

Implementing a parallel version of the Floyd-Warshall algorithm involves distributing the computation of the cost matrix across multiple processors to speed up the computation. Here, I'll provide a high-level explanation of each strategy and discuss why the speedup vs. the number of workers might differ for the two strategies.

Strategy 1: Row- and Column-Wise One-to-All Broadcasts

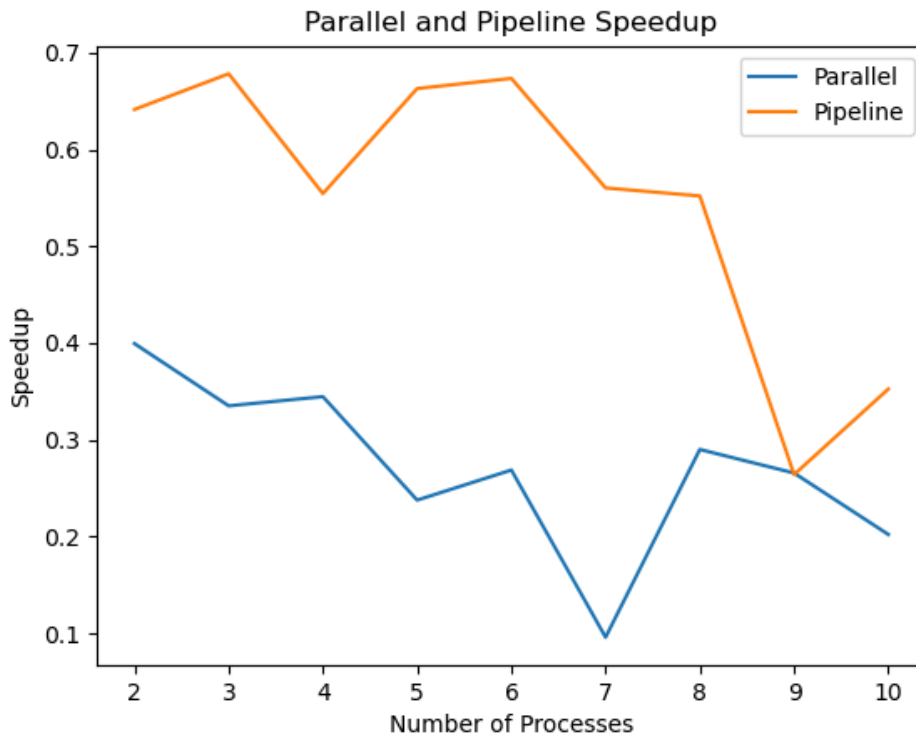
In this strategy, each processor computes a portion of the cost matrix, and during each iteration, the intermediate results are broadcast to all other processors using row- and column-wise one-to-all broadcasts. The algorithm follows these steps:

1. **Initialization:** Distribute the input graph among processors. Each processor is responsible for a subset of rows and columns.
2. **Parallel Computation:** For each intermediate node k (from 0 to $n-1$), each processor broadcasts its subset of rows containing node k to all other processors.
3. **Update Local Cost Matrix:** Each processor updates its local cost matrix using the received information and performs the Floyd-Warshall computation locally.
4. **Iteration:** Repeat steps 2-3 for all nodes.

Strategy 2: Pipelining

In this strategy, one-to-all broadcasts are replaced with a pipelining approach. Instead of broadcasting entire rows/columns at once, each processor broadcasts only the relevant information to its neighbors, and the information is passed along in a pipeline fashion. The algorithm follows these steps:

1. **Initialization:** Similar to Strategy 1, distribute the input graph among processors.
2. **Pipelining Computation:** For each intermediate node k (from 0 to $n-1$), processors communicate with their neighbors, sending and receiving only the necessary information for the computation.
3. **Update Local Cost Matrix:** Each processor updates its local cost matrix using the received information and performs the Floyd-Warshall computation locally.
4. **Iteration:** Repeat steps 2-3 for all nodes.



Speedup vs. Number of Workers

The differences in the speedup vs. the number of workers for the two strategies can be attributed to the communication overhead and the efficiency of utilizing the available resources.

1. Strategy 1 (Broadcasts):

- **Pros:** All processors receive the complete row/column information, reducing the need for repeated communication.
- **Cons:** Higher communication overhead due to broadcasting entire rows/columns, which may become a bottleneck as the number of workers increases. This could result in diminishing returns.

2. Strategy 2 (Pipelining):

- **Pro:** Reduced communication overhead by sending only relevant information. Better scalability as the number of workers increases.
- **Cons:** Increased computation and coordination overhead due to more frequent communication. The pipeline structure might introduce dependencies, limiting parallelism in certain cases.

The observation that pipelining shows higher speedup and exhibits an initial increase in speedup with an increase in the number of workers, followed by a reduction, suggests

interesting characteristics and potential bottlenecks in the parallelization of the Floyd-Warshall algorithm using pipelining.

Here are some factors that could contribute to these observations:

1. Reduced Communication Overhead:

- Pipelining typically involves more frequent but smaller-scale communication between neighboring processors.
- Initially, as the number of workers increases, the reduction in communication overhead contributes to improved parallel efficiency and higher speedup.

2. Communication Saturation:

- As the number of workers continues to increase, there may be a point where communication becomes saturated, leading to diminishing returns.
- Increased contention for communication channels or increased synchronization overhead may limit the scalability.

3. Load Imbalance:

- The pipelining approach may introduce load imbalance if the computational workload is not evenly distributed among processors.
- Load imbalance can impact overall parallel efficiency, especially as the number of workers increases.

4. Pipeline Dependencies:

- The pipeline structure might introduce dependencies between stages, limiting the level of parallelism and affecting the overall efficiency.
- Dependencies can result in processors waiting for data from their neighbors, impacting the pipeline's throughput.

5. Optimal Number of Workers:

- The optimal number of workers for pipelining might be different from that of Strategy 1.
- There may be an optimal configuration where the advantages of reduced communication overhead are maximized before diminishing returns set in.