A1 BASIC(MST3K)

Collaboration Policy: You are encouraged to collaborate with up to 3 other students, but all work submitted must be your own *independently* written solution. List the computing ids of all of your collaborators in the `collabs` command at the top of the tex file. Do not share written notes, documents (including Google docs, Overleaf docs, discussion notes, PDFs), or code. Do not seek published or online solutions for any assignments. If you use any published or online resources (which may not include solutions) when completing this assignment, be sure to cite them. Do not submit a solution that you are unable to explain orally to a member of the course staff. Any solutions that share similar text/code will be considered in breach of this policy. Please refer to the syllabus for a complete description of the collaboration policy.

Collaborators: list your collaborators

Sources: Cormen, et al, Introduction to Algorithms [2]

PROBLEM 1 *Proofs*

Learn how to typeset math and construct proofs by reproducing the second proof below. You will need to use the `eqnarray` or `align` environment, as well as the `eqnarray*` or `align*` environment. Note the reference in red, which should refer correctly to the equation (look up the `ref` command). The first proof is provided as an example.

Definition 1 *A rational number is a fraction $\frac{a}{b}$ where a and b are integers.*

Theorem 1 *$\sqrt{2}$ is irrational.*

Proof. By Contradiction. For a rational number $\frac{a}{b}$, without loss of generality we may suppose that a and b are integers which share no common factors, as otherwise we could remove any common factors (i.e. suppose $\frac{a}{b}$ is in simplest terms). To say $\sqrt{2}$ is irrational is equivalent to stating that 2 cannot be expressed in the form $(\frac{a}{b})^2$. Equivalently, this says that there are no integer values for a and b satisfying

$$a^2 = 2b^2 \tag{1}$$

Assume toward reaching a contradiction that Equation 1 holds for a and b being integers without any common factor between them. It must be that a^2 is even, since $2b^2$ is divisible by 2, therefore a is even. If a is even, then for some integer c

$$\begin{aligned} a &= 2c \\ a^2 &= (2c)^2 \\ 2b^2 &= 4c^2 \\ b^2 &= 2c^2 \end{aligned}$$

therefore, b is even. This implies that a and b are both even, and thus share a common factor of 2. This contradicts our hypothesis, therefore our hypothesis is false. □

Theorem 2 *If $n \in \mathbb{Z}$ is a non-prime integer with $n > 1$, then $2^n - 1$ is not prime [3].*

Proof. Direct Proof. Since n is not prime, $\exists a, b \in \mathbb{Z}$ such that $a < n$ and $b < n$ and $n = ab$. Let

$$x = 2^b - 1$$

and

$$y = 1 + 2^b + 2^{2b} + ... + 2^{(a-1)b}.$$

Then,

$$\begin{aligned} xy &= (2^b - 1)(1 + 2^b + ... + 2^{(a-1)b}) &(2)\\ &= 2^b(1 + 2^b + ... + 2^{(a-1)b}) - (1 + 2^b + ... + 2^{(a-1)b}) &(3)\\ &= 2^{ab} - 1 &(4)\\ &= 2^n - 1. &(5)\end{aligned}$$

Since $b < n$, then $x = 2^b - 1 < 2^n - 1$. Likewise, since $ab = n > a$, we know that $b > 1$ and $x = 2^b - 1 > 2 - 1 = 1$. Therefore, $y < xy = 2^n - 1$ and $2^n - 1$ can be written as the multiplication of x and y by Equation 5. Therefore $2^n - 1$ is not prime. □

PROBLEM 2 *Passages*

Include a passage from **your** favorite book, including a citation. You will need to update the `bibliography.bib` file and include it in your submission. Note that your references will be numbered in alphabetical order. Hint: consider using the `quote` environment.

> "Forty-two!" yelled Loonquawl. "Is that all you've got to show for seven and a half million years' work?"
> "I checked it very thoroughly," said the computer, "and that quite definitely is the answer. I think the problem, to be quite honest with you, is that you've never actually known what the question is." [1]

PROBLEM 3 *Sketchings*

Learn how to include drawings in your documents with the `\includegraphics{file}` command by submitting a caricature of professor Horton or professor Hott.

PROBLEM 4 *Gradescope Submission*

Submit your files to Gradescope. You should only submit your `.pdf`, `.tex`, and `.bib` files.

References

[1] Douglas Adams. *The Hitchhiker's Guide to the Galaxy*. Harmony Books, New York, 1980.

[2] Thomas H Cormen, Charles E Leiserson, Ronald L Rivest, and Clifford Stein. *Introduction to algorithms*. MIT press, 2009.

[3] Daniel J. Velleman. *How to Prove It: A Structured Approach*. Cambridge University Press, New York, 2006.