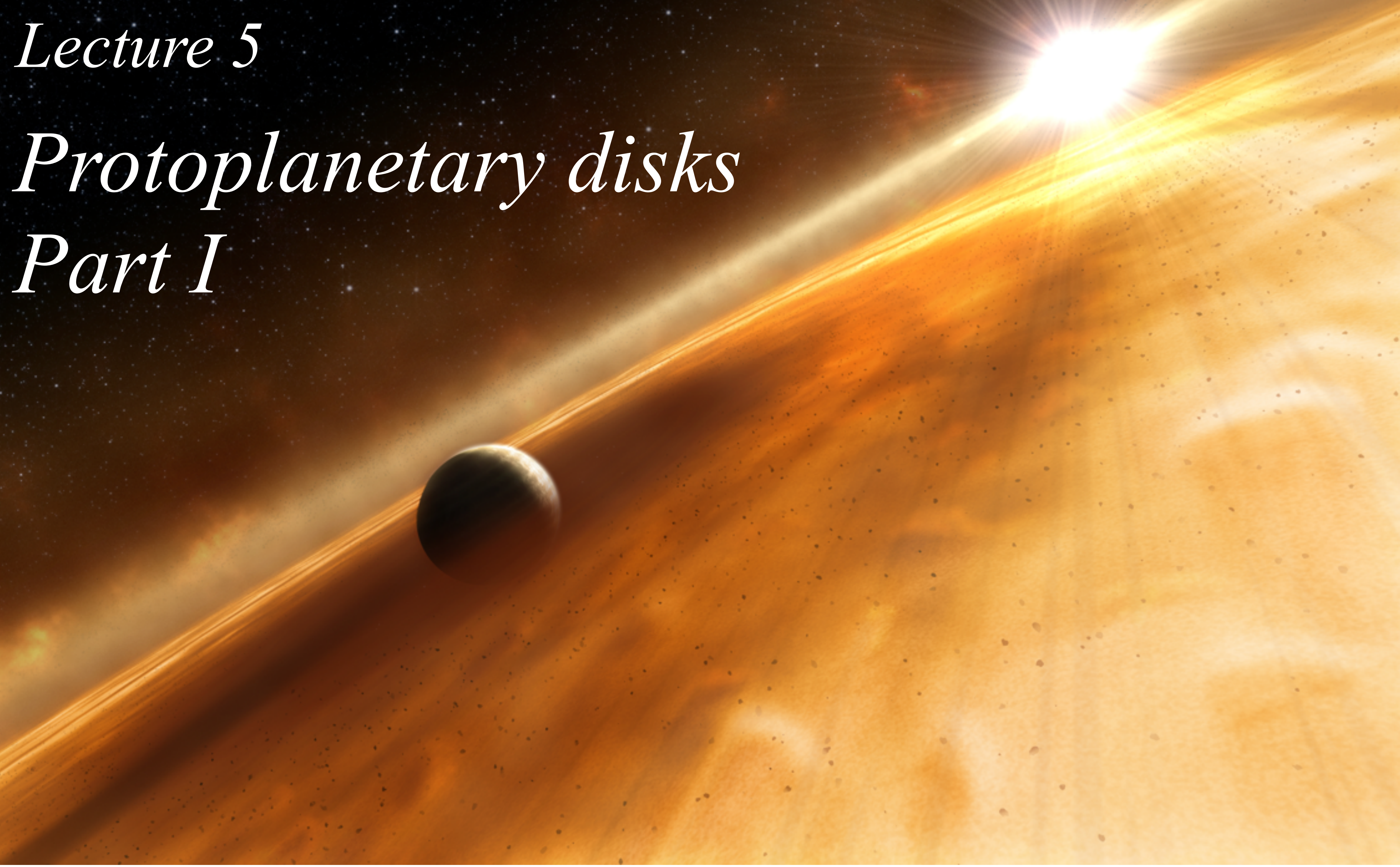
Lecture 5

Protoplanetary disks Part I



Lecture Universität Heidelberg WS 11/12
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Based partially on script of Prof. W. Benz

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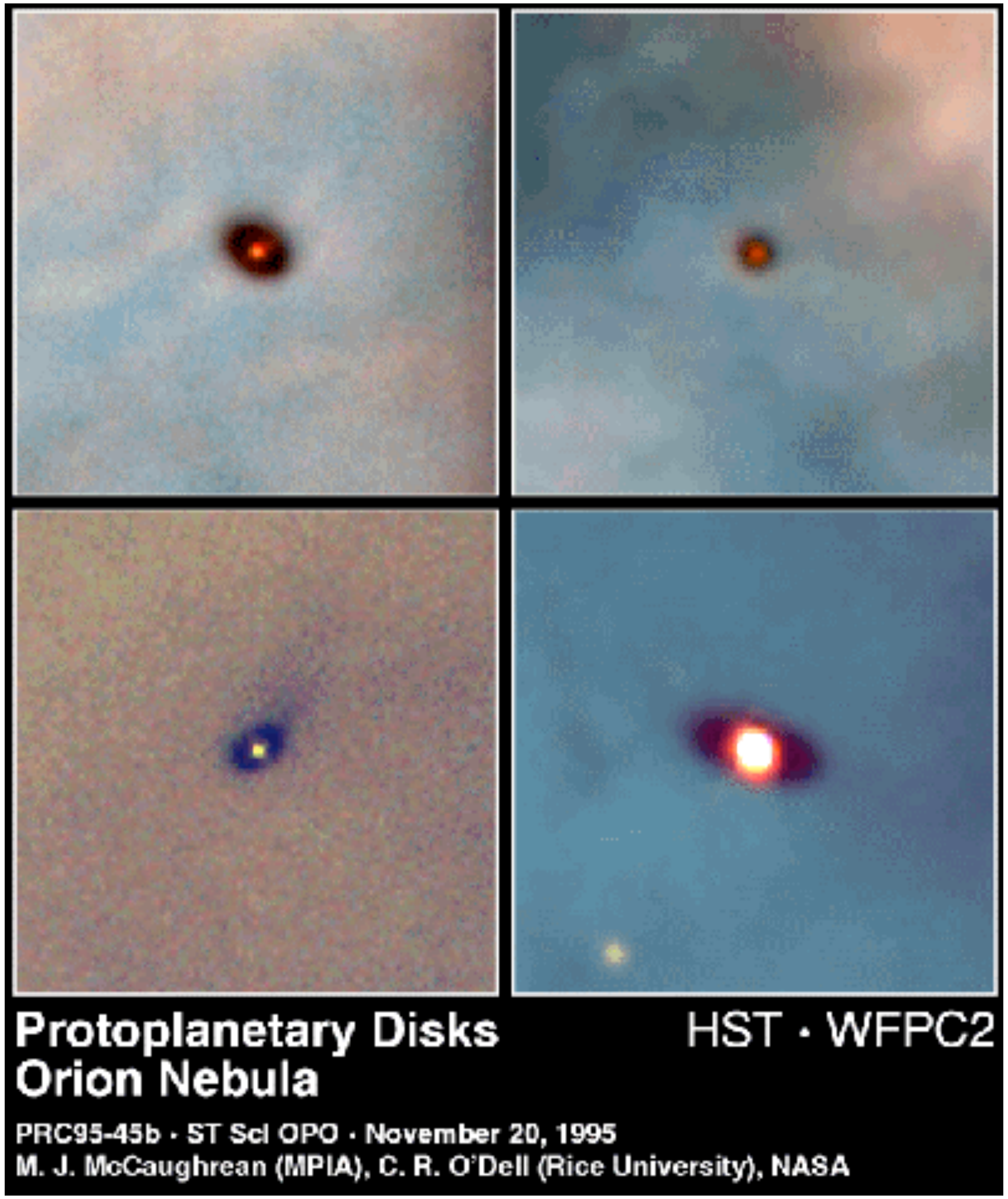
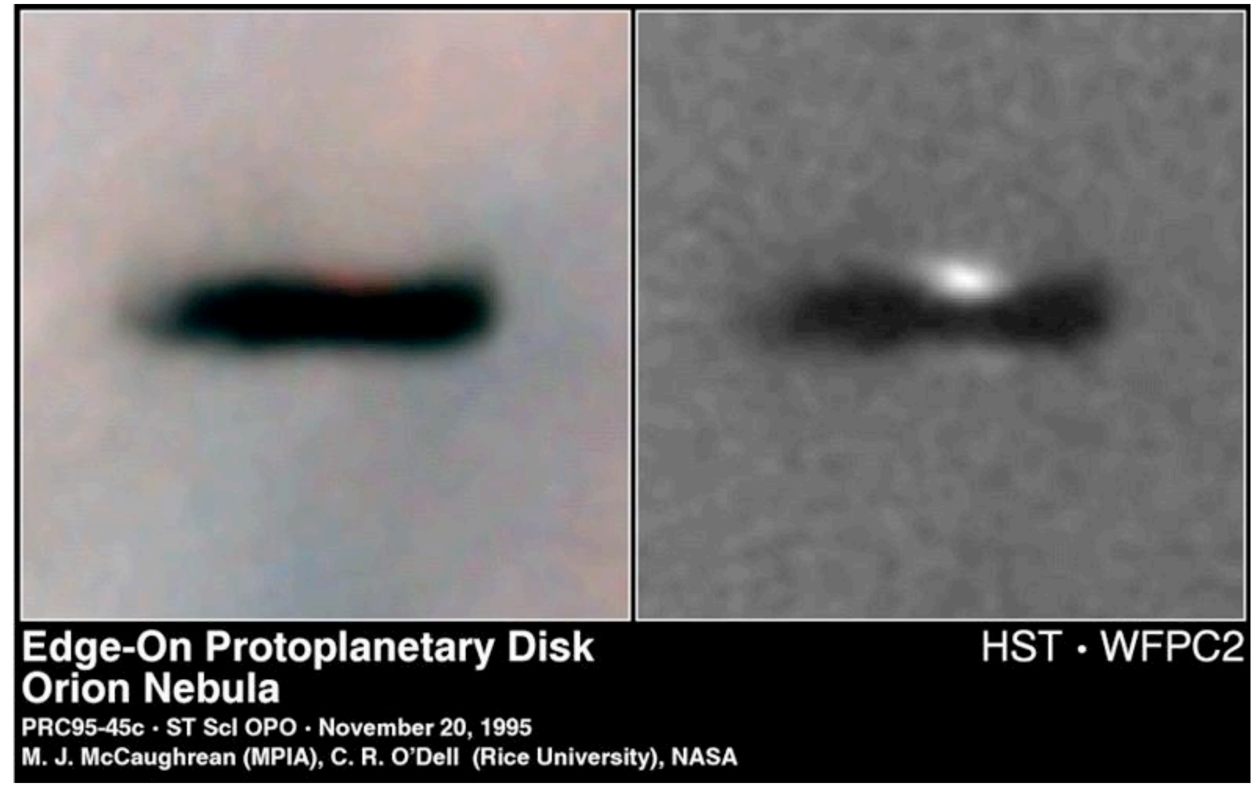
Lecture 5 overview

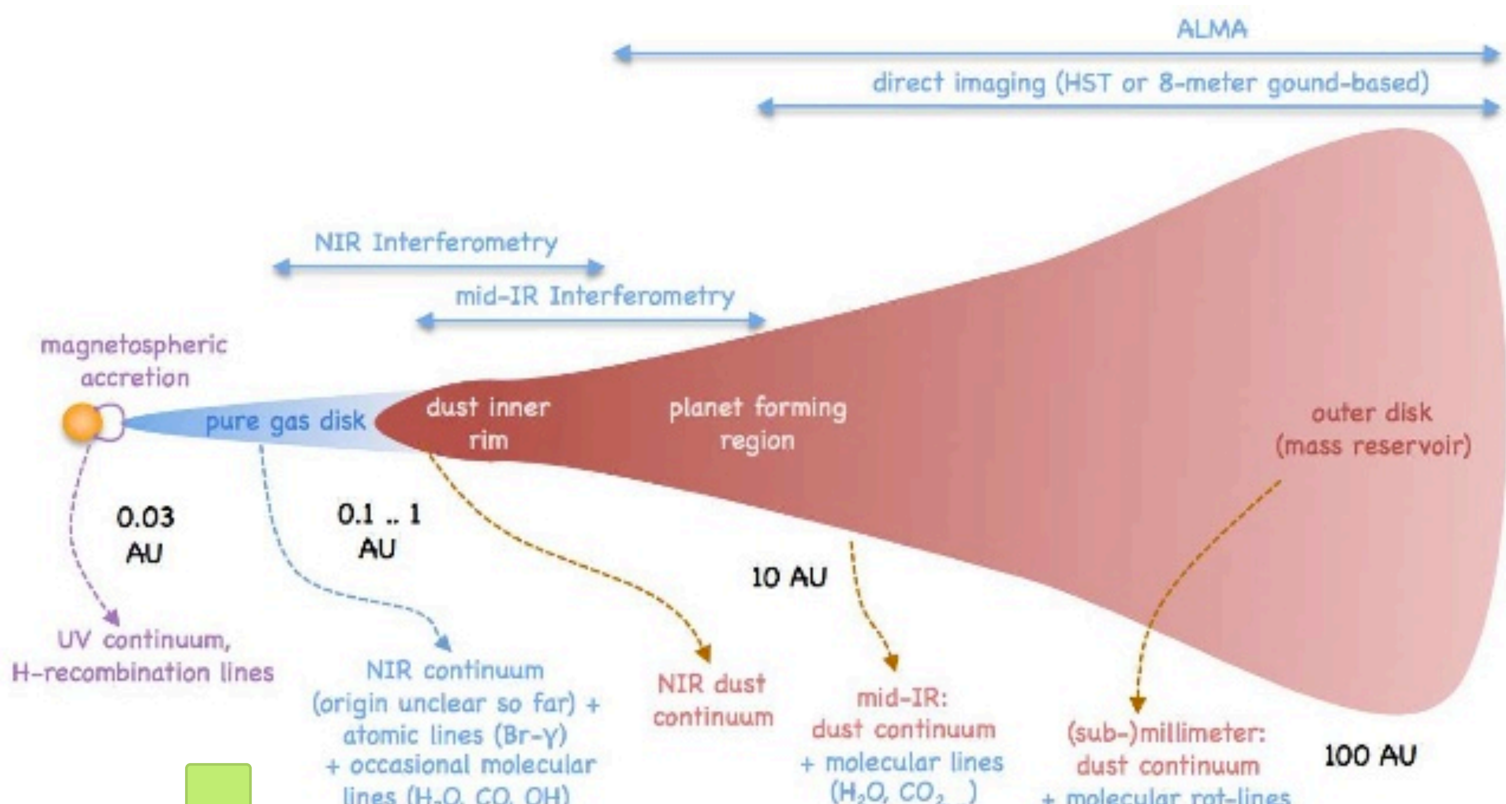
1. Observed features of protoplanetary disks
2. The minimum mass solar nebula
3. Disk structure
   - 3.1 Vertical structure
   - 3.2 Radial structure
   - 3.3 Application for the MMSN
4. Stability of disks
   - 4.1 Formation by direct collapse
   - 4.2 Numerical simulations

1. Observed properties

General structure





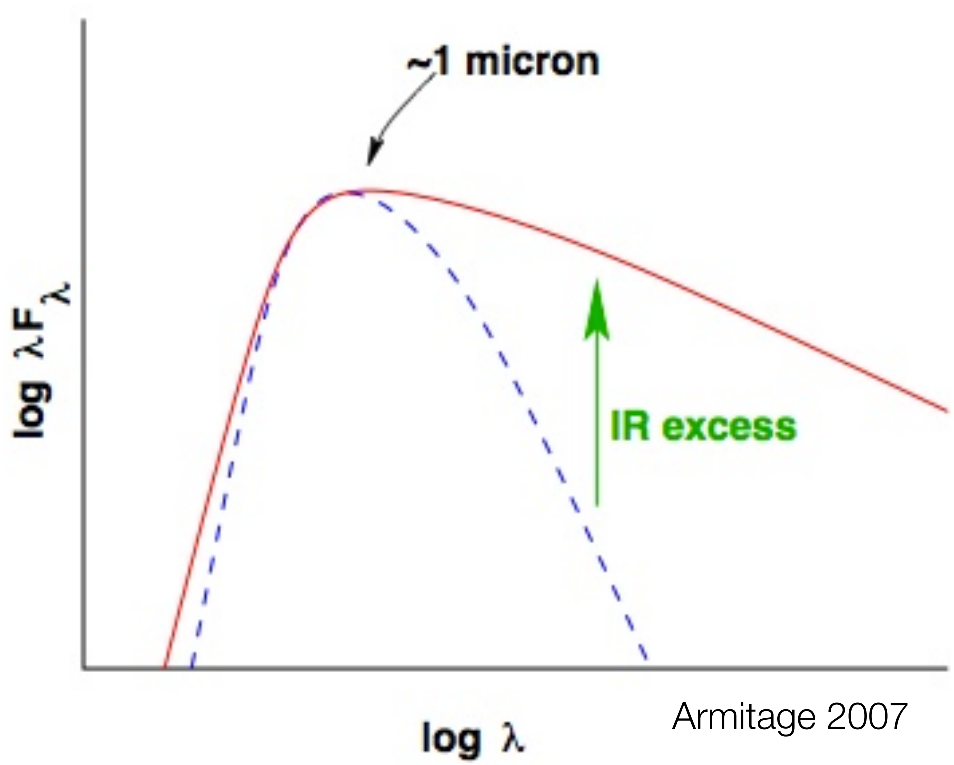


Dullemond & Monnier 2010

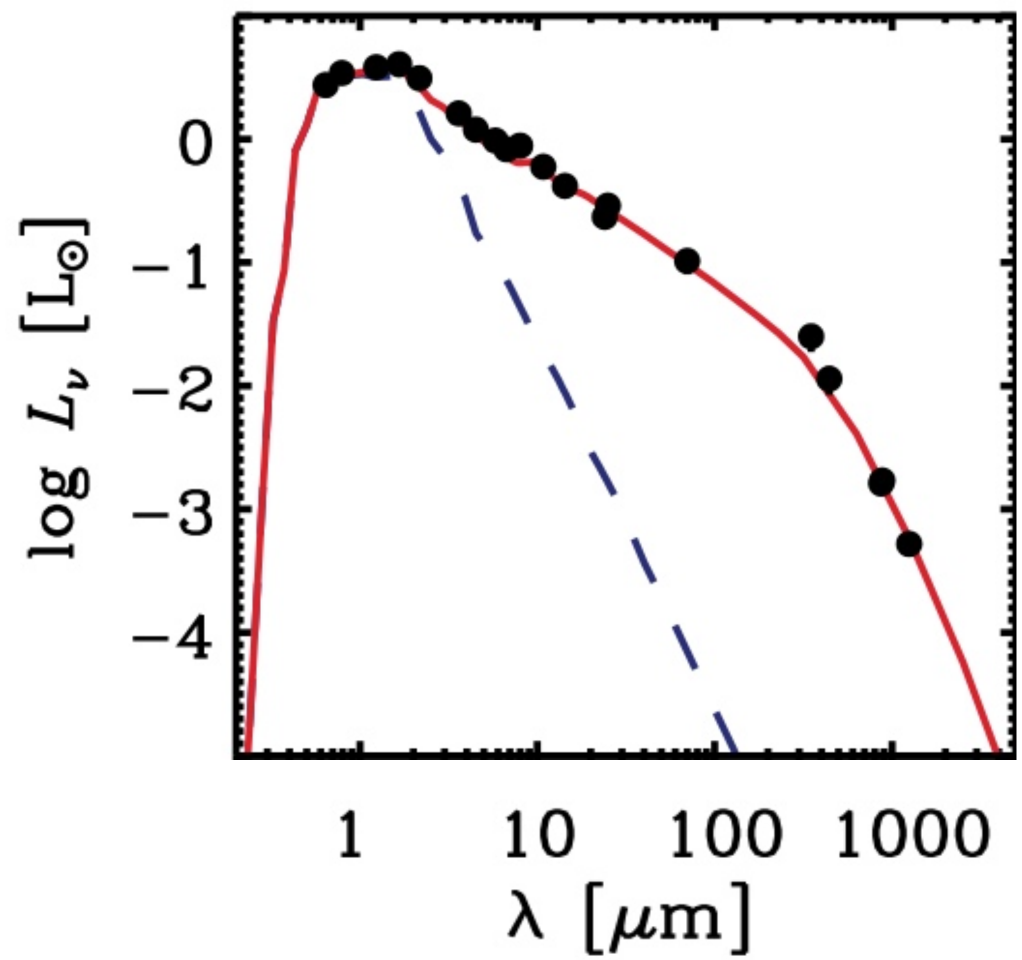
Spectral features

Before direct images of disks were made, it was noted that some stars have peculiarities in their spectra which were attributed to the presence of material in orbit.

1) Infrared excess



Armitage 2007



Andrews+ 2010

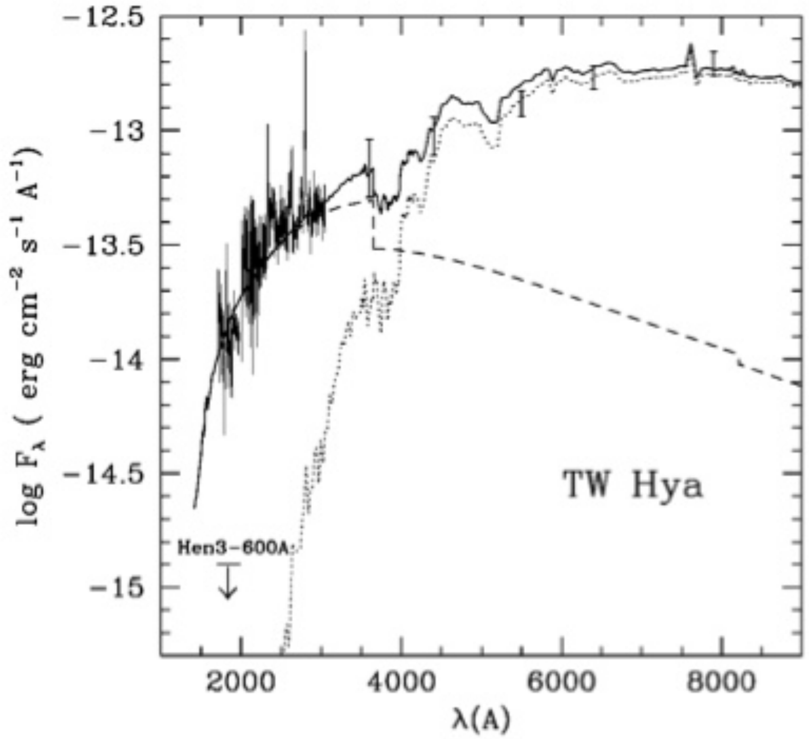
Micrometer sized dust in orbit around the star, radiating in the IR. It is heated by viscous dissipation (liberated potential energy), and by re-radiated stellar irradiation. Excess over the photospheric (stellar) contribution.

The shape of the SED in the IR is used to classify sources (Class 0, I, II, III)

Spectral features II

2) UV excess

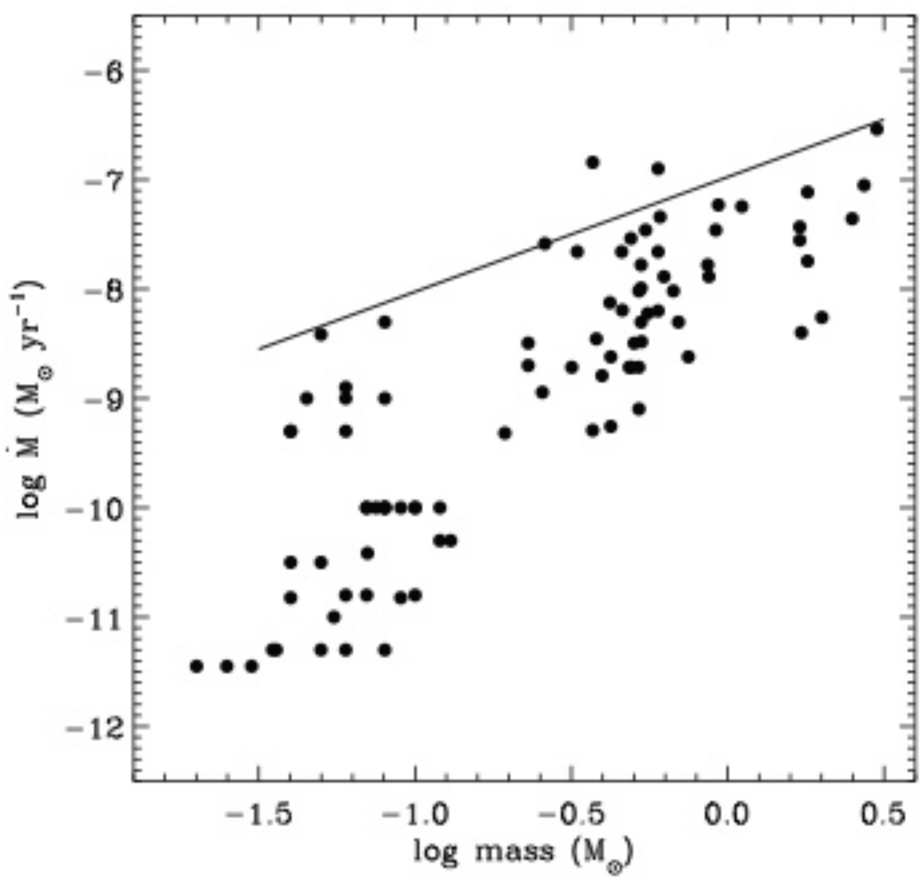
UV& Vis spectrum
solid=observed
dotted=star (model)
dashed=accretion
shock (model)



UV excess emission above the photospheric level: hot spots on the star, shock-heated by accreting material hitting the stellar surface at high velocities.

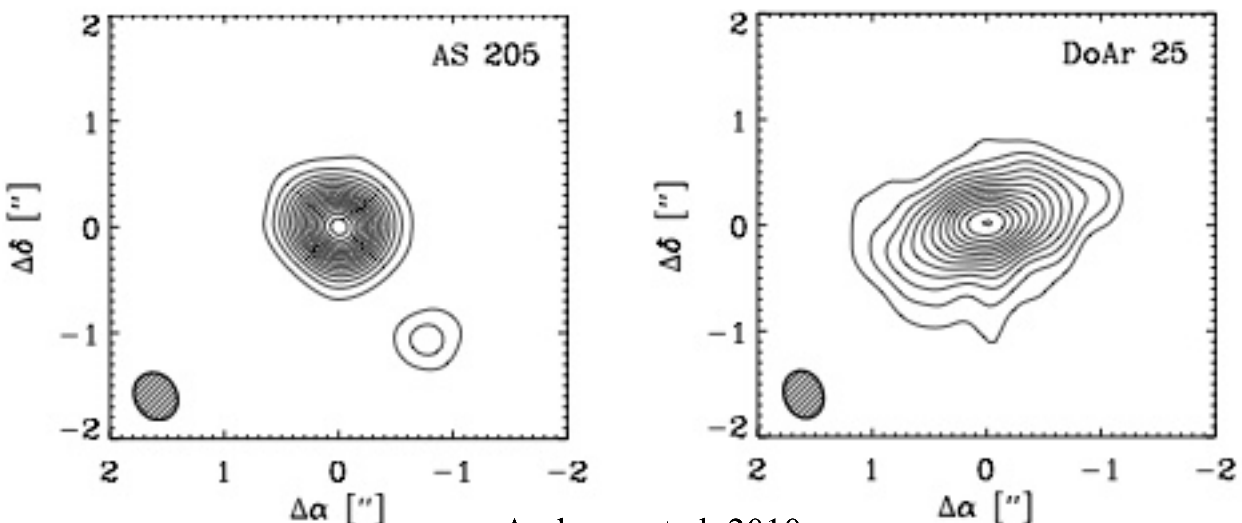
Provides an estimate of the gas accretion rate.

Muzzerolle et al. 2010



Derived accretion rates are between 10^{-11} to 10^{-6} M_{sun}/yr in bursts. The rate seems to be higher for more massive stars.

Characteristic radii and masses

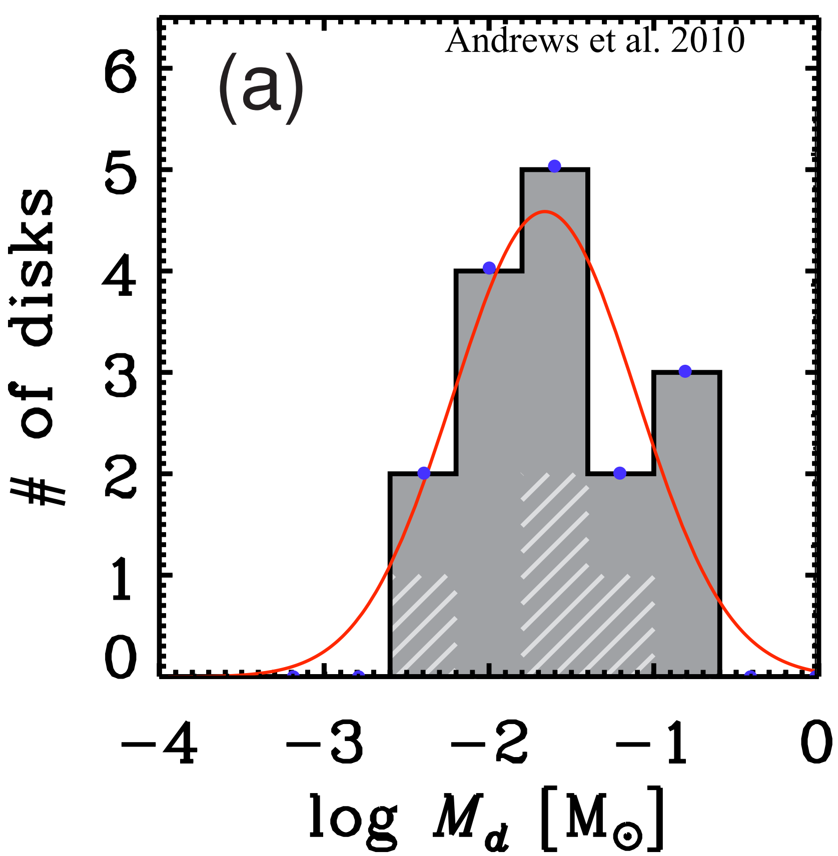


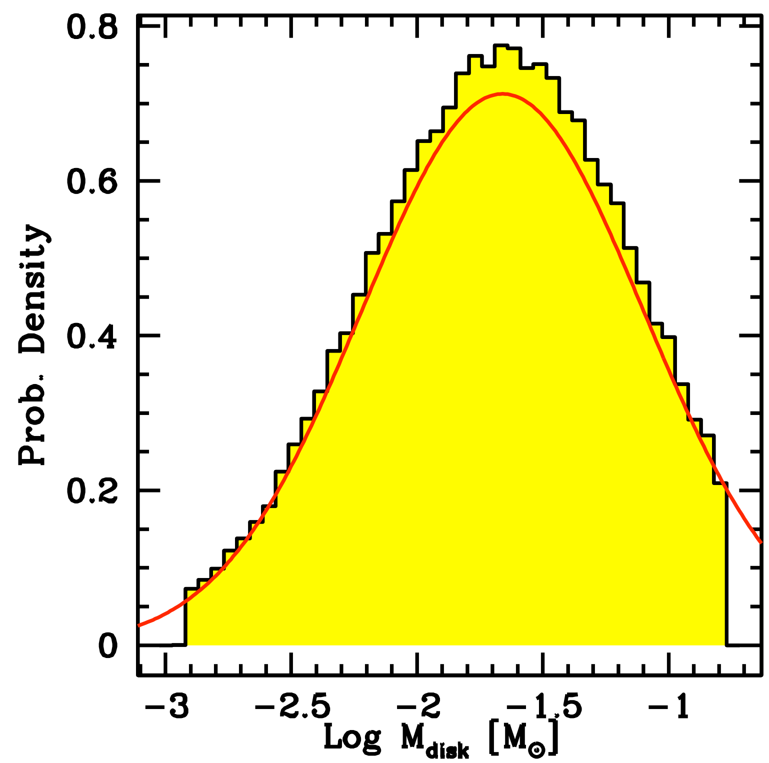
Andrews et al. 2010

Observations in the submm wavelength allow to derive the mass and spatial distribution of cold dust at tens of AU.

Thermal continuum emission from cold dust at mm and submm wavelengths (Ophiuchus nebula).

By doing such observations for many disks, one can derive rough statistics (still small numbers):





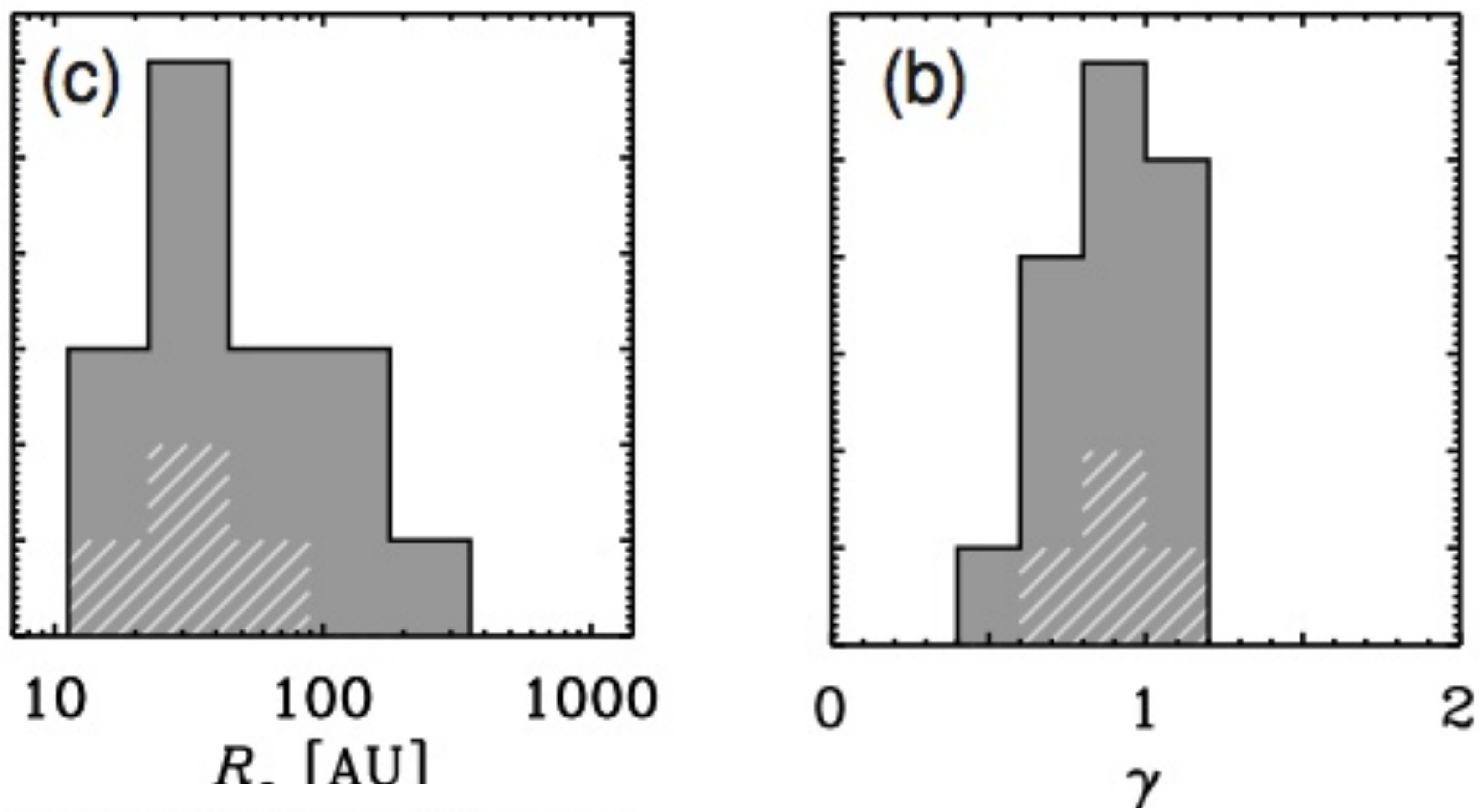
Source	μ	$M_{disk}(\mu)[M_\odot]$	σ
Fit to Taurus	-1.66	0.022	0.74
Fit to Ophiuchus	-1.38	0.042	0.49
Robinson et al. (2006)	-1.3	0.05	0.25
Ida & Lin (2004a)	-1.48	0.033	1.0

Gives total *dust* mass. Multiply by factor ~100 to get gas masses.

Disks have masses between 0.0001 to 0.1 Solar masses, following a lognormal distribution.

We will see later why disks clearly more massive than ~0.1 M_{star} are not expected (grav. instability).

Characteristic radii and masses II

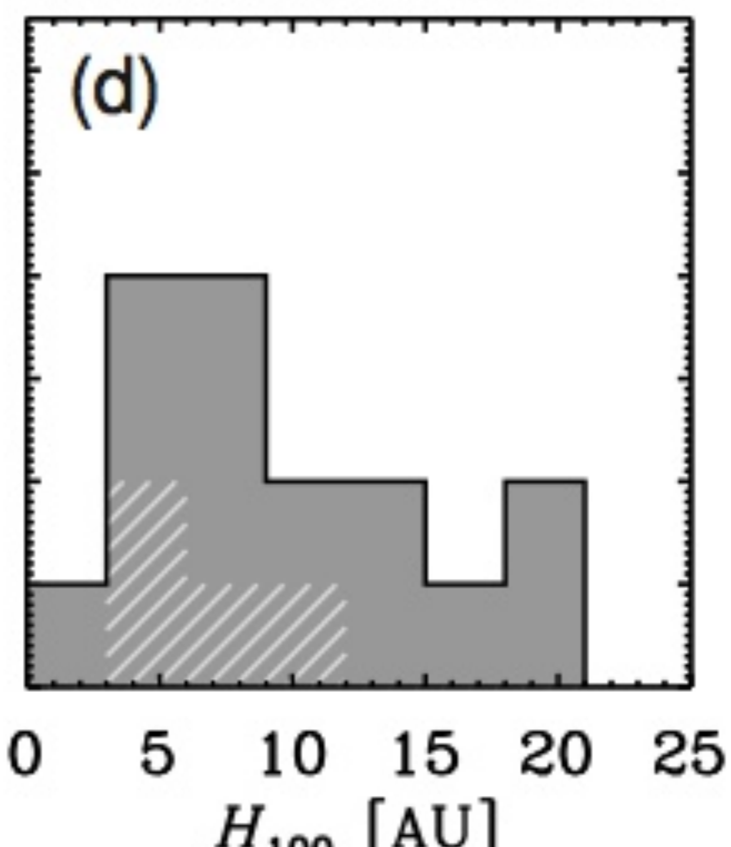


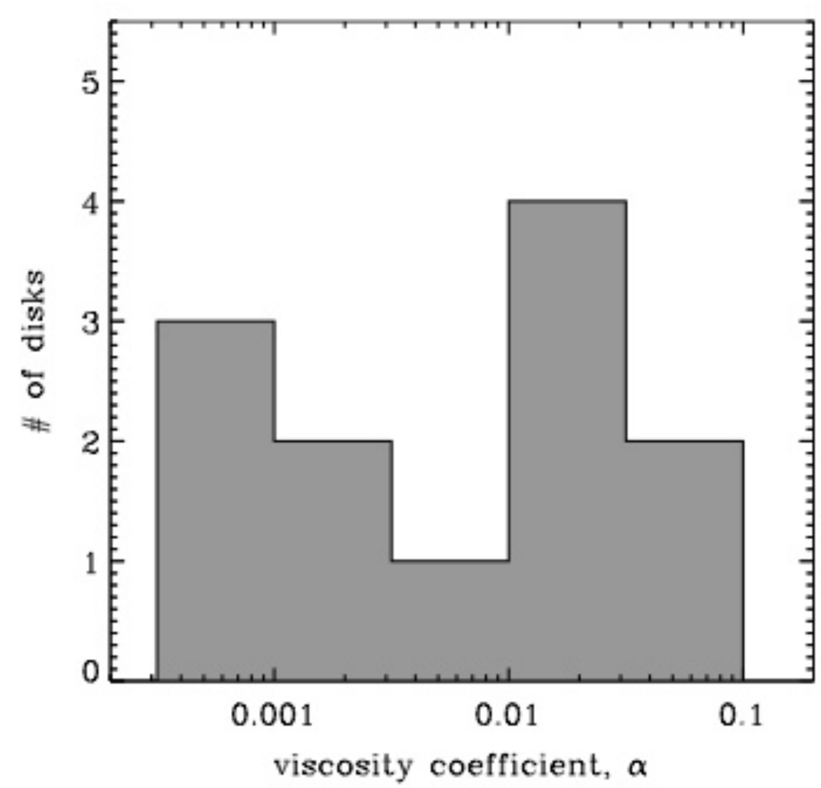
Disks have

1) characteristic radii between tens to hundreds of AU.

2) surface density power law exponents ~1

$$\Sigma(R) \propto \left(\frac{R}{R_c}\right)^{-\gamma}$$





Andrews et al. 2010

3) vertical scale heights at 100 AU of 5-20 AU.

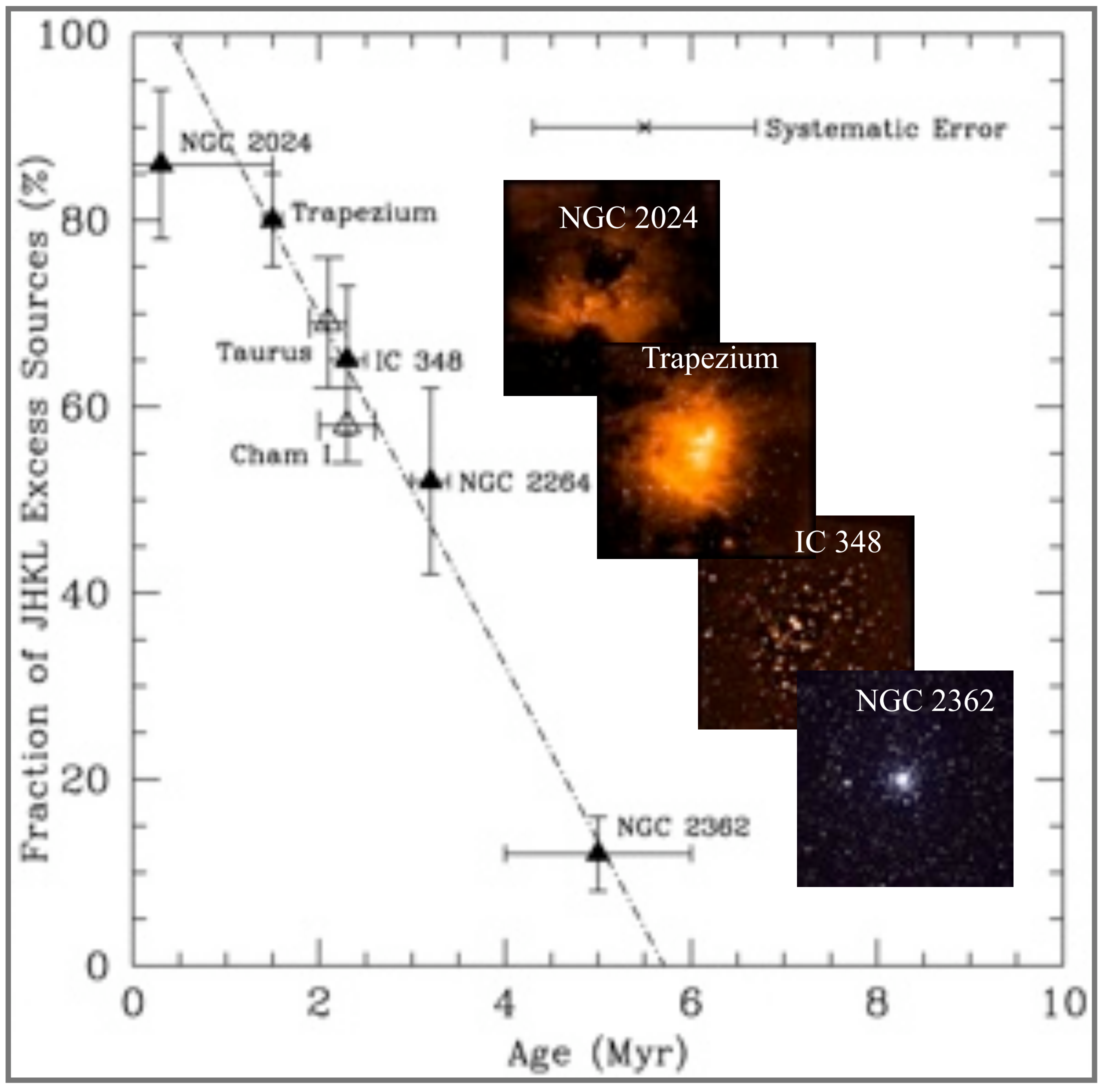
4) alpha viscosity coefficients of ~10^{-3} to 10^{-1}.

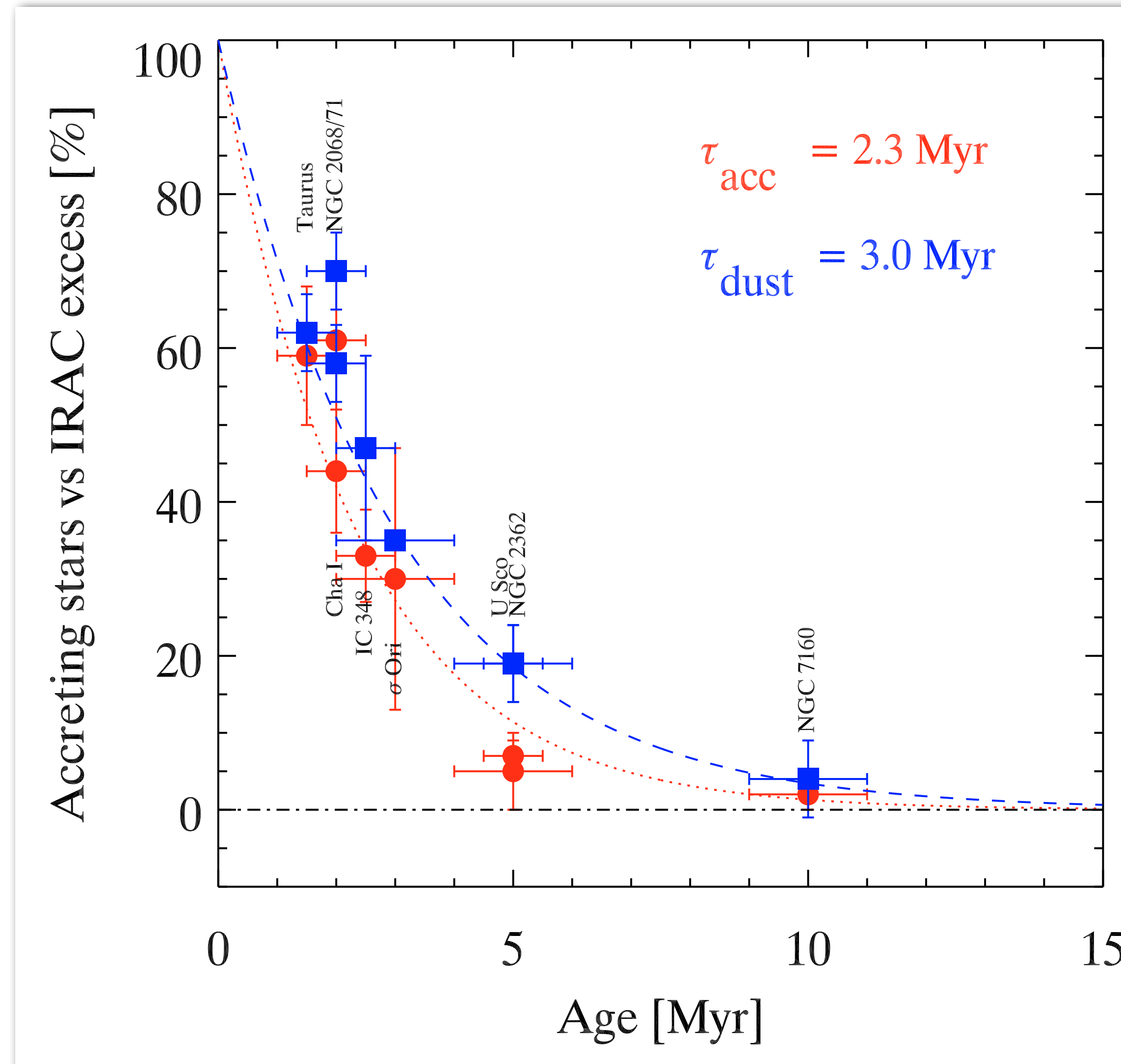
Note: the derivation of these results involves a considerable amount of assumptions and modeling...

Disk lifetimes

The disk lifetime (or more exactly, the lifetime of hot dust close to the star) can be derived by determining at which age of the star the IR excess disappears.

On then studies the fraction of stars with excess as a function of stellar cluster age.





Haisch et al. 2001, Fedele et al. 2010

L-band (3.4 μm) photometry:
- excess caused by μ-sized dust @ ~900K
... ok to < 10 AU

Notes: -mean lifetime ~3 Mrs
-longest lifetime ~10 Myrs
-similar lifetime for dust and gas

Jupiter must form within a few Myrs.

2. The minimum mass solar nebula

MMSN

The basic idea behind the minimum mass solar nebula (Weidenschilling 1977, Hayashi 1981) is to use the structure of the solar system as we observe it now to derive the structure of the disk from which the planets formed.

The solar system was formed from a gas cloud which must have had solar composition. The minimum mass of the nebula is found by replenishing the planet's (observed) composition until they reflect solar abundance. Doing so yields the minimum possible mass hence the term MMSN.

The key ingredient in the analysis is the dust to gas ratio:

$$\zeta = \frac{\text{mass of dust (high Z material)}}{\text{mass of gas}}$$

Note that not all high Z material condenses at temperatures in the nebula. The dust to gas ratio is a function temperature, and thus of the distance from the star.

Ice line

In hot regions close to the star, only refractory "rocky" elements (iron, silicates) remain solid. In colder regions further away from the star both refractory and volatile material (ices of water, methane, ammonia) are condensed.

The two regions are devised by the so called ice line, at T~170 K, at about 2.7 AU. Probably it is no coincidence that this lies between Mars (terrestrial region) and Jupiter (giants).

MMSN II

Solar system inventory

1)Cold region: T<170 K (a>a_{ice}) $\zeta_{r+i} \approx \frac{1}{60}$

2)Hot region: T>170 K (a<a_{ice}) $\zeta_r \approx \frac{1}{240}$

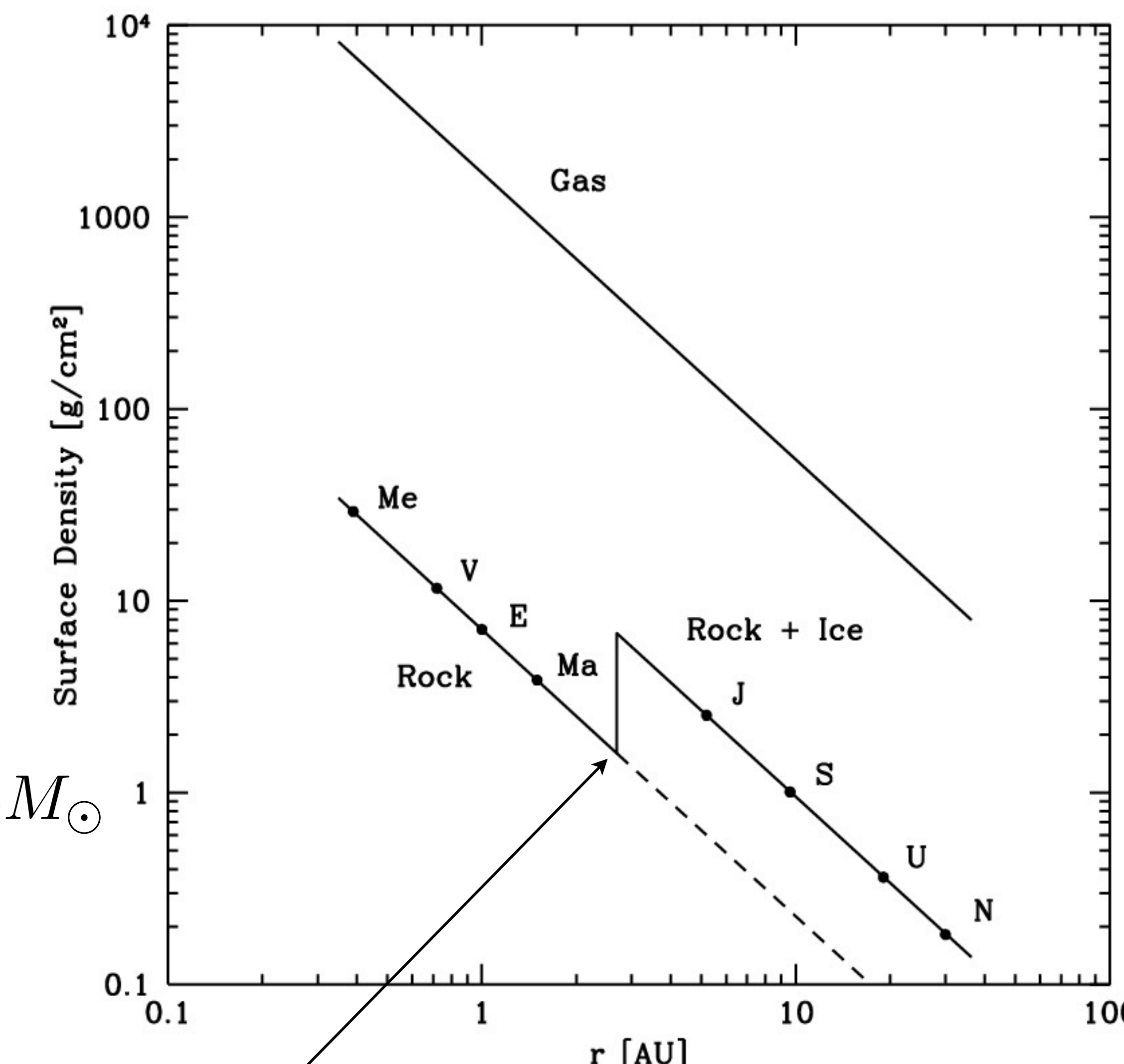
which means a jump by a factor 4 at the iceline.

We can estimate the minimum mass as follows:
-high-Z in giant planets: ~80 Earth masses
-high-Z in terrestrial planets: ~2 Earth masses

$$MMSN \approx \frac{2}{\zeta_r} + \frac{80}{\zeta_{r+i}} \approx 5300 M_\oplus \approx 16.6 M_{jup} \approx 0.016 M_\odot$$

i.e. only 1.6 %.

This is clearly the minimum since it assumes a 100% efficiency in incorporating matter into planets.



a_{ice}

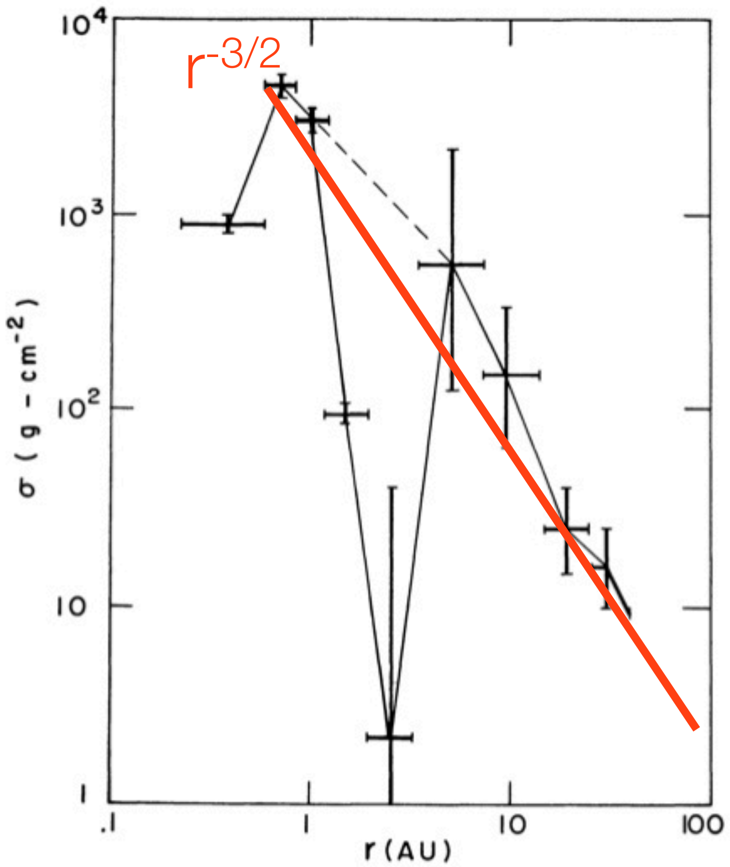
Ruden 2000

Hayashi 1981

Note: the position of the iceline is quite uncertain. It likely moves in time.

MMSN III

A more detailed analysis can be made by spreading each planet out to its nearest neighbors.



Weidenschilling 1977

	Mass ($M_\oplus$)	Fe mass fraction	Solar comp. mass ($M_\oplus$)	Zone limits (AU)	Surface density (g cm^{-2})
Mercury	0.053	0.62	27	0.22	880
Venus	0.815	0.35	235	0.56	4750
Earth	1	0.38	320	0.86	3200
Mars	0.107	0.30	27	1.26	95
Asteroids					
present	0.0005	0.25	0.1	2.0	0.13
original	0.15?		30		40
				3.3	
Jupiter	318	–	600–12 000	7.4	120–2400
Saturn	95	–	1000–6000	14.4	55–330
Uranus	14.6	–	700–2000	24.7	15–40
Neptune	17.2	–	800–2000	35.5	10–25

The zone limits are simply the arithmetic mean between adjacent orbits.

$$\Sigma_{\rm r}(r) = 7\,{\rm g\,cm^{-2}}\left(\frac{r}{\rm AU}\right)^{-3/2} \quad \text{for} \quad 0.35 < r/{\rm AU} < 2.7$$

$$\Sigma_{\rm r+i}(r) = 30\,{\rm g\,cm^{-2}}\left(\frac{r}{\rm AU}\right)^{-3/2} \quad \text{for} \quad 2.7 < r/{\rm AU} < 36$$

$$\Sigma_{\rm g}(r) = 1700\,{\rm g\,cm^{-2}}\left(\frac{r}{\rm AU}\right)^{-3/2} \quad \text{for} \quad 0.35 < r/{\rm AU} < 36$$

The total mass between 0.35 and 36 AU is then

$$M = \int_{r_0}^{r_1} 2\pi r \Sigma_{\rm r+i+g}(r){\rm d}r \approx 0.013 M_\odot$$

Note -this assumes planets do not migrate.
-lies in the domain of observed protoplanetary disk masses.
-the power law slope is 1.5. This is steeper than for the observed disks (~1).
-the actual mass was probably a few times higher (~3 to 4).

Temperature?

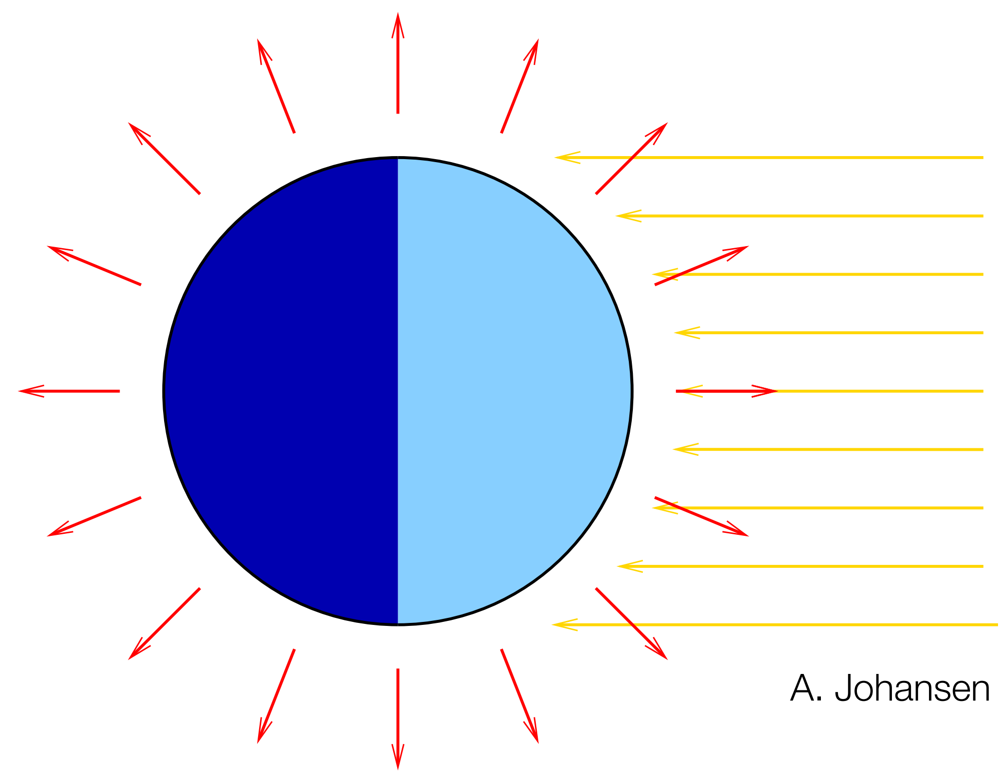
- Much more difficult to determine is the temperature in the solar nebula.
- Two dominant energy sources:
  - -solar irradiation
  - -viscous heating
- Simplest case: only solar irradiation in optically thin nebula. This is a reasonable assumption in the outer parts of the disk and/or at low accretion rates, but not in the inner parts and/or at high accretion rates.

$$F_\odot = \frac{L_\odot}{4\pi r^2}$$

$$P_{\rm in} = \pi \epsilon_{\rm in} R^2 F_\odot \qquad P_{\rm out} = 4\pi R^2 \epsilon_{\rm out} \sigma_{\rm SB} T_{\rm eff}^4$$

$$T_{\rm eff} = \left[\frac{F_\odot}{4\sigma_{\rm SB}}\right]^{1/4}$$

$$\boxed{T_{\rm equi} = 280\,{\rm K}\left(\frac{a}{1{\rm AU}}\right)^{-\frac{1}{2}}\left(\frac{M_*}{M_\odot}\right)}$$



for solar like stars on the main sequence where L goes as $\sim M^4$

3. *Disk structure*

Disk structure

The rotation, density, temperature in the protoplanetary disk are very important for the formation of planets: They are the initial and boundary conditions of planet formation.

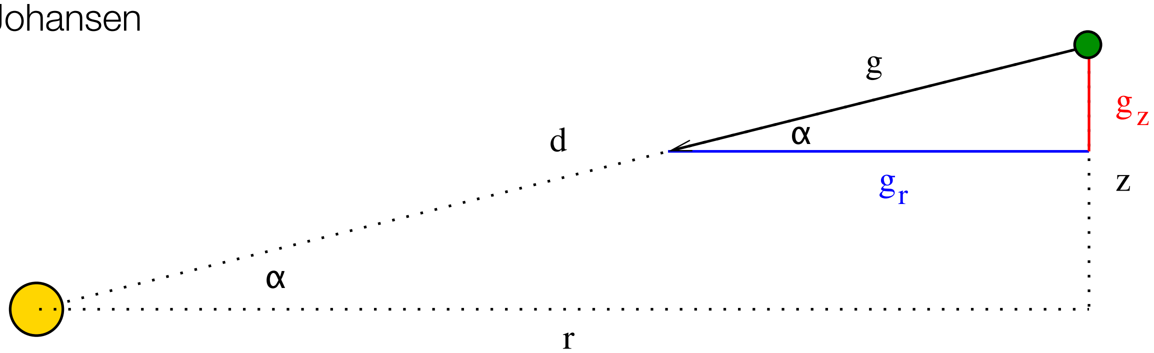
From what we have seen, protoplanetary disks are generally believed to have relatively small mass, typically a few percents or the mass of the star. We will see further down why very massive disks are not expected to exist (for a long time). In what follows, we assume that the disk is axisymmetric and adopt cylindrical coordinates r, ϕ, z.

3.1 Vertical structure

Vertical disk structure

The disk's vertical hydrostatic equilibrium is given in good approximation by:

A. Johansen



$$g_z = g \sin\alpha = \frac{GM_s}{d^2} \sin\alpha \qquad \sin\alpha = \frac{z}{d}$$

$$g_z = \frac{GM_s}{d^2}\frac{z}{d} \approx \frac{GM_s}{r^2}\frac{z}{r} \qquad z<<d$$

The equation of vertical hydrostatic equilibrium is then with $\Omega = \sqrt{\frac{GM_s}{r^3}}$

$$\boxed{\frac{1}{\rho}\frac{\partial p}{\partial z} = -\frac{GM_s}{r^2}\left(\frac{z}{r}\right) = -\Omega^2 z}$$

M_S: mass of the star

We have assumed that the disk is thin (z<<r) and that its own mass is negligible compared to the star's mass.

We can obtain an order of estimate of the thickness of the disk H by recalling that the pressure is given by

$$p = \frac{k}{\mu m_H}\rho T = c^2 \rho$$

then we can estimate from the hydrostatic equilibrium

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{c^2}{\rho}\frac{\partial \rho}{\partial z} \sim \frac{1}{\rho}\frac{c^2 \rho}{H} = \Omega^2 H \rightarrow \boxed{c = H\Omega}$$

Vertical disk structure II

Replacing Ω by v_{rot}/r we obtain $$\frac{H}{r} = \frac{c}{v_{rot}}$$ H/r=h=aspect ratio of the disk

Observations as well as theoretical considerations suggest that H/r is relatively small, typically $H \leq 0.1\, r$. This implies that $c \ll v_{rot}$ and therefore protoplanetary disks are said to be cold. In other words, disks are mainly rotationally supported, pressure gradients are only of secondary importance.

Vertically isothermal structure

For a vertically isothermal disk we can solve the eq. of hydrostatic equilibrium:

$$\frac{c^2}{\rho}\frac{\partial \rho}{\partial z} = c^2 \frac{\partial \ln \rho}{\partial z} = -\Omega^2 z$$

$$\frac{\partial \ln \rho}{\partial z} = -\frac{\Omega^2}{c^2} z = -\frac{z}{H^2}$$

Integrating on both sides and $H = \frac{c}{\Omega}$

$$\ln \rho = \ln \rho_0 - \frac{z^2}{2H^2}$$

$$\boxed{\rho(z) = \rho_0 e^{-\frac{1}{2}\left(\frac{z}{H}\right)^2}}$$

$\rho_0 = \rho(r, z = 0)$ = midplane density

Vertical disk structure III

Because disks are thin, it is convenient to use vertically averaged quantities such as surface density **Σ**:

$$\Sigma = \int_{-\infty}^{\infty} \rho_0 e^{-\frac{1}{2}\left(\frac{z}{H}\right)^2} dz = \sqrt{2} H \rho_0 \int_{-\infty}^{\infty} \rho_0 e^{-x} dx = \sqrt{2\pi} \rho_0 H$$

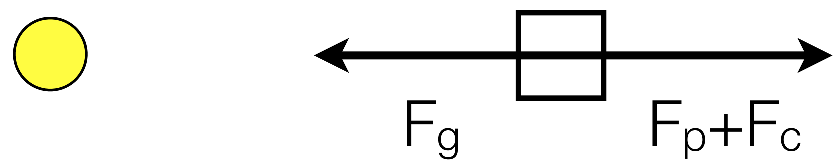
$$\boxed{\rho_0 = \frac{\Sigma}{\sqrt{2\pi} H} \approx \frac{\Sigma}{2H}}$$

For a MMSN like this with H/r~0.05, we find a midplane density of about 10^{-9} g/cm^3 at 1 AU.

3.2 Radial structure

Radial disk structure I

In the radial direction, the gravitational attraction by the star is counteracted by the pressure force (only for the gas) and the centrifugal force.



In the radial direction, the combined hydrostatic and centrifugal equilibrium is given by

$$\boxed{\underbrace{\frac{v^2}{r} = \frac{GM_s}{r^2}}_{\text{solids}} + \frac{1}{\rho}\frac{\partial p}{\partial r}}$$

As the pressure is decreasing towards the exterior, we have $\partial p/\partial r < 0$. We thus see from the equation that $v<v_{keper}$. The gas is going around the sun slower than the solids (dust, planets). This is a consequence of the additional pressure support.

These pressure gradients are however small, as already mentioned. We can estimate them as follows

$$\frac{v^2}{r} \approx \Omega^2 r - \frac{c^2}{r} = \Omega^2 r\left(1 - \frac{c^2}{r^2\Omega^2}\right) = \Omega^2 r\left(1 - \frac{H^2}{r^2}\right)$$

$$\Omega = \sqrt{\frac{GM_s}{r^3}}$$

$$p = c^2\rho$$

$$H = \frac{c}{\Omega}$$

Multiplication with r gives

Radial disk structure II

$$v = v_{Kep}\left[1 - \left(\frac{H}{r}\right)^2\right]$$

This shows that the speed of gas is the same as the one of the solids i.e. the Keplerian velocity to a factor $-(H/r)^2$. As H/r is already small, this is a very small quantity. Let us now write the velocity as $v=\Omega r - \Delta v$ where the first term is the Keplerian speed and the other a correction term. Inserting this in the equilibrium equation and recalling that

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{c^2}{r}\frac{\partial lnp}{\partial lnr}$$

we obtain for the velocity perturbation:

$$\Delta v = \frac{1}{2}\left|\frac{\partial lnp}{\partial lnr}\right|\left(\frac{H}{r}\right)c$$

Since (H/r) << 1 and that the pressure derivative with density is never very big, we conclude that $\Delta v \ll c$. Since c itself is already considerably smaller than Ωr, we conclude that $\Delta v \ll \Omega r$.

In other words, the departure from Keplerian speed is small and the disks are said to be nearly Keplerian. This is a property of thin accretion disks, since thin means a small pressure gradient, and thus a small pressure support.

Still, as we will see, has the difference between the speed of solids and the speed of the gas important implications for the aerodynamics in the nebula (e.g. dust drift).

3.3 Application for the MMSN

Application for the MMSN

With the results form the last chapters, we can set up a simple model based on the MMSN:

$$\Sigma(r) = 1700\,\mathrm{g\,cm^{-2}} \left(\frac{r}{\mathrm{AU}}\right)^{-3/2}$$

$$T(r) = 280\,\mathrm{K} \left(\frac{r}{\mathrm{AU}}\right)^{-1/2}$$

$$\rho(r,z) = \frac{\Sigma(r)}{\sqrt{2\pi}H(r)} \exp\left[-\frac{z^2}{2H(r)^2}\right]$$

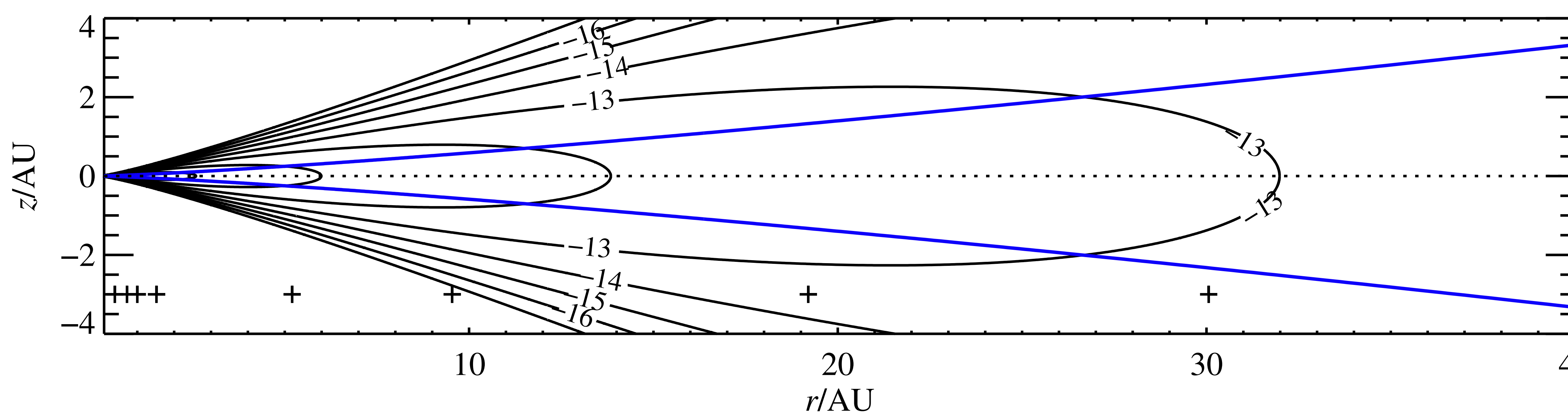
$$H(r) = \frac{c_\mathrm{s}}{\Omega_\mathrm{K}} \qquad \Omega_\mathrm{K} = \sqrt{\frac{GM}{r^3}}$$

$$c_\mathrm{s} = 9.9 \times 10^4\,\mathrm{cm\,s^{-1}} \left(\frac{2.34}{\mu}\frac{T}{280\,\mathrm{K}}\right)^{1/2}$$

$$H/r = \frac{c_\mathrm{s}}{v_\mathrm{K}} = 0.033 \left(\frac{r}{\mathrm{AU}}\right)^{1/4}$$

- Mid-plane gas density varies from 10^{-9} g/cm^3 in the terrestrial planet formation region down to 10^{-13} g/cm^3 in the outer nebula
- Blue line (not a straight line) shows location of z = H: The aspect ratio H/r increases with r, so solar nebula is said to be (slightly) "flaring".
- The most serious drawback of this model is the temperature structure (passive irradiation only). Still, this model is quite of used for first estimates.

Density contours



A. Johansen

4. Stability of disk

Stability of an uniformingly rotating sheet

We consider the stability of a fluid disk or sheet of zero thickness and with constant surface density $\mathbf{\Sigma}_0$ and temperature T. The sheet is located in the z=0 plane and is rotating with constant angular velocity $\Omega = \Omega_{\mathbf{z}}$. The equations governing the evolution of the sheet in the rotating frame of reference are:

$$(1)\ \frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \mathbf{v}) = 0$$

$$(2)\ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\Sigma}\nabla p - \nabla \phi - 2(\mathbf{\Omega} \times \mathbf{v}) + \Omega^2 (x\mathbf{e_x} + y\mathbf{e_y})$$

$$a_{cor} = 2(\overrightarrow{v} \times \overrightarrow{\Omega})$$
$$a_{cent} = -\overrightarrow{\Omega} \times (\overrightarrow{\Omega} \times \overrightarrow{r})$$
$$\overrightarrow{\Omega} = (0, 0, \Omega)$$

$$(3)\ \Delta \phi = 4\pi G \Sigma \delta(z)$$

(mass is in the z plane)

Because the sheet is assumed to be isothermal, the vertically integrated pressure is given by:

$$p = p(\Sigma) = c^2 \Sigma$$

In the unperturbed state, we assume an equilibrium solution given by:

$$\Sigma = \Sigma_0;\ \mathbf{v} = 0;\ p = p_0 = c^2 \Sigma_0 \overset{(2)}{\longrightarrow} \nabla \phi_0 = \Omega^2 (x\mathbf{e_x} + y\mathbf{e_y});\ \Delta \phi_0 = 4\pi G \Sigma_0 \delta(z)$$

(rot. frame!)

Note that since the sheet is uniform, the gradient of the potential cannot really lie in the (x,y) plane as indicated by the above equation. So the adopted initial conditions are not strictly speaking a solution to the set of equations. Remember the Jeans swindle....

Stability of an uniformingly rotating sheet II

We now introduce small perturbations in the equilibrium quantities:

$$\Sigma(x,y,t) = \Sigma_0 + \epsilon\Sigma_1(x,y,t);\;\; \mathbf{v}(x,y,t) = \epsilon\mathbf{v_1}(x,y,t);\;\; \ldots\;\; ;\;\; \epsilon \ll 1$$

We keep only the terms linear in **ε**. We obtain the linearized equations for the evolution of the perturbations:

$$(4)\;\; \frac{\partial\Sigma_1}{\partial t} + \Sigma_0\nabla\cdot(\mathbf{v_1}) = 0$$

$$(5)\;\; \frac{\partial\mathbf{v_1}}{\partial t} = -\frac{c^2}{\Sigma_0}\nabla\Sigma_1 - \nabla\phi_1 - 2(\mathbf{\Omega}\times\mathbf{v_1})$$

$$(6)\;\; \Delta\phi_1 = 4\pi G\Sigma_1\delta(z)$$

As in the stability analysis for the Jeans mass, we now look for solutions of the type:

$$\Sigma_1(x,y,t) = \Sigma_a e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

$$\mathbf{v_1}(x,y,t) = (v_{ax}\mathbf{e_x} + v_{ay}\mathbf{e_y})e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

Five unknowns

$$\phi_1(x,y,t) = \phi_0 e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

Stability of an uniformingly rotating sheet III

To simplify but without loss of generality, we chose the x-axis to be parallel to the propagation of the perturbation k. In other words, we chose: $\mathbf{k} = k\mathbf{e_x}$

Consider first the Poisson equation. For points outside the sheet, we must have $\Delta\phi_1 = 0$ whereas for points in the z=0 plane we have the solution given above. The only function that satisfies these constraints and that vanishes at infinity is given by:

$$\phi_1 = \frac{2\pi G \Sigma_a}{|k|} e^{-i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

This solution substituted back into the linearized equation yields:

$$(7) \quad -i\omega\Sigma_a = -ik\Sigma_0 v_{ax}$$

$$(8) \quad -i\omega v_{ax} = \frac{c^2 ik\Sigma_a}{\Sigma_0} + \frac{2\pi G i\Sigma_a k}{|k|} + 2\Omega v_{ay}$$

$$-i\omega v_{ay} = -2\Omega v_{ax}$$

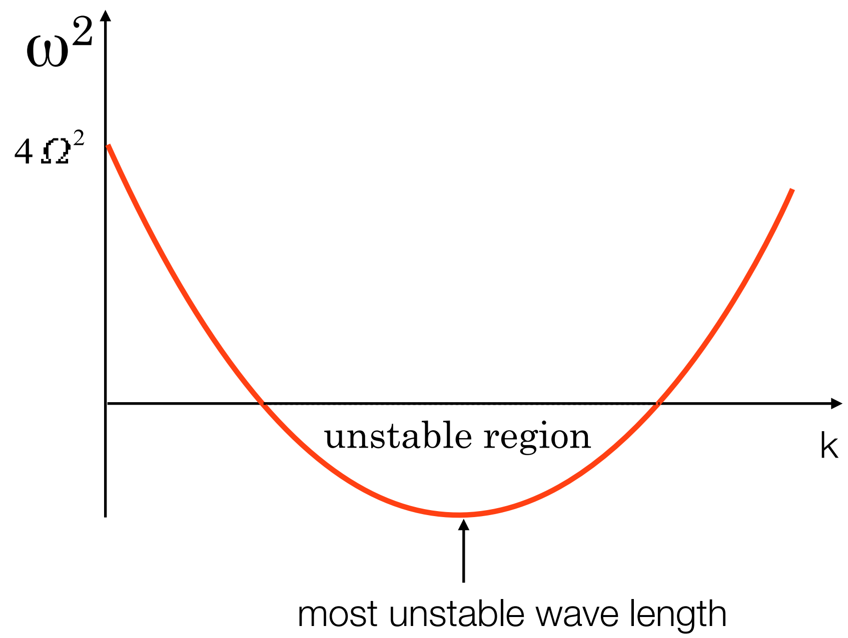
This set of equations can be written in form of a matrix. It has a non trivial solution only when

$$\boxed{\omega^2 = 4\Omega^2 - 2\pi G\Sigma_0|k| + k^2c^2 \geq 0}$$

Dispersion relation for the uniformingly rotating sheet.

Stability of an uniformingly rotating sheet IV

$$\omega^2 = 4\Omega^2 - 2\pi G\Sigma_0|k| + k^2c^2 \geq 0$$



Note: - long wavelengths (small k) are stabilized by rotation
- short wavelength (large k) are stabilized by pressure

This result is equivalent to the one found for the rotating cloud.

Overall stability is achieved if $\omega(k) \geq 0$ everywhere, i.e. the minimum -determined by setting the derivative equal zero - must still be positive. This condition implies that the condition necessary for stability of the uniformly rotating sheet is given by the so called Toomre criterion

$$\boxed{Q = \frac{2c\Omega}{\pi G\Sigma_0} > 1}$$

stability criterion for the uniformly rotating sheet
cold, massive disks are instable

A very similar criterion can be derived for the differentially rotating sheet. In this case, we can show that stability is given by:

$$Q = \frac{2c\kappa}{\pi G\Sigma_0} > 1$$

with the epicyclic frequency defined by: $\kappa^2 = r\frac{d\Omega}{dr} + 4\Omega^2$

The same criterion also applies for spiral galaxies.

Q_{Toomre} *from intuitive arguments*

In the last paragraph, we have derived Q from perturbation theory. An intuitive, order of magnitude derivation is found by comparing the involved energies.

Energies per unit mass:

Gravity $$\left|E_g\right| \approx \frac{GM}{R} = \frac{G\pi\Sigma R^2}{R} = G\pi\Sigma R \propto R$$

Rotation $$\left|E_r\right| \approx \tfrac{1}{2}(\Omega R)^2 \propto R^2$$

Thermal $$\left|E_r\right| \approx \tfrac{3}{2}T \approx {c_s}^2 \propto \text{constant}$$

For instability, gravity must be dominant both over the rotational and the thermal energy:

$$\frac{E_{therm}}{\left|E_{grav}\right|}\frac{E_{rot}}{\left|E_{grav}\right|} < 1$$

Plugging in the expression above yields

$$\frac{c_s^2}{G\pi\Sigma R}\frac{\frac{1}{2}\Omega^2 R^2}{G\pi\Sigma R} = \frac{1}{2}\left(\frac{c_s\Omega}{\pi\Sigma G}\right)^2$$

This corresponds, to numerical constants of order unity, to the same as the conditions we derived before, i.e. $Q = \frac{2c\Omega}{\pi G\Sigma_0}$

4.1 Formation by direct collapse

Formation by direct collapse

The Toomre criterium describes the stability of the disk agains the development of axi-symmetric radial rings. Hydrodynamical simulations show that disks become unstable to non-axisymmetric perturbations (spiral waves) already at slightly higher Q values of about Q_{crit} = 1.4 to 2.

Cooling criterion

In order to additionally allow *fragmentation into bound clumps*, the timescale τ_{cool} on which a gas parcel in the disk cools and thus contracts must be short compared to the shearing timescale, on which the clump would be disrupted otherwise, which is equal the orbital timescale $\tau_{orb} = 2\pi/\Omega$. This means that

$$\boxed{\tau_{cool}\Omega \lesssim \xi}$$

Here, ξ is of order unity (Gammie, 2001). If this conditions is not fulfilled, only spiral waves develop leading to a gravoturbulent disk. The spiral waves efficiently transport angular momentum outwards, and therefore let matter spiral rapidly inwards to the star. This process liberates gravitational binding energy, which increases the temperature, and reduces the gas surface temperature, which both reduce Q, so that the disk evolves to a steady state of marginal instability only, without fragmentation.

This also explains why no disk with a mass similar as the central mass exist (at least not for a long time).

Only if *both* criteria are fulfilled, the formation of self-gravitating, bound gas clumps can occur.

Formation by direct collapse

For thin disks we can write the sound speed as: $c = \frac{H}{r} v_{rot}$

$$H = \frac{c}{\Omega}$$

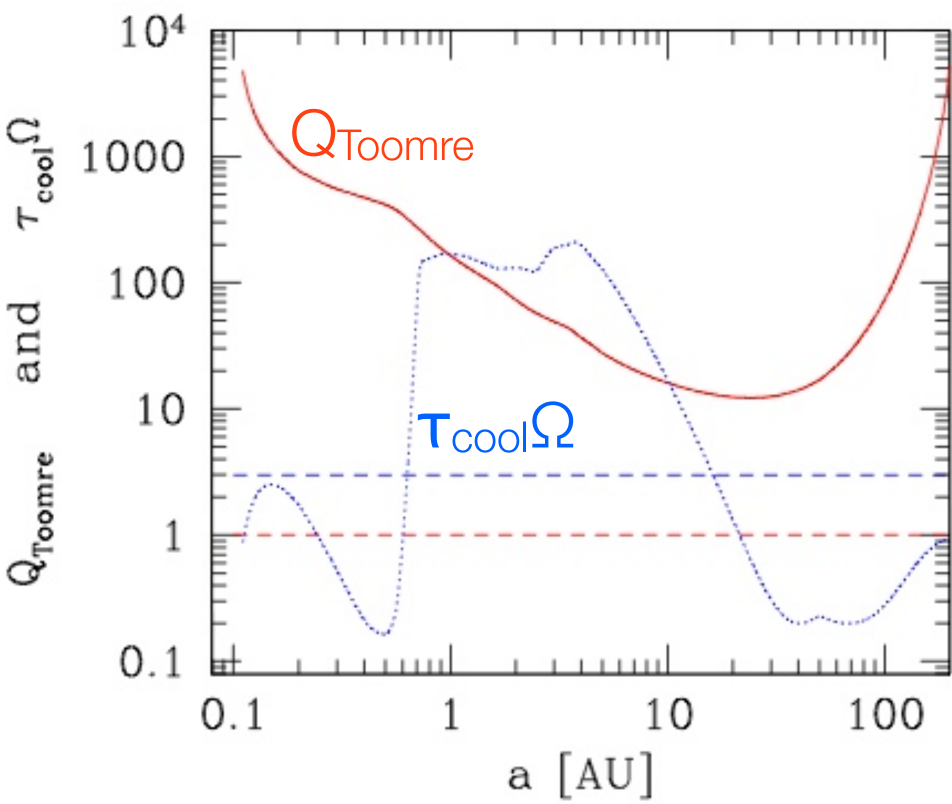
$$\Omega = \frac{2\pi}{1 \text{ year}} \sim 2 \times 10^{-7} 1/s$$

For a MMSN disk we thus find at 1 AU

$$Q = \frac{H}{r} \frac{2\Omega^2 r}{\pi G \Sigma_0} = \frac{H}{r} \frac{(2 \times 10^{-7})^2 \cdot 1.5 \times 10^{13}}{6.67 \times 10^{-8} \cdot 1700} \approx \frac{H}{r} \cdot 3400 \gg 1$$

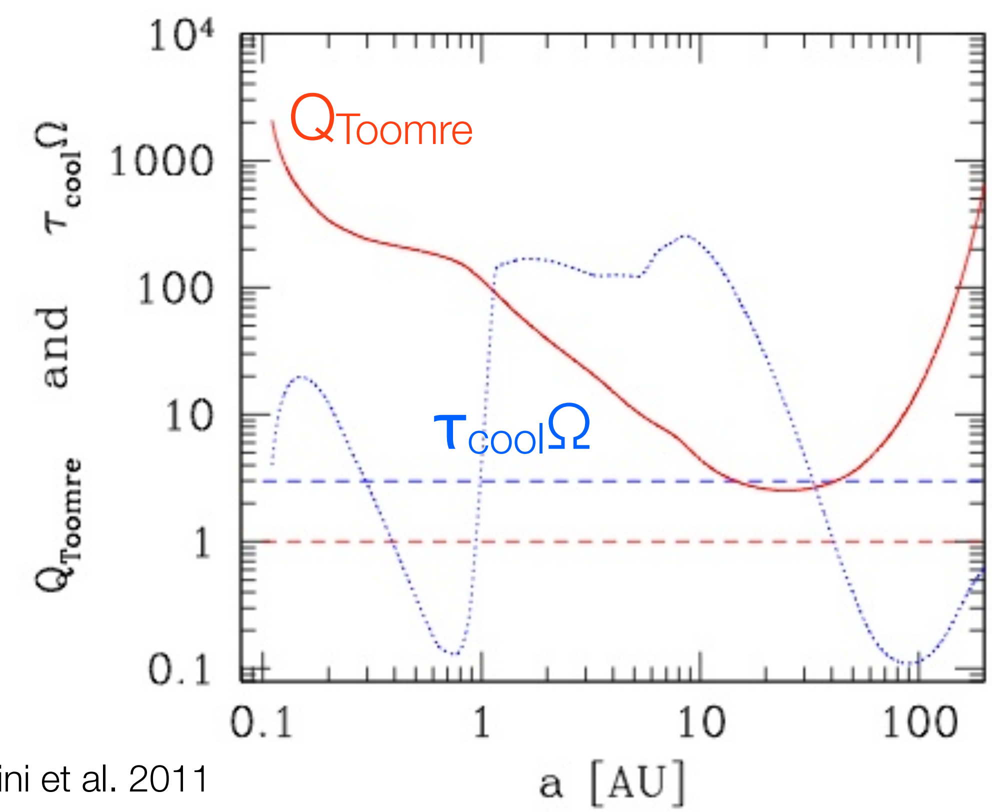
The disk corresponding to the MMSN is therefore very stable. Planets do not form through direct collapse, at least not at 1 AU.

In a more detailed disk model (alpha disk with irradiation), Q reaches minimum at about 30 AU. $\tau_{cool}\Omega$ has a peculiar form coming from the variation of the opacity with temperature.



M_{disk}=0.024 M_{sun}

Mordasini et al. 2011



M_{disk}=0.1 M_{sun}

Note:

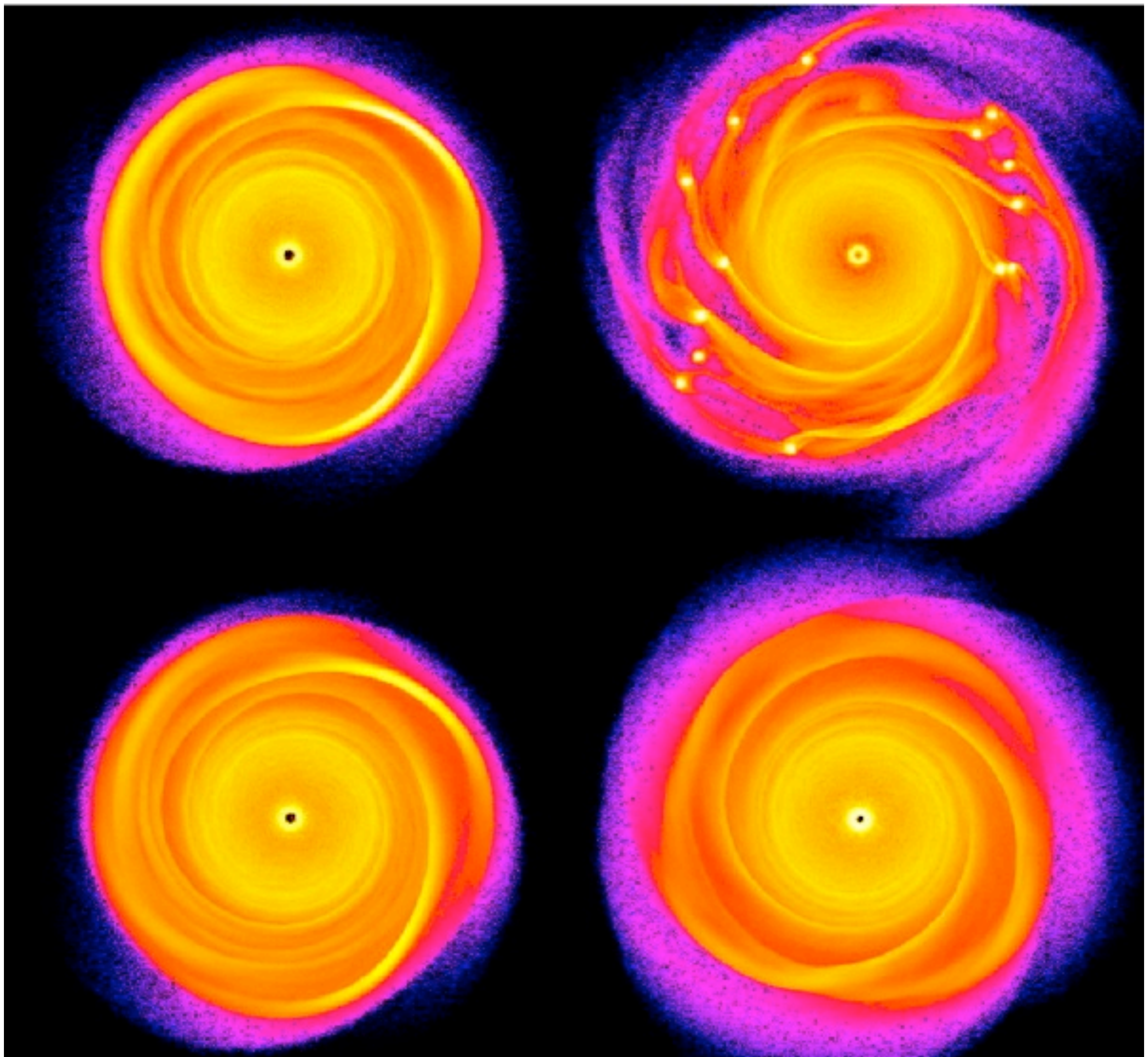
The more massive the disk, the smaller Q.

But it nowhere falls below 1, so fragmentation does not happen.

4.2 Numerical simulations

Numerical simulations

Early simulations assumed a locally isothermal equation of state. This corresponds to an infinitely short cooling timescale, as it means that when the density increases, the temperature remains constant, as compressive work is immediately radiated away. This is of course not correct ...



Mayer et al. 2004

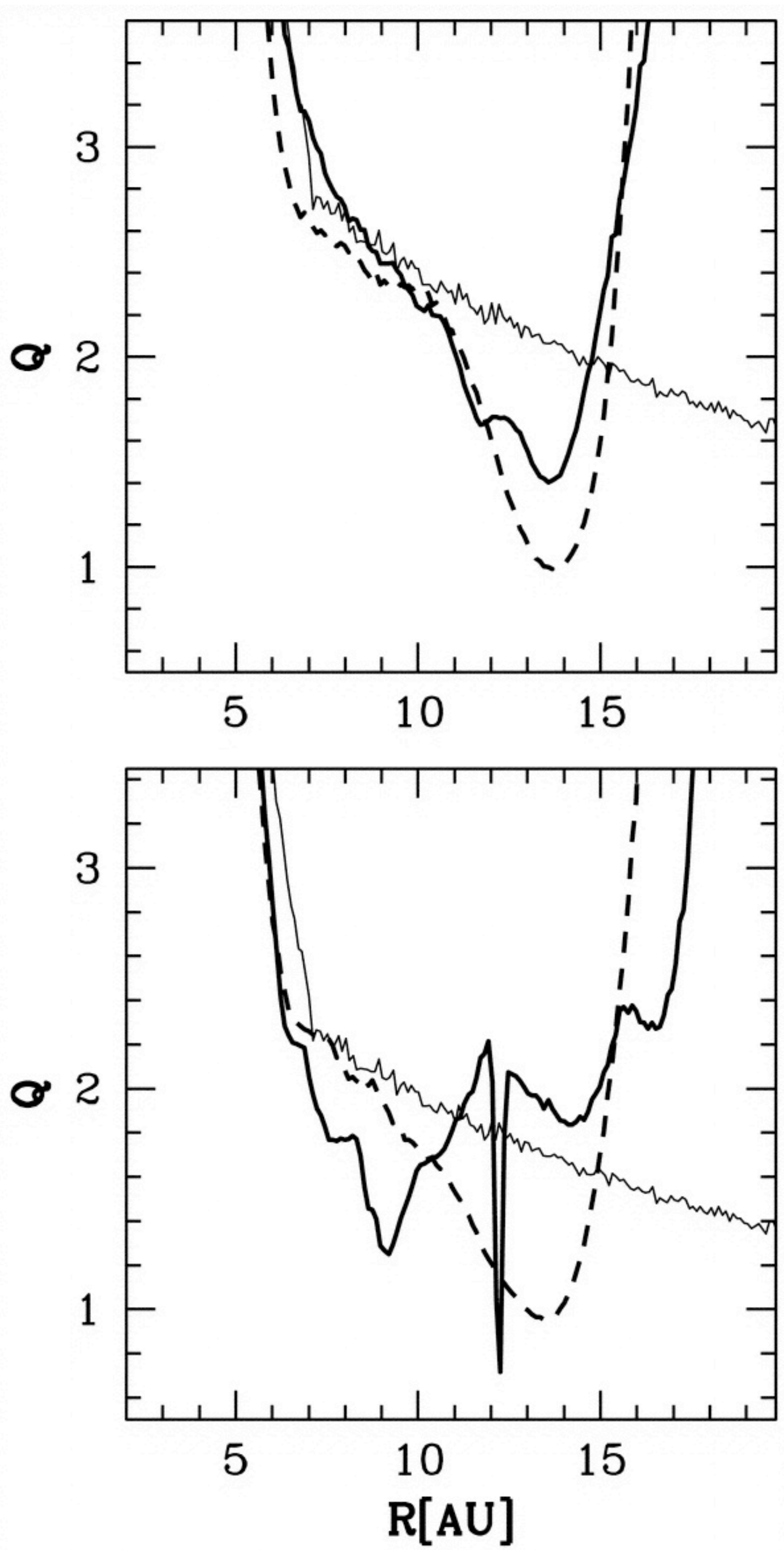
Size: 20 AU M_{disk}: 0.085 M_{sun}

Color-coded face-on density maps of a run with Q_{min}~1.3 (top) and Q_{min}~1.5 (bottom), at 200 (left panels) and 350 yr (right panels). The equation of state is isothermal but switched to adiabatic close to fragmentation. Brighter colors are for higher densities, and the disks are shown out to 20 AU.

We see that both disks develop strong spiral waves, but only the upper one fragments into bound clumps.

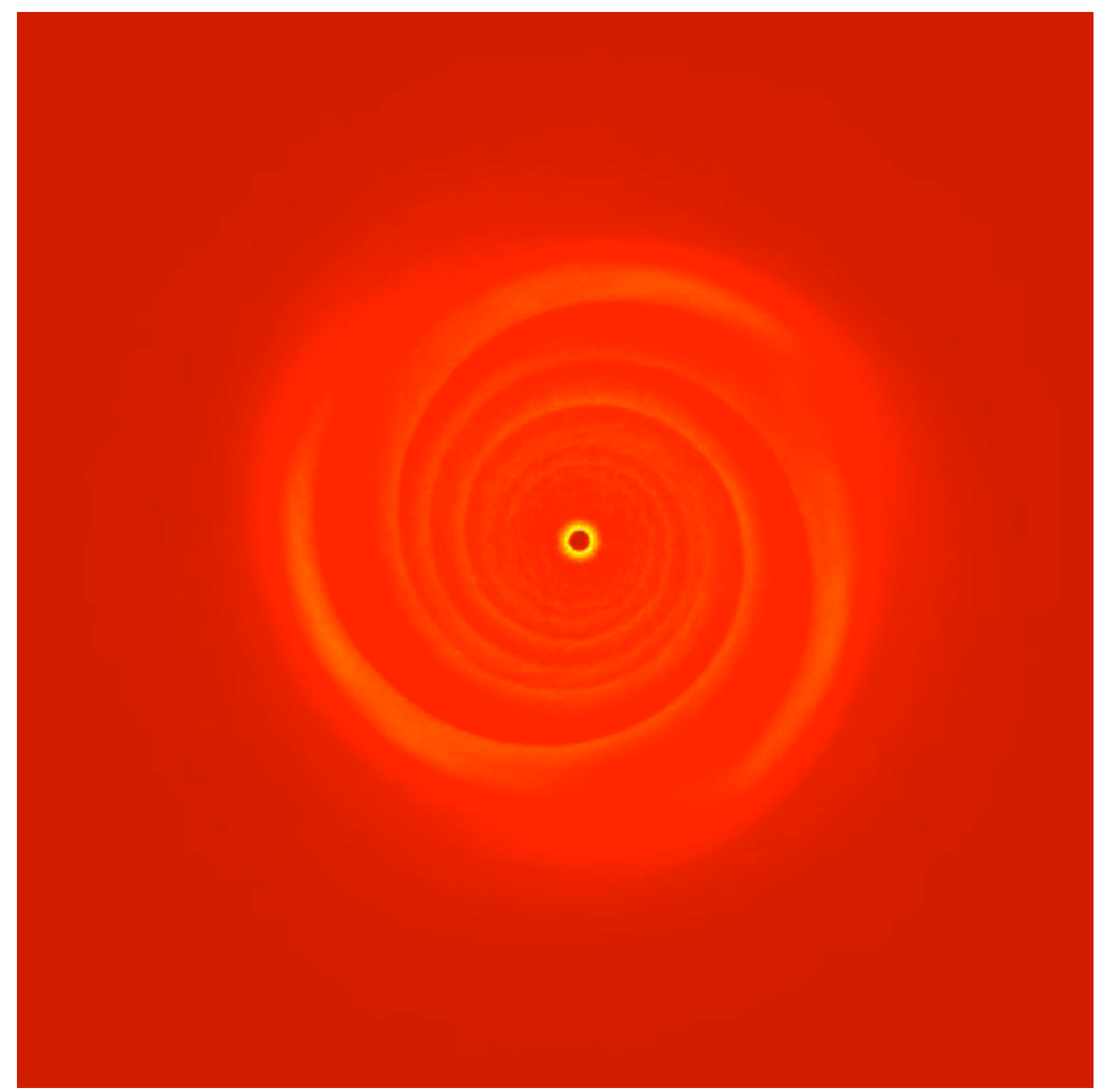
This happens very fast!

Numerical simulations II

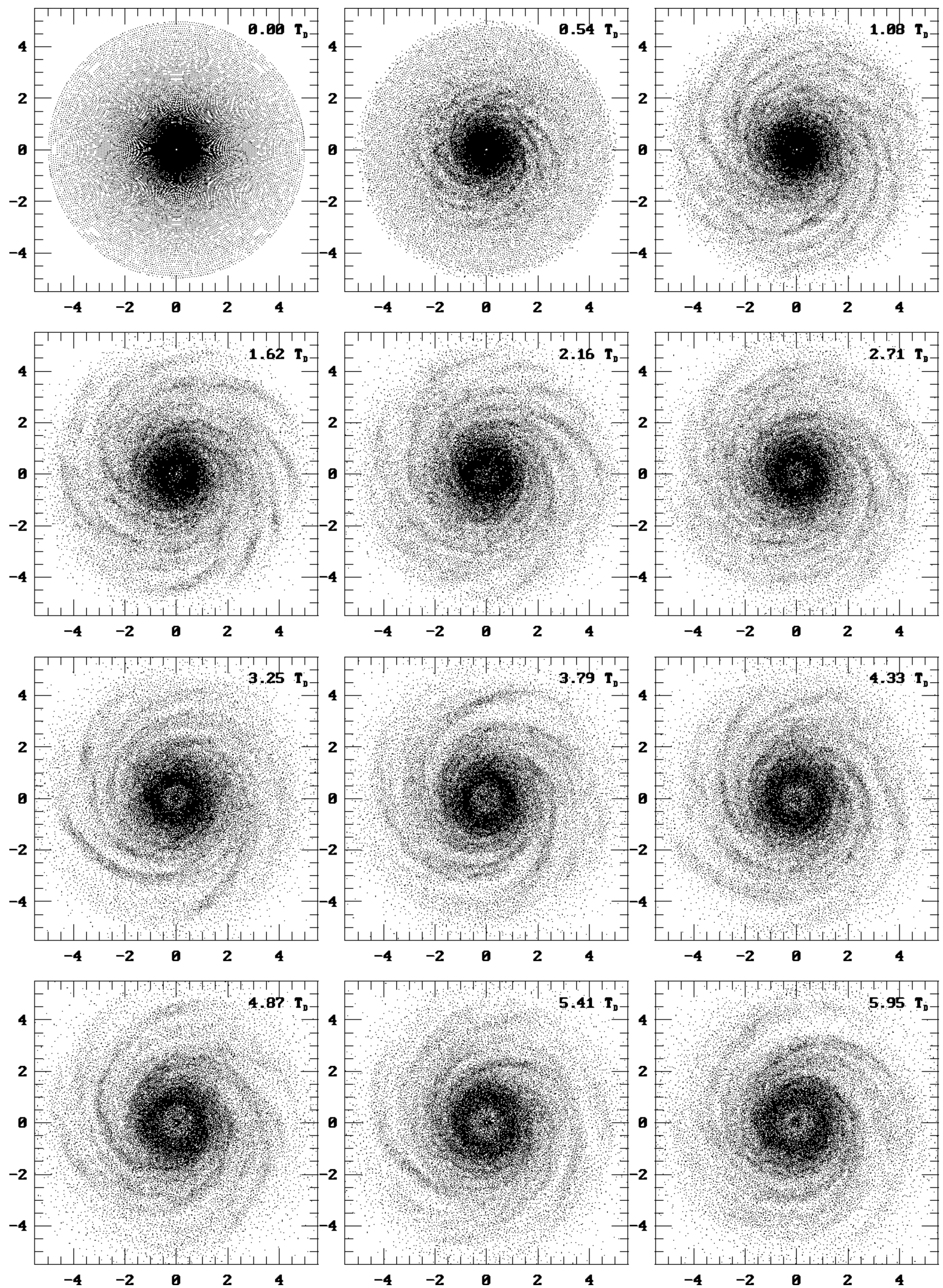


Mayer et al. 2004

Evolution of Q-profiles. Profiles at t = 0 (thin solid line), 160, (dashed line), and 240 yr (thick solid line). Fragmentation occurs between 160 and 240 yr in the model shown below (M_{disk}=0.1 M_{sun}, Q_{min}=1.38) while model shown in the upper panel (M_{disk}=0.08 M_{sun}, Q_{min}=1.65) develops only strong spiral arms.



Numerical simulations III



Nelson, Benz et al. 2000

Uses realistic cooling by computing the structure of the disk and taking the photospheric temperature for computing the radiative losses.

The plot shows a time series of SPH particle positions for a disk of mass M_{disk}/M_{star}=0.2 and initial minimum Q_{min}=1.5. Spiral structure varies strongly over time. Length units are defined as 1=10 AU. With the assumed mass of the star of 0.5 M⊙ and the radius of the disk of 50 AU, TD≈ 500 yr.

In this simulation, T rises when the density rises. Spiral arms still develop but there is no evidence for local collapse. A correct treatment of the thermodynamics is essential for accurate dynamics.

To date, no general agreement exists on the feasibility of direct collapse as planetary formation mechanism.


Questions?