

CS301 :: Homework 1

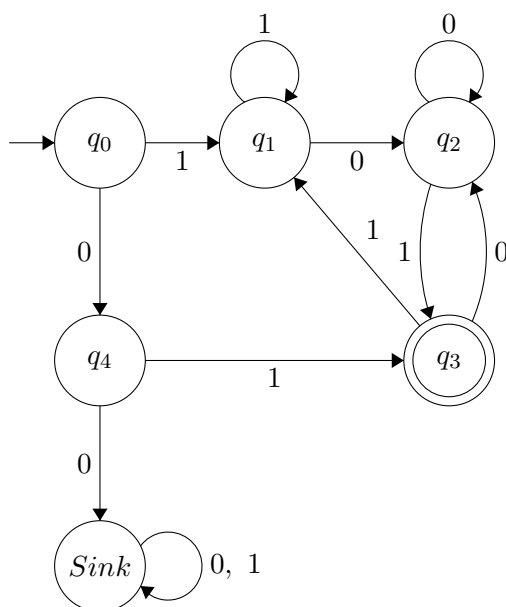
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Problem 1. DFAs

- a) Generate the state diagram for a DFA which decides the following language L . $\Sigma = \{0, 1\}$.
 $L = \{w \in \Sigma^* : w \text{ does not start with '00' and } w \text{ ends with '01'}\}$

Solution ::



- b) Give the 5-tuple which represents the DFA from 1a). You may use a table to represent the transition function δ .

Solution ::

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

$$\Sigma = \{0, 1\}$$

$$q_0 = q_0$$

$$F = \{q_4\}$$

$$\delta =$$

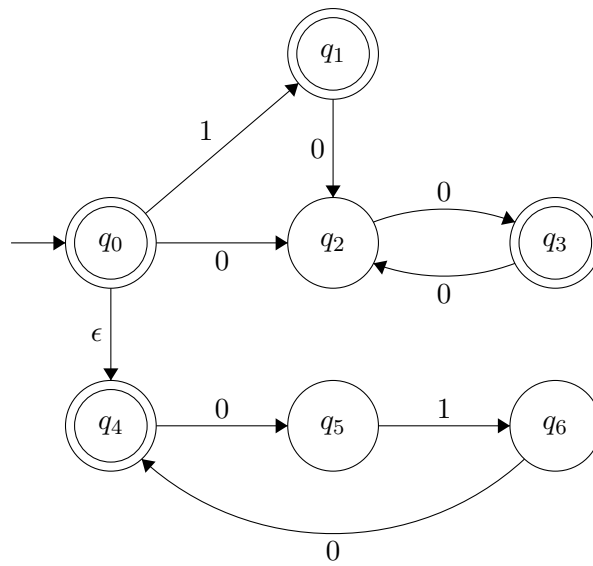
	0	1
q_0	q_4	q_1
q_1	q_2	q_1
q_2	q_2	q_3
q_3	q_2	q_1
q_4	<i>sink</i>	q_3
<i>sink</i>	<i>sink</i>	<i>sink</i>

Problem 2. NFAs

- a) Generate the state-diagram for an NFA which decides the following language L . $\Sigma = \{0, 1\}$.

$$L = (010)^* \cup 1^*(00)^*$$

Solution ::



- b) Give the 5-tuple which represents the NFA from 2a). You may use a table to represent the transition function δ .

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$$

$$\Sigma = \{0, 1\}$$

$$q_0 = q_0$$

$$F = \{q_0, q_1, q_3, q_4\}$$

$$\delta =$$

	0	1	ϵ
q_0	$\{q_2\}$	$\{q_1\}$	$\{q_4\}$
q_1	$\{q_2\}$	$\{q_1\}$	\emptyset
q_2	$\{q_3\}$	\emptyset	\emptyset
q_3	$\{q_2\}$	\emptyset	\emptyset
q_4	$\{q_5\}$	\emptyset	\emptyset
q_5	\emptyset	$\{q_6\}$	\emptyset
q_6	$\{q_4\}$	\emptyset	\emptyset

Problem 3. Closure

Given the 5-tuple for an NFA $M_L = (Q, \Sigma, \delta, q_0, F)$ which decides, L , describe how to produce the 5-tuple for an NFA $M_{L^R} = (Q_R, \Sigma, \delta_R, q_{0R}, F_R)$ which decides L^R , the reverse of L .

The reverse, L^R is the recursive operation given below which gives the reverse of a string.

e.g. $(110)^R = 011$

- $\epsilon^R = \epsilon$
- For string w and character a , $(wa)^R = a(w^R)$

Solution ::

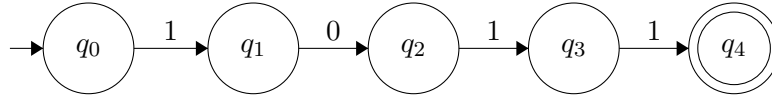
$$Q^R = Q$$

$$\Sigma^R = \Sigma$$

$$q_{0R} = F$$

$$F^R = q_0$$

For the transition function let's use the following string example:
 $L = 1011, L^R$ should become 1011.



L transition function δ :

	1	0
q_0	q_1	\emptyset
q_1	\emptyset	q_1
q_2	q_3	\emptyset
q_3	q_4	\emptyset
q_4	\emptyset	\emptyset

To retrieve the string we need to invert the starting and final state.
(As noted with $q_{0^R} = F$, $F^R = q_0$). After that we must reverse the transition function of L ; δ .

One method to reverse δ would be to enter the original starting state, in this example q_0 , and enter each state sort of recursively, so we go:

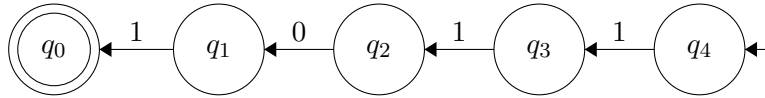
$$q_0 \rightarrow q_1 \rightarrow q_2 \rightarrow q_3$$

and stop when the next state only has the empty set on all transitions, this would be our base case.

In this example we would stop at q_3 because q_4 terminates fully. We can now reverse and in the end the L^R transition function δ^R should look like this:

	1	0
q_0	\emptyset	\emptyset
q_1	q_0	\emptyset
q_2	\emptyset	q_1
q_3	q_2	\emptyset
q_4	q_3	\emptyset

The final NFA L^R should now be reverse in all strings cases like so:



Extra note: Doing the steps listed above again should result in the original L , that is:

$$L = (L^R)^R$$