Optimal Storage Arbitrage under Net Metering using Linear Programming

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***Abstract*—We formulate the optimal energy arbitrage problem for a piecewise linear cost function for energy storage devices using linear programming (LP). The LP formulation is based on the equivalent minimization of the epigraph. This formulation considers ramping and capacity constraints, charging and discharging efficiency losses of the storage, inelastic consumer load and local renewable generation in presence of net-metering which facilitates selling of energy to the grid and incentivizes consumers to install renewable generation and energy storage. We consider the case where the consumer loads, electricity prices, and renewable generations at different instances are uncertain. These uncertain quantities are predicted using an Auto-Regressive Moving Average (ARMA) model and used in a model predictive control (MPC) framework to obtain the arbitrage decision at each instance.**

I. Introduction

Energy storage devices provide flexibility to alter the consumption behavior of an electricity consumer. This feature of storage devices would be more lucrative for storage owners with the growth of intermittent generation sources which would increase volatility is generation side in power network [1]. Furthermore, storage devices are witnessing the deterioration of cost of battery making several applications of storage devices financially viable. Electric consumer bills vary based on policies applicable locally, however, primary variable component of electricity bills worldwide is the cost of energy consumption. Energy storage could perform arbitrage of energy with time varying consumer load, distributed generation production and electricity price. Furthermore, utilities promote inclusion of distributed generation and storage deployment by introducing net-metering. Net energy metering (NEM) or net-metering refers to the rate consumers receive for feeding power back to the grid. Most NEM policies indicate that consumers receive a rate at best equal to the buying price of electricity [2]. Authors in [3] consider storage operation under equal buy and sell price case. This framework is generalized in [4], covering cases where the ratio of buy and sell price could arbitrarily vary between 0 and 1. For equal buying and selling price, the storage control becomes independent of inelastic load and renewable generation of the consumer [3], [5]. Authors in [4] show that the cost function based on selection of optimization variable is convex and piecewise linear. This cost function considers inelastic load, renewable generation and storage charging and discharging efficiency and ramping and capacity constraints. The focus of this work is to formulate optimal arbitrage problem for a electricity consumer with renewable generation with NEM using Linear Programming (LP).

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Authors in [6] provide a summary of storage control methodologies used in power distribution networks among which LP based formulations can be solved efficiently using commercially available solvers. The complexity of LP based algorithms is polynomial time [7]. Therefore, these algorithms can be used to efficiently solve the arbitrage problem for the duration of a day divided into smaller time steps ranging from 5 minutes to an hour. A day is the typical time horizon over which arbitrage is performed [8], [9]. However, with a significantly longer time horizon LP might not be tractable.

LP techniques for energy storage arbitrage have been used in several prior works: [10], [11], [12], [13], [14], [15], [16]. Authors in [14], [11], [16] consider storage operation in presence of time-varying electricity price. However, in these formulations no renewable energy source or consumer load is assumed to be present. Authors in [12], [13] consider optimal scheduling of storage battery for maximizing energy arbitrage revenue in presence of distributed energy resources and variable electricity price. Formulations presented in [15], [10] consider storage performing arbitrage in a residential setting with inelastic load and local generation. Most common LP formulations for energy arbitrage such as in [10], [16], [13], [11] consider separation of charging and discharging components, with no constraint enforcing either the charging or the discharging component to be active at any particular time as the inclusion of such a constraint makes their for formulation nonlinear. Therefore, in these formulations optimal results cannot be guaranteed. Authors in [12], [14] do not consider energy storage charging and discharging efficiencies in the cost minimization, making it straight forward to apply LP. Authors in [15] consider a special case of optimization with zero-sum aggregate storage power output. For such a case LP tools could be used, however, generalizing the formulations needs to be explored further.

The key contributions of this paper are as follows:

• *LP formulation for storage control*: We formulate LP optimization problem for piecewise linear convex cost function, for storage with efficiency losses, ramping and capacity constraints and a consumer with inelastic load and renewable generation. The buying and selling price of electricity are varying over time. The selling price is assumed to be at best equal to buying price for each time instant, this assumption is in sync with most net-metering policies worldwide. We present LP formulations for lossy battery with inelastic consumption, renewable generation and selling price less than or equal to buying price. The reduction of this formulation for cases (a) lossless battery with equal buying and selling price of electricity and (b) lossy battery with selling price less than or equal to buying price, is trivial and not included in this paper. Based on the structure of cost function we apply an epigraph based minimization described in [17] and apply it to the arbitrage problem.

• *Real-time implementation:* We implement auto-regressive based forecast model along with model predictive control and numerically analyze their effect on arbitrage gains using real data from a household in Madeira in Portugal and electricity price from California ISO [18]. The effect of uncertainty on arbitrage gains is more pronounced for cases where selling price is higher compared to cases where selling price is closer to zero.

• *Sensitivity of ratio of selling and buying price*: We numerically analyze the effect of the ratio of buying and selling price of electricity on the value of storage integration for inelastic load and renewable generation. We observe that the value of storage performing arbitrage significantly increases in the presence of load and renewable generation with the increasing disparity of selling and buying price of electricity, compared to only storage performing arbitrage.

The paper is organized as follows. Section II provides the description of the system. Section III presents linear programming formulation of storage performing arbitrage with inelastic load, renewable generation and net-metering based compensation. Section IV presents an online algorithm using the proposed optimal arbitrage algorithm along with auto-regressive forecasting in the MPC framework. Section V discusses numerical results. Finally, Section VI concludes the paper.

II. System Description

We consider the operation of a single residential user of electricity over a fixed period of time. The user is assumed to be equipped with a rooftop solar PV and a battery to store excess generation. It is also connected to the electricity grid from where it can buy or to which it can sell energy. The objective is to find an efficient algorithm for a user to make optimal decisions over a period of varying electricity prices considering variations in the solar generation and end user load. The total duration, T, of operation is divided into N steps indexed by $\{1, ..., N\}$. The duration of step $i \in \{1, ..., N\}$ is denoted as h_i. Hence, $T = \sum_{i=1}^{N} h_i$. The price of electricity, $p_{\text{elec}}(i)$, equals the buying price, $p_b(i)$, if the consumption is positive; otherwise $p_{\text{elec}}(i)$ equals the selling price, $p_s(i)$. Note p_{elec} is ex-ante and consumer is a price taker. The ratio of selling and buying price is denoted as $\kappa_i = \frac{p_s(i)}{p_b(i)}$. The end user inelastic consumption is denoted as d_i and generates r_i units of energy through renewable sources in time step i. Net energy consumption without storage is denoted as $z_i = d_i - r_i \ \in \mathbb{R}$.

The efficiency of charging and discharging of the battery are denoted by $\eta_{\text{ch}}, \eta_{\text{dis}} \in (0, 1]$, respectively. We denote the change in the energy level of the battery at i^{th} instant by $x_i = h_i\delta_i$, where δ_i denotes the storage ramp rate at i^{th} instant such that $\delta_i \in [\delta_{\min}, \delta_{max}]$ $\forall i$ and $\delta_{\min} \leq 0, \delta_{\max} \geq 0$ are the minimum and the maximum ramp rates (kW); $\delta_i > 0$ implies charging and $\delta_i < 0$ implies discharging. Energy consumed by the storage in the i^{th} instant is given by $s_i = f(x_i) = \frac{1}{\eta_{\text{ch}}}[x_i]^+ - \eta_{\text{dis}}[x_i]^-$, where x_i must lie in the range from $X_{\min}^i = \delta_{\min}h_i$ to $X_{\max}^i = \delta_{\max}h_i$. Note $[x_i]^+ = \max(0, x_i)$ and $[x_i]^- = -\max(0, -x_i)$. Alternatively, we can write $x_i = \eta_{\text{ch}}[s_i]^+ - \frac{1}{\eta_{\text{dis}}}[s_i]^-$. The limits on s_i are given as $s_i \in [S_{\min}^i, S_{\max}^i]$, where $S_{\min}^i = \eta_{\text{dis}}\delta_{\min}h_i$ and $S_{\max}^i = \frac{\delta_{\max}h_i}{\eta_{\text{ch}}}$.

Let b_i denote the energy stored in the battery at the i^{th} step. Then, $b_i = b_{i-1} + x_i$. The capacity of the battery imposes the constraint $b_i \in [b_{\min}, b_{\max}], \forall i$, where $b_{\min}, b_{\max}$ are the minimum and the maximum battery capacity. The total energy consumed between time step i and $i{+}1$ is given as $L_i = z_i{+}s_i$.

Energy storage battery operational life is often quantified using cycle and calender life which decides the cycles a battery should perform over a time period. Friction coefficient denoted as $\eta_{\text{fric}} \in [0, 1]$ is introduced in [19] assists in reducing the operational life of the battery such that only low returning transactions of charging and discharging are eliminated, in effect increasing the operational life of the battery. In subsequent work, authors in [20] propose a framework to tune the value of friction co-efficient for increasing operational life of battery. In prior work, [21], we show that redefining η_{ch} equal to $\eta_{\text{ch}}\eta_{\text{fric}}$ and η_{dis} equal to $\eta_{\text{dis}}\eta_{\text{fric}}$, we can control the cycles of operation by eliminating the low returning transactions by reducing the value of η_{fric}.

A. Arbitrage under Net-Metering

The optimal arbitrage problem (denoted as (P)) is defined as the minimization of the cost of the total consumed energy, $\min \sum_{i=1}^{N} L_i p_{\text{elec}}(i)$, subject to the battery constraints. It is given as follows:

$$\text{(P)} \ \min \sum_{i=1}^{N} C_{nm}^i(x_i),$$

subject to, $b_{\min} - b_0 \leq \sum_{j=1}^{i} x_j \leq b_{\max} - b_0, \forall i \in \{1, .., N\}$, and $x_i \in \left[X_{\min}^i, X_{\max}^i\right] \forall i \in \{1, .., N\}$. $C_{\text{nm}}^i(x_i)$ denotes the energy consumption cost function at instant i and is equal to $[z_i + f(x_i)]^+ p_b(i) - [z_i + f(x_i)]^- p_s(i)$. Now we will show that the optimal arbitrage problem is convex in $x = (x_i, i = 1 : N)$. For this convexity to hold we require $p_b(i) \geq p_s(i)$ for all $i = 1 : N$, i.e., $\kappa_i \in [0, 1]$. Proposed framework is applicable for case where selling price of electricity for the end user is lower than the buying price. This assumption is quite realistic as this is generally the case in most practical net metering policies [2].

Theorem II.1. *If $p_b(i) \geq p_s(i)$ for all $i = 1 : N$, then problem (P) is convex in x.*

Proof. Let $\psi(t) = a[t]^+ - b[t]^-$ with $a \geq b \geq 0$. Using $t = [t]^+ - [t]^-$ we have $\psi(t) = (a - b)[t]^+ + bt$. Since both $[t]^+$ and t are convex in t and $a - b, b \geq 0$ we have that ψ is convex since it is the positive sum of two convex functions.

Now let $f(x) = \frac{1}{\eta_{ch}}[x]^+ - \eta_{dis}[x]^-$ and $h_i(s) = [z_i + s]^+ p_b(i) - [z_i + s]^- p_s(i)$. Then by the above reasoning we have that for $p_b(i) \geq p_s(i) \geq 0$ and $\eta_{ch}, \eta_{dis} \in (0, 1]$, h_i

is convex in s and f is convex in x. Also, note that h_i is non-decreasing in s. Hence, for $\lambda \in [0, 1]$ we have

$$h_i\big(f(\lambda x + (1-\lambda)y)\big) \leq h_i\big(\lambda f(x) + (1-\lambda)f(y)\big) \quad (1)$$
$$\leq \lambda h_i(f(x)) + (1-\lambda)h_i(f(y)) \quad (2)$$

In the above, the first inequality follows from the convexity of f and non-decreasing nature of h_i and the second inequality follows from convexity of h_i. Therefore, we have that $h_i \cdot f = h_i(f())$ is a convex function in x. This shows that the objective function of (P) is convex in x since $C_{nm}^i = h_i \cdot f$. Since the constraints are linear in x thus problem (P) is convex. $\square$

III. Optimal Arbitrage with Linear Programming

The optimal arbitrage problem, (P), could be solved using linear programming as the cost function is (i) convex and (ii) piecewise linear and (iii) the associated ramping and capacity constraints are linear. In this section we provide LP formulations for storage device for performing arbitrage. We apply the epigraph based minimization presented in [17] for optimal storage arbitrage. A summary of epigraph based formulation for piecewise linear convex cost function is presented in Appendix A.

The optimal arbitrage formulation for storage under net-metering and consumer inelastic load and renewable generation using the epigraph formulation is presented in this section. Fig. 1 shows the two cost functions depending on the net-load without storage output, i.e. for $z_i > 0$ and $z_i < 0$. Note that there are 4 unique segments which formulate the cost function $C_{nm}(i)$. The slope, x-intercept and y-intercept of these linear segments are listed in Table I.

TABLE I: Cost-function for storage with load under NEM

Segment	Slope	x-intercept	y-intercept
Segment 1	$p_b(i)/\eta_{ch}$	$-z_i\eta_{ch}$	$z_i p_b(i)$
Segment 2	$p_s(i)\eta_{dis}$	$-z_i/\eta_{dis}$	$z_i p_s(i)$
Segment 3	$p_b(i)\eta_{dis}$	$-z_i/\eta_{dis}$	$z_i p_b(i)$
Segment 4	$p_s(i)/\eta_{ch}$	$-z_i\eta_{ch}$	$z_i p_s(i)$

The epigraph based LP formulation is possible as irrespective of sign of load the cost function is given as

$$C_{nm}(i) = \max\big(\text{Segment 1, Segment 2, Segment 3, Segment 4}\big). \quad (3)$$

Since, Eq.3 is independent of the sign of load and based on the intercepts, Eq.3 is valid for $p_b(i) \geq p_s(i)$ and for $\eta_{ch}, \eta_{dis} \in (0, 1]$ (conditions of convexity), therefore, we could formulate this problem into a LP. Using the epigraph equivalent formulation for piecewise linear convex cost function we formulate the optimal arbitrage problem using LP.

$$\min \quad \{t_1 + t_2 + ... + t_N\},$$
subject to,
$$\text{(a) Segment 1: } \frac{p_b^i}{\eta_{ch}} x_i - t_i \leq -z_i p_b^i, \ \forall i$$
$$\text{(b) Segment 2: } p_s^i \eta_{dis} x_i - t_i \leq -z_i p_s^i, \ \forall i$$
$$\text{(c) Segment 3: } p_b^i \eta_{dis} x_i - t_i \leq -z_i p_b^i, \ \forall i$$
$$\text{(d) Segment 4: } \frac{p_s^i}{\eta_{ch}} x_i - t_i \leq -z_i p_s^i, \ \forall i$$
$$\text{(e) Ramp constraint: } x_i \in [X_{\min}, X_{\max}], \ \forall i$$
$$\text{(f) Capacity constraint: } \sum x_i \in [b_{\min} - b_0, b_{\max} - b_0], \ \forall i.$$

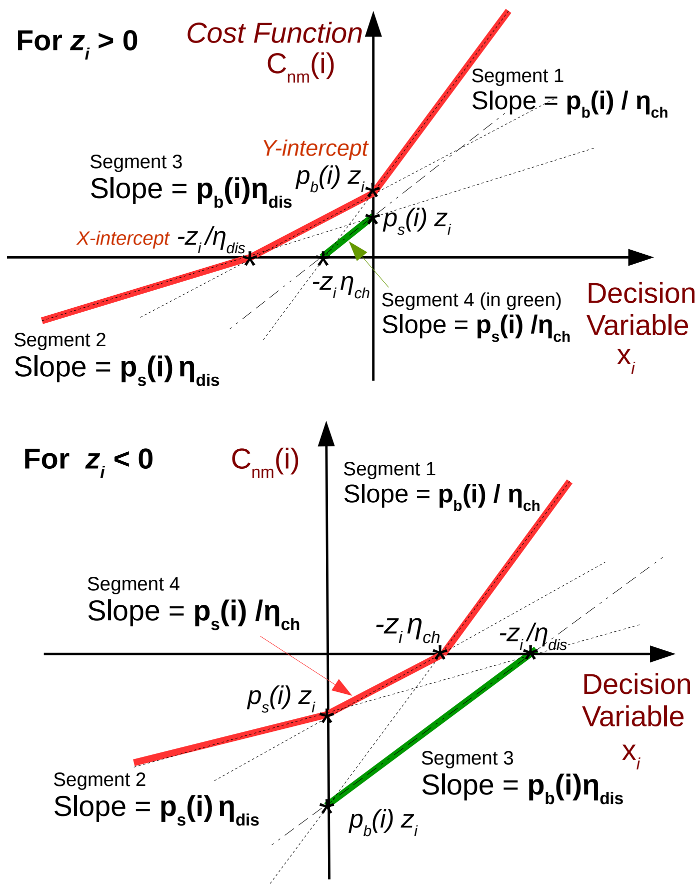

Fig. 1: The cost function segment wise for positive and negative z [4]. The decision variable is storage change in charge level, x_i, and cost function, $C_{nm}(i)$ is formed with 4 unique line segments.

The cost function for only lossy storage operation under NEM would have two-piecewise linear segments. LP formulation would be linear for equal buying and selling price of electricity with lossless battery. Authors in [12], [14] present this case in their LP formulation. These above mentioned formulations could be obtained by simplifying the more general case depicted in Fig. 1.

We make our code open source on formulating optimal arbitrage problem using linear programming. The lack of algorithm benchmarks encourages us to present this package of codes for other people working in the field[1].

IV. Real-time implementation

The previous section discussed optimal storage arbitrage under complete knowledge of future net loads and prices. In this section, we consider the setting where future values may be unknown. To that end, we first develop a forecast model for net load without storage (which includes inelastic consumer load and consumer distributed generation) and electricity price for future times, where the forecast is updated after each time step. We develop the forecasting model for net load with solar generation using AutoRegressive Moving Average (ARMA) model and electricity price forecast using AutoRegressive Integrated Moving Average (ARIMA).

The forecast models based on ARMA and ARIMA model developed in [22] are used in this work. The forecast values are fed to a Model Predictive Control (MPC) scheme to identity the optimal modes of operation of storage for the current time-instance. Any of the developed schemes from the previous section can be used for the optimization inside MPC. These steps (forecast and MPC) are repeated sequentially and highlighted in online Algorithm 1: `ForecastMPClinearProgram`.

Algorithm 1 `ForecastMPClinearProgram`

Global Inputs: $\eta_{\text{ch}}, \eta_{\text{dis}}, \delta_{\max}, \delta_{\min}, b_{\max}, b_{\min}, b_0$
Inputs: $h, N, T, i = 0$

while $i < N$ **do**
 $i = i + 1$
 Forecast $\hat{z}$ from time step i to N using ARMA
 Forecast $\hat{p}_b$ and $\hat{p}_s$ from time step i to N using ARIMA
 Calculate $\hat{\kappa}$ as the ratio of $\hat{p}_s$ and $\hat{p}_b$,
 Formulate LP matrices,
 Solve the Linear Optimization problem for vector $\hat{x}^*$,
 $b^{i^*} = b^{i-1} + \hat{x}^*(1)$ and Update $b_0 = b^{i^*}$
end while

V. Numerical Results

For the numerical evaluation, we use battery parameters listed in Table II. The performance indices used for evaluating simulations are: • *Arbitrage Gains* denotes the gains (made in absence of load and renewable) or reduction in the cost of consumption (made in presence of load and renewable) due to storage performing arbitrage • *Cycles of operation*: In our prior work [20] we develop a mechanism to measure the number of cycles of operation based on depth-of-discharge (DoD) of energy storage operational cycles. Equivalent cycles of 100% DoD are identified. This index provides information about how much the battery is operated.

We use xC-yC notation to represent the relationship between ramp rate and battery capacity. xC-yC implies battery takes 1/x hours to charge and 1/y hours to discharge completely. We perform sensitivity analysis with (a) four battery models with the different ramping capability listed in Table II and (b) 5 levels of the ratio of selling price and buying price of electricity, i.e., $\kappa \in \{1, 0.75, 0.5, 0.25, 0\}$. In this work we assume the selling price is equal to the product of scalar variable κ and the buying price of electricity.

[1]https://github.com/umar-hashmi/linearprogrammingarbitrage

TABLE II: Battery Parameters

$b_{\min}$, $b_{\max}$, b_0	200Wh, 2000 Wh, 1000 Wh
$\eta_{\text{ch}} = \eta_{\text{dis}}$	0.95
$\delta_{\max} = -\delta_{\min}$	500 W for 0.25C-0.25C, 1000 W for 0.5C-0.5C 2000 W for 1C-1C, 4000 W for 2C-2C

A. Deterministic Simulations

The price data for our simulations in this subsection is taken from NYISO [23]. The load and generation data is taken from data collected at Madeira, Portugal. Fig. 2 shows the electricity price and energy consumption (includes inelastic load and rooftop solar generation) data used for deterministic simulations.

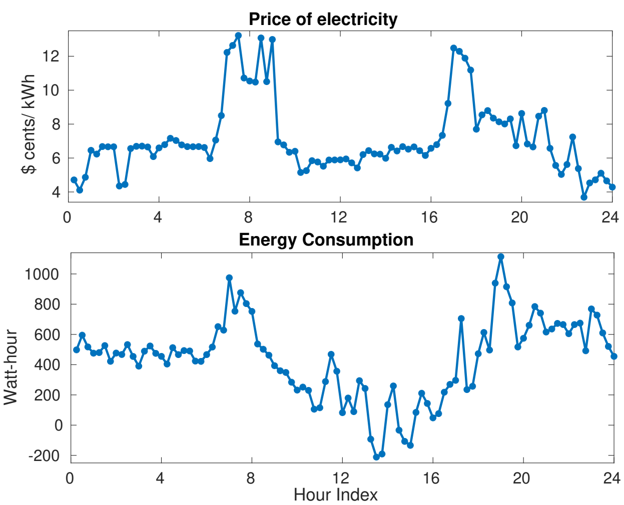

Fig. 2: Electricity price and consumer net load data used for deterministic simulations.

Table III and Table IV lists the energy storage arbitrage without and with energy consumption load for the electricity price data shown in Fig. 2. The observations are:

- The value of storage in presence of load and renewable increases as κ decreases. Note that for $\kappa = 0$, the only storage operation provides zero gain (see Table III), however, for the same buying and selling levels, the consumer would make significant gains when operated with inelastic load and renewable generation (see Table IV),
- The cycles of operation for faster ramping batteries are higher compared to slower ramping batteries. This implies that faster ramping batteries should be compared on gains per cycle with slower ramping batteries. Observing only gains could be misleading.
- As κ decreases, the cycles of operation decreases, thus the effect on storage operation in the cases presented is similar to η_{fric}.

- Note that for $\kappa = 1$, the arbitrage gains with and without load are the same. This observation is in sync with claims made in [3]. Authors in [3] observe that storage operation becomes independent of load and renewable variation for equal buying and selling case.

TABLE III: Performance indices for only storage

κ	2C-2C	1C-1C	0.5C-0.5C	0.25C-0.25C
Arbitrage gains in \$ cents for 1 day				
1	44.445	33.760	25.636	17.536
0.75	18.842	17.668	14.077	9.921
0.5	7.682	7.088	6.253	5.219
0.25	2.513	2.502	2.483	2.422
0	0	0	0	0
Cycles of operation for 1 day				
1	6.586	3.856	2.237	1.620
0.75	2.401	1.742	1.484	0.795
0.5	1.539	1.099	0.714	0.386
0.25	0.182	0.171	0.164	0.160
0	0	0	0	0

TABLE IV: Performance indices for storage + load

κ	2C-2C	1C-1C	0.5C-0.5C	0.25C-0.25C
Arbitrage gains in \$ cents for 1 day				
1	44.445	33.760	25.636	17.536
0.75	37.848	33.023	26.469	18.337
0.5	39.045	34.105	27.696	19.344
0.25	40.272	35.332	28.923	20.351
0	41.500	36.560	30.150	21.358
Cycles of operation for 1 day				
1	6.586	3.835	2.263	1.620
0.75	5.986	4.039	2.338	1.652
0.5	5.986	4.033	2.364	1.660
0.25	5.986	4.033	2.364	1.660
0	5.986	4.033	2.364	1.660

B. Results with Uncertainty

The forecast model is generated for load with solar generation and for electricity price. The ARMA based forecast use 9 weeks of data (starting from 29th May, 2019) for training and generates forecast for the next week. `ForecastMPClinearProgram` is implemented in receding horizon. The electricity price data used for this numerical experiment is taken from CAISO for the same days of load data. To compare the effect of forecasting net load and electricity prices with perfect information, we present average arbitrage gains and cycles of operation starting from 1st June 2019.

The deterministic results for without and with load are presented in Table V and Table VI. Compare the deterministic results with stochastic results presented in Table VII and Table VIII. The primary numerical observations are:

- Effect of uncertainty on gains for faster ramping battery is greater compared to slower ramping battery [24],
- Clubbing storage with inelastic load with renewable generation provides greater gains for decreasing κ. Fur-

TABLE V: Deterministic arbitrage gains for only storage

κ	2C-2C	1C-1C	0.5C-0.5C	0.25C-0.25C
Arbitrage gains in \$ for 1 week				
1	9.411	7.059	4.784	3.065
0.75	5.729	4.491	3.168	2.082
0.5	3.166	2.550	1.833	1.217
0.25	1.124	0.941	0.688	0.456
0	0	0	0	0
Cycles of operation for 1 week				
1	58.729	37.257	21.324	12.107
0.75	23.462	16.341	10.746	7.519
0.5	12.689	9.770	7.579	6.174
0.25	7.727	6.229	4.558	3.464
0	0	0	0	0

TABLE VI: Deterministic arbitrage gains for storage with load

κ	2C-2C	1C-1C	0.5C-0.5C	0.25C-0.25C
Arbitrage gains in \$ for 1 week				
1	9.411	7.059	4.784	3.065
0.75	7.462	6.269	4.540	3.025
0.5	6.641	5.987	4.468	3.019
0.25	6.350	5.904	4.451	3.019
0	6.313	5.902	4.451	3.019
Cycles of operation for 1 week				
1	58.700	37.294	21.324	12.107
0.75	28.583	20.809	14.382	10.229
0.5	19.296	16.629	13.007	9.971
0.25	16.591	15.348	12.498	9.968
0	16.041	15.201	12.484	9.968

TABLE VII: Stochastic indices for only storage

κ	2C-2C	1C-1C	0.5C-0.5C	0.25C-0.25C
Arbitrage gains in \$ for 1 week				
1	6.035	4.684	3.469	3.000
0.75	5.024	4.118	3.081	1.904
0.5	3.004	2.367	1.692	1.110
0.25	1.067	0.891	0.618	0.442
Cycles of operation for 1 week				
1	64.323	38.979	22.622	12.850
0.75	24.870	16.169	10.570	7.733
0.5	11.393	8.891	7.013	6.099
0.25	6.429	5.557	4.359	3.395

TABLE VIII: Stochastic indices for storage with load

κ	2C-2C	1C-1C	0.5C-0.5C	0.25C-0.25C
Arbitrage gains in \$ for 1 week				
1	6.034	4.684	3.496	3.000
0.75	4.827	4.075	3.400	2.987
0.5	4.168	3.711	3.292	2.975
0.25	4.204	3.943	3.348	3.002
0	4.427	3.896	3.396	3.009
Cycles of operation for 1 week				
1	64.322	38.979	22.622	12.850
0.75	41.613	30.322	19.948	11.980
0.5	34.658	27.627	18.744	11.348
0.25	31.429	26.370	18.476	11.396
0	32.958	28.255	19.845	11.372

thermore, the effect of uncertainty for lower κ is lower compared to higher values of κ.
- Profitability of operating only storage deteriorates sharply with decrease of κ. For only storage case under zero selling price case ($\kappa = 0$) no arbitrage would be possible and the gain remains zero.

VI. Conclusion

We formulate energy storage arbitrage problem using linear programming. The LP formulation is possible due to piecewise linear convex cost functions. In this formulation we consider: (a) net-metering compensation (with selling price at best equal to buying price) i.e. $\kappa_i \in [0, 1]$, (b) inelastic load, (c) consumer renewable generation, (d) storage charging and discharging losses, (e) storage ramping constraint and (f) storage capacity constraint. Using numerical results we perform sensitivity analysis of energy storage batteries for varying ramp rates and varying ratio of selling and buying price of electricity. We observe that the value of storage in presence of load and renewable increases as the ratio of selling and buying price decreases. We also compare stochastic simulation for real-time implementation. Net-load and electricity price is modeled with AutoRegressive models for model predictive control.

In future work we aim to tune the friction coefficient with different κ, such that the battery is not over-used which otherwise would lead to reduction in battery operational life. Authors of [25] approximate the link cost in a network, which is a convex monotonically increasing function, with a piecewise linear function and they formulate the routing problem as a linear program. Similar to this application, we wish to extend this formulation for convex cost function which needs to be approximated as piecewise linear components.

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Appendix

An unconstrained minimization problem of a convex piecewise-linear function, $h(x)$, could be transformed to an equivalent linear programming problem by forming the epigraph problem [17], [26]. Consider the convex piecewise cost function minimization problem is denoted as $(P_{org}) \quad \min h(x)$, where $h(x) = max_{i=1,...,m}(a_i^T x + b_i)$. For cases where decision variable x is scaler in such cases a_i^T is a scaler. Thus $a_i x + b_i$ denotes a two-dimensional line with b_i denoting the y-intercept and a_i denotes the slope of the line. The equivalent epigraph problem for the original problem P_{org} is denoted as $(P_{epi}) \quad \min t$, subject to, $a_i x + b_i \leq t, \quad i = 1, ..., m,$ where t denotes auxiliary scalar variable. The LP matrix notation for the optimization problem P_{epi} is denoted as: minimize $\tilde{f}^T\tilde{x}$, subject to $\tilde{A}\tilde{x} \leq \tilde{b}$; where

$$\tilde{f} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \tilde{x} = \begin{bmatrix} x \\ t \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} a_1 & -1 \\ : & : \\ a_m & -1 \end{bmatrix}, \quad \tilde{b} = \begin{bmatrix} -b_1 \\ : \\ -b_m \end{bmatrix}.$$

Now consider extending this minimization problem for two time instants with a unique cost function for each time instant. The optimization problem is denoted as $(P_{epi}) \quad \min \quad t_1 + t_2,$ s.t., (i) $a_{1i}x + b_{1i} \leq t_1,$ (ii) $a_{2i}x + b_{2i} \leq t_2,\ i = 1, ..., m,$ The equivalent LP matrices are denoted as

$$\tilde{f} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \tilde{x} = \begin{bmatrix} x_1 \\ x_2 \\ t_1 \\ t_2 \end{bmatrix}, \tilde{A} = \begin{bmatrix} a_{11} & 0 & -1 & 0 \\ : & : & : & : \\ a_{1m} & 0 & -1 & 0 \\ 0 & a_{21} & 0 & -1 \\ : & : & : & : \\ 0 & a_{1m} & 0 & -1 \end{bmatrix}, \tilde{b} = \begin{bmatrix} -b_{11} \\ : \\ -b_{1m} \\ -b_{21} \\ : \\ -b_{2m} \end{bmatrix}.$$

A similar LP formulations for N time steps with piecewise linear cost function could be formulated.

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