CS301 :: Homework 5

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Problem 1. Turing Machines

a) Give a *high-level* description of a Turing Machine that decides the langauge:

$$L = \{0^n 1^m \, | \, n \ge 2m\}$$

Solution ::

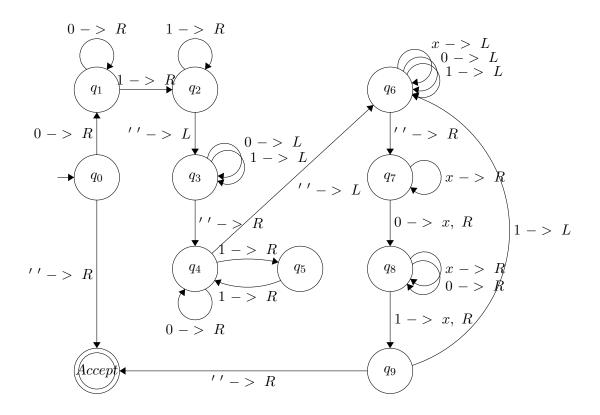
M = "On input string w:

- 1. If the input string is empty (starts on ' ') then accept, else go to step 2.
- 2. Scan the input from left to right and make sure that the input is of the form "0*1*", and reject if it isn't.
- 3. Return tape head to the front of the input tape.
- 4. Move head to the first '1' character on tape.
- 5. Repeat the following until head is over a '', if so go to step 8.
- 6. Scan right once and check it is a '1', else reject.
- 7. Move head right once, go to step 5 loop condition check step.
- 8. Return tape head to the front of the input tape and scan right until first 0.
- 9. Repeat the following until no more 1s are on the tape, if no 1s go to step 14.
- 10. Replace the leftmost '0' with a 'x'.
- 11. Scan right until a '1' occurs. If there are no 1s, reject.
- 12. Replace the leftmost '1' with a 'x'.
- 13. Return tape head to front end of tape, and go to step 9.
- 14. Accept."

b) Give a state diagram for the Turing Machine you described in part (a), for the language:

$$L = \{0^n 1^m \, | \, n \ge 2m\}$$

Solution ::



Problem 2. Decidable Languages

Recall that to show a language is decidable, you must show either a halting algorithm which decides it, or a halting recution to a known decidable problem. In either case, you must show that either the algorithm or the reduction halts.

Show that the language $ALL_{DFA} = \{\langle D \rangle \mid D \text{ is a DFA and } L(D) = \sum^* \}$ You may use any problem we have shown to be decidable in lecture for a reduction.

Solution ::

Algorithm / machine using E_{DFA} : $M = \text{"On input } \langle D \rangle$:

- 1. Construct a new DFA A such that the language of A, L(A) is the complement of L(D).
- 2. Use a TM T that decides E_{DFA} on input $\langle A \rangle$.
- 3. Accept if T accepts, else reject.