Y \tag{7}$$
$$Y \longrightarrow S \mid 0 \mid \epsilon \tag{8}$$

BIN ::

$$S_0 \longrightarrow S \tag{9}$$
$$S \longrightarrow 0S_1 \mid Y \tag{10}$$
$$S_1 \longrightarrow XY \tag{11}$$
$$X \longrightarrow 1X \mid Y \tag{12}$$
$$Y \longrightarrow S \mid 0 \mid \epsilon \tag{13}$$

DEL ::

$$S_0 \longrightarrow S \mid \epsilon \tag{14}$$
$$S \longrightarrow 0S_1 \mid Y \mid 0 \tag{15}$$
$$S_1 \longrightarrow XY \mid X \mid Y \tag{16}$$
$$X \longrightarrow 1X \mid Y \mid 1 \tag{17}$$
$$Y \longrightarrow S \mid 0 \tag{18}$$

UNIT ::

$$S_0 \longrightarrow \epsilon \mid 0S_1 \mid 0 \tag{19}$$
$$S \longrightarrow 0S_1 \mid 0 \tag{20}$$
$$S_1 \longrightarrow XY \mid 1X \mid 0S_1 \mid 1 \mid 0 \tag{21}$$
$$X \longrightarrow 1X \mid 0S_1 \mid 1 \mid 0 \tag{22}$$
$$Y \longrightarrow 0S_1 \mid 0 \tag{23}$$

TERM ::

$$S_0 \longrightarrow \epsilon \mid U_0S_1 \mid 0 \tag{24}$$
$$S \longrightarrow U_0S_1 \mid 0 \tag{25}$$
$$S_1 \longrightarrow XY \mid U_1X \mid U_0S \mid 1 \mid 0 \tag{26}$$
$$X \longrightarrow U_1X \mid U_0S_1 \mid 1 \mid 0 \tag{27}$$
$$Y \longrightarrow U_0S_1 \mid 0 \tag{28}$$
$$U_0 \longrightarrow 0 \tag{29}$$
$$U_1 \longrightarrow 1 \tag{30}$$

Problem 3. Non-context free Proof

Prove that the following language is not context free. Let $\sum = \{a, b, c\}$

$$L = \{a^n b^n c^i : n < i < 2n\}$$

Solution ::

I assume $n = 0$ isn't possible because $0 < 0 + 1 < 2(0)$ isn't true.
I assume $n = 1$ isn't possible because $0 < 1 + 1 < 2(1)$ isn't true.

Suppose, for the sake of contradiction, that L is context free. Then by definition, there must be a CFG G with pumping length p that generates it. Let $s = a^p b^p c^{p+1}$.
By the pumping lemma, s can be partitioned into u, v, x, y, z such that $|vxy| \leq p$ and $|vy| \geq 1$. We proceed on cases for v and y:

If v and y consist of the same characters:

- If v and y consist of only a's or only b's then vy consist of k a's or k b's, for some $k \geq 1$. Because $|vxy| \leq p$, vxy is contained within the p a's or p b's. Then uxz is equal to either $a^{p-k}b^p c^{p+1}$ or $a^p b^p c^{p-k+1}$, none of which are within L when v/y is pumped down, thus uv^0xy^0z is not within L, which is a contradiction.

- If v and y consist of only c's then vy must consist of k c's, for some $k \geq 1$. Because $|vxy| \leq p$, vxy is contained within the $p + 1$ c's. Therefore uxz is equal to $a^p b^p c^{p-k+1}$. From this we can see that c^{p-k+1} will make it not be contained within L due to k needing to be ≥ 1 when we pump down, this we've reached a contradiction.

If v and y consist of different characters:

- If v consists of only a's and y consists of only b's, for simplicity sake let's say they contain the same number of their respective character, say k characters for each, for some $k \geq 1$. Because $|vxy| \leq p$, vxy should be contained within the last k characters of a, and the first k characters of b.

  From this we can see that uxz is equal to $a^{p-k}b^{p-k}c^{p+1}$. From this now, we can see that this will not be within L if we pump down and are left with $a^{p-k}b^{p-k}c^{p+1}$, this is not within L because k must be ≥ 1, causing a contradiction.

- If v consists of only b's and y consists of only c's, for simplicity sake let's say they contain the same number of their respective characters, say k characters for each, for some $k \geq 1$.
  Because $|vxy| \leq p$, vxy should be contained within the last k characters of b, and the first k characters of c. From this we can see that uxz is equal to $a^p b^{p-k} c^{p-k+1}$, when we pump down we are left with that result, which falls out of L, thus causing a contradiction.

All cases result in a contradiction of the pumping lemma.
$\therefore L$ is not context-free.