

A similar recursive approach would work for the  $\ell^p$  norm of  $\mathbf{x}$ , where  $p \geq 1$  and

$$\|\mathbf{x}\|_p = \left( \sum_{i=1}^n |x_i|^p \right)^{1/p},$$

if a function  $\text{normp}(x, y)$  were available, where

$$\text{normp}(x, y) = (|x|^p + |y|^p)^{1/p},$$

such that unwarranted under/over-flows are avoided, and relative accuracy and monotonicity with respect to  $|x|$  and  $|y|$  are guaranteed.

For  $p = 1$ ,  $\text{norm1}(x, y) = |x| + |y|$ , for  $p = 2$ ,  $\text{norm2}(x, y) = \text{hypot}(x, y)$ . Else, as the first attempt,

$$\text{normp}(x, y) = M (1 + q^p)^{1/p}, \quad M = \max\{|x|, |y|\}, \quad m = \min\{|x|, |y|\}, \quad q = m/M \text{ or } 0 \text{ if } m = 0.$$

Let  $z = q^{p/2}$ . Then  $1 + q^p = 1 + z^2 \approx \text{fma}(z, z, 1)$ . With `cr_pow`, the algorithm might be as follows:

$$\begin{aligned} X &= \text{fabs}(x), \quad Y = \text{fabs}(y); \\ M &= \text{fmax}(X, Y), \quad m = \text{fmin}(X, Y); \\ q &= m/M, \quad Q = \text{fmax}(q, 0); \\ z &= \text{cr\_pow}(Q, p/2), \quad Z = \text{fma}(z, z, 1); \\ C &= \text{cr\_pow}(Z, 1/p); \\ \text{normp}(x, y) &\approx M \cdot C. \end{aligned}$$

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If  $p = 1$  or  $p = \infty$ , the computation can be vectorized, with the per-lane operations

$$\text{add\_abs}(x, y) = |x| + |y|, \quad \text{max\_abs}(x, y) = \max\{|x|, |y|\},$$

respectively, instead of  $\text{hypot}(x, y)$  from the paper.