CS 401 - UIC
Homework 1
Instructor: Anastasios Sidiropoulos

Problem 1. [20 points] Order the following functions by asymptotic dominance. That is, give an ordering $f_1, f_2, \ldots$, such that for all $i \geq 1$, we have $f_i(n) = O(f_{i+1}(n))$.

- $f(n) = 2^n$.
- $f(n) = 1000 \cdot n$.
- $f(n) = n^{\log n}$.
- $f(n) = n^2$.
- $f(n) = 2^{2^n}$.
- $f(n) = \log(n)$.
- $f(n) = \log(\log(n))$.
- $f(n) = n \cdot \log(n)$.

Problem 2. [10 points] Prove or disprove that $2^{n+1} = O(2^n)$. Prove or disprove that $2^{2n} = O(2^n)$.

Problem 3. [10 points] Give the asymptotic running time of the following algorithm in Θ notation. Briefly justify your answer. Be sure to justify both the upper and the lower bound.

```
Func1(n)
s ← 0;
i ← 5;
while (i < n^2 + 7) do
    for j ← i to i^3 log i do
        s ← s + 1;
    end
    i ← i × 4;
end
```

Problem 4. [30 points]

(a) You are given a set of n items of sizes $a_1, \ldots, a_n \in \mathbb{N}$, and a bin of size $B \in \mathbb{N}$. Your goal is to find a maximum cardinality subset of items that all fit inside the bin. That is, you want to find a set of distinct indices $I = \{i_1, \ldots, i_k\} \subseteq \{1, \ldots, n\}$, such that

$$a_{i_1} + \ldots + a_{i_k} \leq B,$$

maximizing k.

For example, if the sizes are $a_1 = 7$, $a_2 = 4$, $a_3 = 5$, and the bin size is $B = 10$, then the optimum solution is $I = \{2, 3\}$ (that is, picking the second and the third item).

Design a greedy algorithm for this problem. The running time of your algorithm should be polynomial in n.

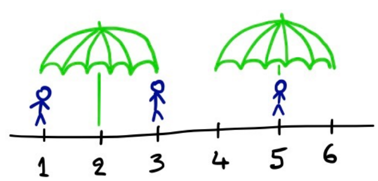
(b) Suppose that instead of maximizing k, we want to maximize the total size of the items in the bin; that is, we want to maximize the quantity

$$\text{size}(I) = a_{i_1} + \ldots + a_{i_k}.$$

Show that your greedy algorithm does not work in this case.

Problem 5: A day at the beach. [30 points] A group of n people are lying on the beach. The beach is represented by the real line $\mathbb{R}$ and the location of the i-th person is some integer $x_i \in \mathbb{Z}$. Your task is to prevent people from getting sunburned by covering them with umbrellas. Each umbrella corresponds to a closed interval $I = [a, a + L]$ of length $L \in \mathbb{N}$, and the i-th person is covered by that umbrella if $x_i \in I$. Design a greedy algorithm for covering all people with the minimum number of umbrellas. The input consists of the integers $x_1, \ldots, x_n$, and L. The output of your algorithm should be the positions of umbrellas.

For example, if the input is $x_1 = 1$, $x_2 = 3$, $x_3 = 5$, and $L = 2$, then an optimum solution is the set of two umbrellas placed at positions 2 and 5, covering intervals $[1, 3]$ and $[4, 6]$.



Prove that your algorithm is correct and that it runs in time polynomial in n.