Theoretical considerations on the dose rate calibration of NaI:Tl detectors

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1 Dose rate estimation

For a NaI:Tl detector the dose rate $\dot{D}_M$ from count rate $\dot{N'}_M$ can be estimated using the following linear relationship

$$\dot{D}_M = m \cdot \dot{N'}_M + \dot{D}_0 \tag{1}$$

with m being the slope and $\dot{D}_0$ the intercept. Both these parameters must be estimated by calibration measurements, which must be representative for the designated use case (e.g. in terms of geometry, material and source spectrum). The count rate $\dot{N'}_M$ is calculated from

$$\dot{N'}_M = \dot{N}_M - \dot{N}_B \tag{2}$$

where the measured count rate $\dot{N}_M$ is subtracted a measured background count rate $\dot{N}_B$

Both count rates are calculated with

$$\dot{N} = \frac{N}{T} = \frac{1}{T} \cdot \sum_{i=k}^{k'} N_i \tag{3}$$

where T is the measurement time and N_i is the number of measured counts in the ith channel. The lowest summation index k is set to a channel closest
to a corresponding energy $E = 500$ keV, while index k' is set to the channel closest to $E = 4$ MeV. These default values were chosen as the differences in photon interaction cross sections in quartz and NaI:Tl are smallest in this energy range and thus a similar deposition of dose can be expected.

2 Calibration

Parameters m and $\dot{D}_0$ of (1) are estimated by separate calibration measurements, where the count rates $\dot{N'}_{C1}$ and $\dot{N'}_{C2}$ of two known dose rates $\dot{D}_{C1}$ and $\dot{D}_{C2}$ must be determined using (2) and (3). With these results the coefficients m and $\dot{D}_0$ can be calculated with

$$m = \frac{\dot{D}_{C1} - \dot{D}_{C2}}{\dot{N'}_{C1} - \dot{N'}_{C2}} \tag{4}$$

and

$$\dot{D}_0 = \dot{D}_{C1} - m \cdot \dot{N'}_{C1} \tag{5}$$

As both count rates are background corrected the calibration curve can be expected to include the pair of values $\dot{D} = 0$ and $\dot{N'} = 0$. If only one calibration spectrum (i.e., $\dot{D}_{C1}$ and $\dot{N'}_{C1}$) is available one could simplify (1) by assuming intercept $\dot{D}_0 = 0$.

3 Estimating the uncertainty

3.1 Count rate

The uncertainty on the count rates $\Delta\dot{N}$ is assumed to follow Poisson statistics:

$$\Delta\dot{N} = \frac{1}{T} \cdot \sqrt{N} = \frac{1}{T} \cdot \sqrt{\dot{N} \cdot T} = \frac{\sqrt{\dot{N}} \cdot \sqrt{T}}{\sqrt{T} \cdot \sqrt{T}} = \sqrt{\frac{\dot{N}}{T}} \tag{6}$$

For the uncertainty on (2) it follows

$$\Delta\dot{N'}_M = \sqrt{(\Delta\dot{N}_M)^2 + (\Delta\dot{N}_B)^2} \tag{7}$$

which can also analogously be applied to all other count rates.

3.2 Calibration

Deduced from (7) and known $\dot{D}_{C1}$ and $\dot{D}_{C2}$ the uncertainty on (4) is estimated by

$$\Delta m = m \cdot \sqrt{(\frac{\Delta\dot{D}_{C1}}{\dot{D}_{C1} - \dot{D}_{C2}})^2 + (\frac{\Delta\dot{D}_{C2}}{\dot{D}_{C1} - \dot{D}_{C2}})^2 + (\frac{\Delta\dot{N'}_{C1}}{\dot{N'}_{C1} - \dot{N'}_{C2}})^2 + (\frac{\Delta\dot{N'}_{C2}}{\dot{N'}_{C1} - \dot{N'}_{C2}})^2} \tag{8}$$

In consideration of (8) the uncertainty on (5) is estimated by

$$\Delta\dot{D}_0 = \sqrt{(\Delta\dot{D}_{C1})^2 + (\dot{N'}_{C1} \cdot \Delta m)^2 + (m \cdot \Delta\dot{N'}_{C1})^2} \tag{9}$$

3.3 Measured dose rate

Finally, with (8) and (9) the uncertainty on the measured dose rate $\dot{D}_M$, estimated in (1), is calculated with

$$\Delta\dot{D}_M = \sqrt{(\dot{N'}_M \cdot \Delta m)^2 + (m \cdot \Delta\dot{N'}_M)^2 + (\Delta\dot{D}_0)^2} \tag{10}$$