

CS301 :: Homework 3

Ryan Magdaleno

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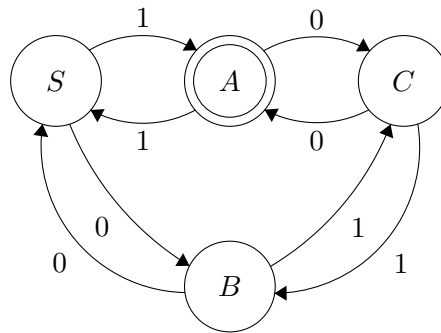
Problem 1. Regular Grammars

Consider the following language L . $\Sigma = \{0,1\}$. **You do not need to produce any tuples.**

$$L = \{w : w \text{ has an even number of 0s and an odd number of 1s}\}$$

a) Give the NFA which decides L .

Solution ::



b) Produce the right-linear, single-step CFG which is equivalent to your NFA in a).

Solution ::

$$S \longrightarrow 1A \mid 0B \quad (1)$$

$$A \longrightarrow 1S \mid 0C \mid \epsilon \quad (2)$$

$$B \longrightarrow 0S \mid 1C \quad (3)$$

$$C \longrightarrow 0A \mid 1B \quad (4)$$

Problem 2. Context Free Grammars

Produce the CFG for the following languages. $\Sigma = \{a, b, c\}$. *You do not need to produce the 4-tuples.*

a) $L_a = \{w : \text{the } \# \text{ of } a\text{'s is equal to the } \# \text{ of } b\text{'s and } c\text{'s combined} \}$

Solution ::

$$S \longrightarrow aSx \mid xSa \mid SaX \mid XaS \mid SXa \mid XSa \quad (5)$$

$$S \longrightarrow \epsilon \quad (6)$$

$$X \longrightarrow b \mid c \quad (7)$$

b) $L_b = \{a^i b^{2j} c^{i+j} : i, j \geq 1\}$

Solution ::

$$S \longrightarrow aSc \mid abbXcc \quad (8)$$

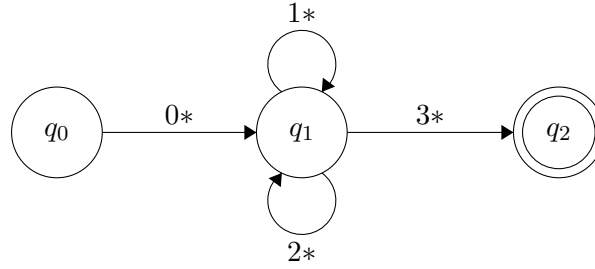
$$X \longrightarrow \epsilon \mid bbXc \quad (9)$$

$$(10)$$

Problem 3. Pushdown Automata

- a) Produce a PDA which decides the following language. $\Sigma = \{a, b, c, d\}$
 $L = \{w : \text{the } \# \text{ of } a\text{'s is equal to the } \# \text{ of } b\text{'s and } c\text{'s combined} \}$
Note: unlike L_a from Q2, strings in this L may contain any $\#$ of d 's.

Solution ::



$$0* = \epsilon, \epsilon \rightarrow \$ \quad (11)$$

$$1* = d, \epsilon \rightarrow \epsilon \mid c, x \rightarrow \epsilon \mid b, x \rightarrow \epsilon \mid a, \epsilon \rightarrow x \quad (12)$$

$$2* = b, \epsilon \rightarrow y \mid c, \epsilon \rightarrow y \mid a, y \rightarrow \epsilon \quad (13)$$

$$3* = \epsilon, \$ \rightarrow \epsilon \quad (14)$$

- b) Give the 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$ for your PDA from a)
 You do **not** need to provide the transition function δ .

Solution ::

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b, c, d\}$$

$$\Gamma = \{x, y\}$$

$$q_0 = q_0$$

$$F = \{q_2\}$$