CS301 :: Homework 1

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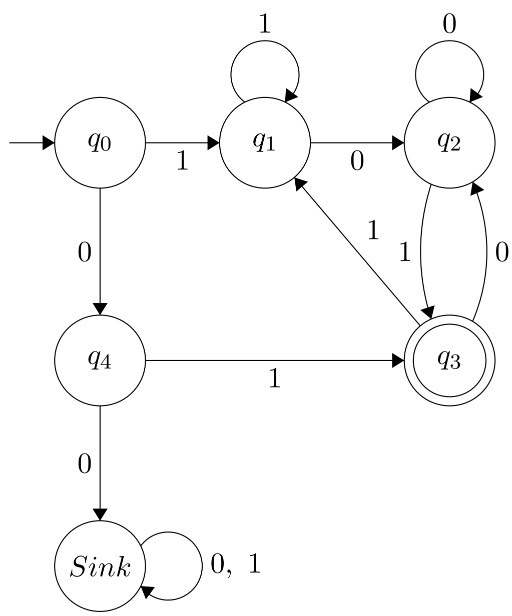
Problem 1. DFAs

a) Generate the state diagram for a DFA which decides the following language L. $\sum = \{0, 1\}$.
$L = \{w \in \sum^* : w \text{ does not start with } '00' \text{ and } w \text{ ends with } '01' \}$

Solution ::



b) Give the 5-tuple which represents the DFA from 1a). You may use a table to represent the transition function δ.

Solution ::

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$
$$\sum = \{0, 1\}$$
$$q_0 = q_0$$
$$F = \{q_4\}$$

$\delta =$

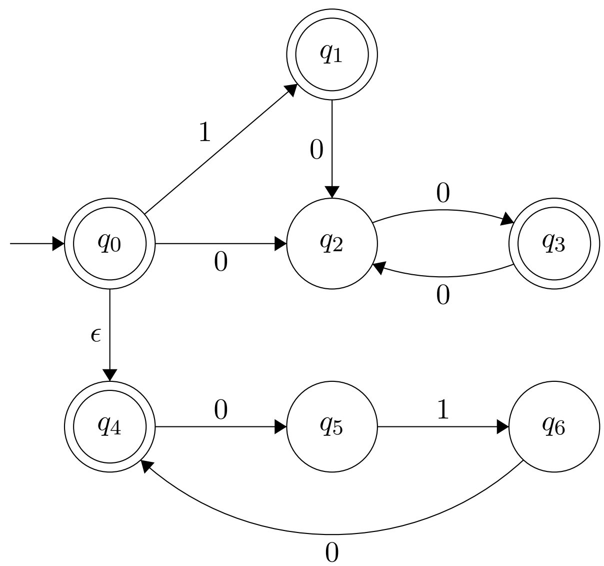
	0	1
q_0	q_4	q_1
q_1	q_2	q_1
q_2	q_2	q_3
q_3	q_2	q_1
q_4	q_5	q_3
q_5	q_5	q_5

Problem 2. NFAs

a) Generate the state-diagram for an NFA which decides the following language L. $\sum = \{0, 1\}$.

$$L = (010)^* \cup 1^*(00)^*$$

Solution ::



b) Give the 5-tuple which represents the NFA from 2a). You may use a table to represent the transition function δ.

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$$

$$\sum = \{0, 1\}$$

$$q_0 = q_0$$

$$F = \{q_0, q_1, q_3, q_4\}$$

$\delta =$

	0	1	ϵ
q_0	$\{q_2\}$	$\{q_1\}$	$\{q_4\}$
q_1	$\{q_2\}$	$\{q_1\}$	$\emptyset$
q_2	$\{q_3\}$	$\emptyset$	$\emptyset$
q_3	$\{q_2\}$	$\emptyset$	$\emptyset$
q_4	$\{q_5\}$	$\emptyset$	$\emptyset$
q_5	$\emptyset$	$\{q_6\}$	$\emptyset$
q_6	$\{q_4\}$	$\emptyset$	$\emptyset$

Problem 3. Closure
Given the 5-tuple for an NFA $M_L = (Q, \sum, \delta, q_0, F)$ which decides, L, describe how to produce the 5-tuple for an NFA $M_{L^R} = (Q_R, \sum, \delta_R, q_{0_R}, F_R)$ which decides L^R, the reverse of L.

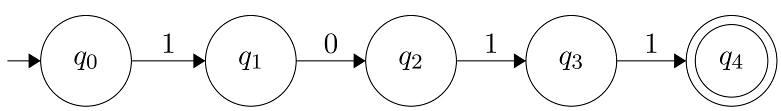
The reverse, L^R *is the recursive operation given below which gives the reverse of a string.*
e.g. $(110)^R = 011$

- $\epsilon^R = \epsilon$
- For string w and character $a, (wa)^R = a(w^R)$

Solution ::

$$
\begin{aligned}
Q^R &= Q \\
\sum^R &= \sum \\
q_{0^R} &= F \\
F^R &= q_0
\end{aligned}
$$

For the transition function let's use the following string example:
$L = 1011, L^R$ should become 1011.



L transition function δ :

	1	0
q_0	q_1	$\emptyset$
q_1	$\emptyset$	q_1
q_2	q_3	$\emptyset$
q_3	q_4	$\emptyset$
q_4	$\emptyset$	$\emptyset$

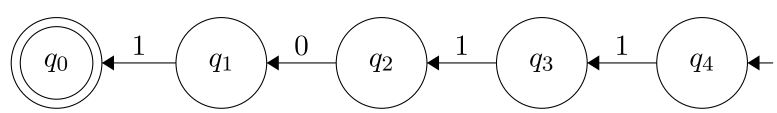
To retrieve the string we need to invert the starting and final state.
(As noted with $q_{0^R} = F$, $F^R = q_0$). After that we must reverse the transition function of $L; \delta$.
One method to reverse δ would be to enter the original starting state, in this example q_0, and enter each state sort of recursively, so we go:

$$q_0 \to q_1 \to q_2 \to q_3$$

and stop when the next state only has the empty set on all transitions, this would be our base case.
In this example we wold stop at q_3 because q_4 terminates fully. We can now reverse and in the end the L^R transition function δ^R should look like this:

	1	0
q_0	$\emptyset$	$\emptyset$
q_1	q_0	$\emptyset$
q_2	$\emptyset$	q_1
q_3	q_2	$\emptyset$
q_4	q_3	$\emptyset$

The final NFA L^R should now be reverse in all strings cases like so:



Extra note: Doing the steps listed above again should result in the original L, that is:

$$L = (L^R)^R$$