CS301 :: Homework 6

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Problem 1. A FINITE Language
Let $FINITE_{TM} = \{\langle M \rangle \,|\, M$ is a TM and M accepts a finite language$\}$.
Prove that $FINITE_{TM}$ is undecidable.

Solution ::
Suppose $FINITE_{TM}$ is deciable.
Therefore there is a TM C that decides it.

$A = $ "On input $\langle M, w \rangle$,
      construct $M' = $ "on input x,
         Simulate M on input w.
         If M halts, reject, else accept."
      Pass M' to C.
      If C accepts M', reject, else accept."

C decides similar to A_{TM} which is undecidable, a contradiction.
$\therefore FINITE_{TM}$ is undecidable.

Problem 2. There Can Only Be ONE
Let $ONE_{TM} = \{\langle M\rangle \mid M$ is a TM that accepts exactly 1 string$\}$.
Prove that ONE_{TM} is undecidable.

Solution ::
Suppose for the sake of contradiction that ONE_{TM} is deciable. Therefore a
TM S decides it.

$A =$ "On input $\langle M, w\rangle$,
      Construct $M' =$ "On input x,
         If $x \neq w$, reject,
         else, return $M(w)$."
      Pass M' to S. If S accepts M', accept, else reject."

TM A decides A_{TM} which is undecidable, we have a contradiction.
$\therefore ONE_{TM}$ is undecidable.

Problem 3. A LENGTH-y Decision
Let $LENGTHK_{TM} =$
$\{\langle M, k\rangle \mid M$ is a TM that accepts all length-k strings$\}$.
Prove that $LENGTHK_{TM}$ is undecidable.

Solution ::
Suppose for the sake of contradiction that $LENGTHK_{TM}$ is deciable.
Therefore a TM Z decides it.

$A =$ "On input $\langle M, w\rangle$,
      Construct $M' =$ "On input k,
         If w does not have length k, reject.
         else, return $M(w)$."
      Pass $\langle M', k\rangle$ to Z and run this machine for $LENGTHK_{TM}$,
      if M' accepts, Z accepts, else Z rejects."

TM Z decides similar to A_{TM} which is undecidable, a contradiction.
$\therefore LENGTHK_{TM}$ is undecidable.