Department of Physics and Astronomy
University of Heidelberg

Bachelor Thesis in Physics
submitted by

Vincent C. Mader

born in Ulm (Germany)

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Gas accretion onto eccentric planets

This Bachelor Thesis has been carried out by Vincent C. Mader at the
Max Planck Institute for Astronomy in Heidelberg
under the supervision of
Dr. Bertram Bitsch

Abstract

Zusammenfassung

Acknowledgements

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Chapter 1

Introduction

- one of the oldest questions: where do we come from? what is our relevance in the cosmos? is there a connection between our home (home planet) and the vastness of the night sky? are we alone? (maybe quote from/reference to Carl Sagan)
- ancient Babylonians identified the planets (meaning wanderer) as different from the 'fixed' background stars
- now, we know this is due to the planets' inependent motion around their parent star, our Sun, but:
- for a long time ⇒ geocentric model of the universe (e.g. Ptolemy)
- Aristarchus of Samos had suggested it as early as 250 BC, but the theory was not widely accepted until the 17th century
- Renaissance: heliocentric model (Copernicus), planets travel on concentric circles around their parent star
- observations of Mars orbit by Johannes Kepler and his mentor Tycho Brahe eventually lead to a mathematical description of the astronomical motions
- improved model relative to Copernicus, ellipses instead of circles
- Kepler's discoveries lay foundation for Newton's work later on
- Kepler problem leads to the development of calculus by Isacc Newton and Gottfried Wilhelm Leibniz
- Newtonian theory of gravity (same laws govern here and there)
- from the heliocentric world view, questions arise:
  are all stars suns like our own?
  are there other planets out there?
- more planets inside our own solar system were discovered after the invention of (semi-)modern optical telescopes
- first observations of extrasolar planets (exoplanets) in the late 20th century
- many have been discovered since then, most notably by the Kepler space telescope, (about 2600 planets detected by KST alone, 5000 total)
- observations show that about 1 out of every 5 Sun-like stars has an Earth-sized planet inside the Goldilocks zone (where liquid water can exist in a stable form)
  ⇒ there could be many dozens of billions of Earth-like planets in the Milky Way alone

- masses of observed planets range between about twice the Moon and about 30 times Jupiter (multiple orders of magnitude)
  ⇒ very different material compositions (terrestrial vs. gas giant)
- not only exoplanets observed, but also newly forming solar systems in various stages of development (protoplanetary disks, show images)
- since the early 1980s studies of young stars have shown them to be surrounded by cool discs of dust and gas, as so-called nebular hypothesis states
- first image of Protoplanetary disk (PDS 70b) was reported in July 2018
- gives us insights into the evolution of extra-solar systems and the planets within ⇒ information about our own home planet/solar system (, its significance in the grander scheme of things) and ultimately, deep philosophical questions like whether there is extraterrestrial life somewhere out there
- formation of solar systems/planets not entirely understood yet ⇒ active field of study, complex topic (gravity, magneto-hydro-dynamics, material science (?), ...)
- temporal limits, this bachelor's thesis can only cover a tiny portion of what there is to be said about this complex and actively evolving field of study
- this work utilizes construct a highly simplified model of a protoplanetary disk (2D) with the help of computer simulations. it is assumed that a ... planet has already formed (circumventing problem of early accretion, cite) that is orbiting a central proto star
- the evolution of the planet is obviously greatly influenced/characterized by the rate of mass accretion.
- variation of disk and planet parameters ⇒ influence on evolution of planet

Chapter 2

Theory

2.1 Prerequisites

2.1.1 Classical Theory of Gravity

Kepler's Laws

- formulated by Johannes Kepler between 1609 and 1619
- empirical laws from studies of Tycho Brahe's observational data
- first two laws were published in 1609, *Astronomia Nova*
  1. A planet travels around its parent star on an ellipse.
  2. The planet sweeps out equal areas in equals times
- additionally, Kepler regognized that neither velocity nor angular velocity are constant, but area velocity is (closely related to angular momentum)
- ellipses are conical sections, characterized by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad \text{with } a > b \tag{2.1}$$

- ellipse characterized by two numbers (a & b or one axis and e)
- Earth's orbit has an eccentricity of about 0.017

$$e = \sqrt{1 - \frac{b^2}{a^2}} \tag{2.2}$$

- give equation for trajectory
- circles can be seen as special cases of ellipses, or as a separate class of conical section
- can also be parabolas or hyperbolas ($e = 1$ or $e \geq 1$)
- third law published in 1619
- square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

Newton's Law of Gravitation

- published in Isaac Newton's famous *Principia*

$$\mathbf{F} = G \cdot \frac{m_1 \cdot m_2}{|\mathbf{r}_2 - \mathbf{r}_1|^2} \cdot \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|} \tag{2.3}$$

- from this equation, Kepler's empirical laws could be derived mathematically/theoretically (theoretical underpinning)
- inverse square law is responsible for first and third Kepler law, conservation of angular momentum for second one
- Vis Viva equation [1]

2.1.2 Fluid Dynamics

2.2 Protoplanetary Disks

- rotating circumstellar disk (newly formed star in the center)
- consists mostly of gas and dust
- most of the mass is in the star (how much)
- often accompanied by jets

2.2.1 Disk Formation

- initial molecular gas cloud (nebular hypothesis)
- mainly H_2, about 98%
- also small amounts of He and Li from Big Bang, as well as miniscule amounts of heavier elements created in earlier generation(s) of stars
- nebular hypothesis formulated in the 1700s by Emanuel Swedenborg, Immanuel Kant, and Pierre-Simon Laplace
- gravitational collapse (which criteria/external influences)
- statistical motion averages out in favour of cloud's net angular momentum
- conservation of angular momentum: particles speed up as they fall towards the center of the disk
- formation of relatively thin disk (gravity vs. centripetal force)
- can be seen partially as accretion disk onto star
- outcome: thin disk supported in vertical direction by gas pressure
- can be modeled as ideal gas

2.2.2 Star Formation in the Center of the Disk

- accretion of gas onto star
- star forming in the center of the disk
- continues for about 3-10 million years
-
-
-

2.2.3 Geometry, Timescales & other Properties

- radii of up to 1000 AU
- much more wide than thick
- collapse takes about 100.000 years $\Rightarrow$ then, star has similar temperature to main sequence star of same mass, becomes visible
- oldest disk ever observed: 25 million years
- after that, gas is either blown away by stellar wind or simply stops emitting radiation
- gas density can be modeled as

$$\Sigma = \Sigma_0 \left(\frac{r}{r_c} \right)^{-\gamma} \tag{2.4}$$

  with $\gamma \approx 1$ (flaring index (?), geometry)
- cutoff radius (not r_c, is it?)
- gas is supported by gas pressure $\Rightarrow$ orbits slightly slowly as it would if it were purely Keplerian

2.3 Formation of Planets in the Disk

- inner terrestrial planets, outer gaseous planets (gaseous planets beyond the frost line much larger, since there is much more ice than heavier elements $\Rightarrow$ could grow large enough to capture abundant H_2 and He)
- formation of circumplanetary disks (similar to protoplanetary disk, much smaller scale, how many AU approximately?)
- Roche lobe
-
-

2.3.1 Planet Core Formation

Formation of Planetesimals

- collisions, gravitational capture
- still not entirely known how
- electrostatic and gravitational interactions lead accretion into planetesimals
- planetesimals are building blocks for both terrestrial and giant planets
- problem: how does early core not fragment into smaller pieces at collisions? (meter size barrier, still unclear

2.3.2 Accretion Mechanisms

Machida Accretion

Kley Accretion

Chapter 3

Numerical Techniques

3.1 The FARGO2D1D Algorithm

The FARGO algorithm was originally introduced by who, 1999, what advantages relative to what?. FARGO stands for *Fast Advection in Rotating Gaseous Objects* [**paper introducing fargo algorithm**].

FARGO2D1D is an extension of the standard version of FARGO, in which the 2D grid is surrounded by a simplified 1D grid, made of elementary rings non azimuthally resolved. [**what is FARGO2D1D**]

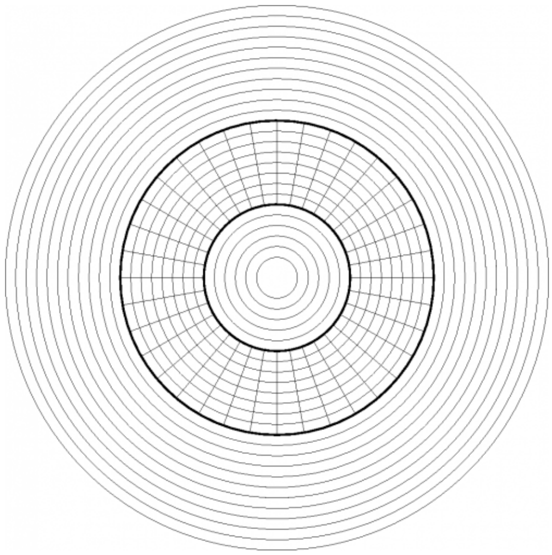

Figure 3.1: example of a grid used by the FARGO2D1D algorithm[**2D1D grid**]

parameters $G = 1, M_{\odot} = 1, R_0 = 1$
disk is modeled partially as a 2D array, partially as a 1D array
$\Rightarrow$ make plot showing this (look in FARGO documentation?)

1. n-body solver with 5th order Runge-Kutta algorithm
2. fluid dyn for gas

3.1.1 Runge-Kutta

1. family of implicit and explicit iterative methods
2. developed around 1900 by German mathematicians Carl Runge and Wilhelm Kutta
3. include the well-known routine called the Euler Method

4.

5.

Chapter 4

Simulation Procedure

4.1 First Runs

4.2 Choosing the Resolution of the 2D1D-Grid

4.3 Variation of the Parameters of the Protoplanetary Disk

4.4 Variation of the Initial Parameters of the Planet

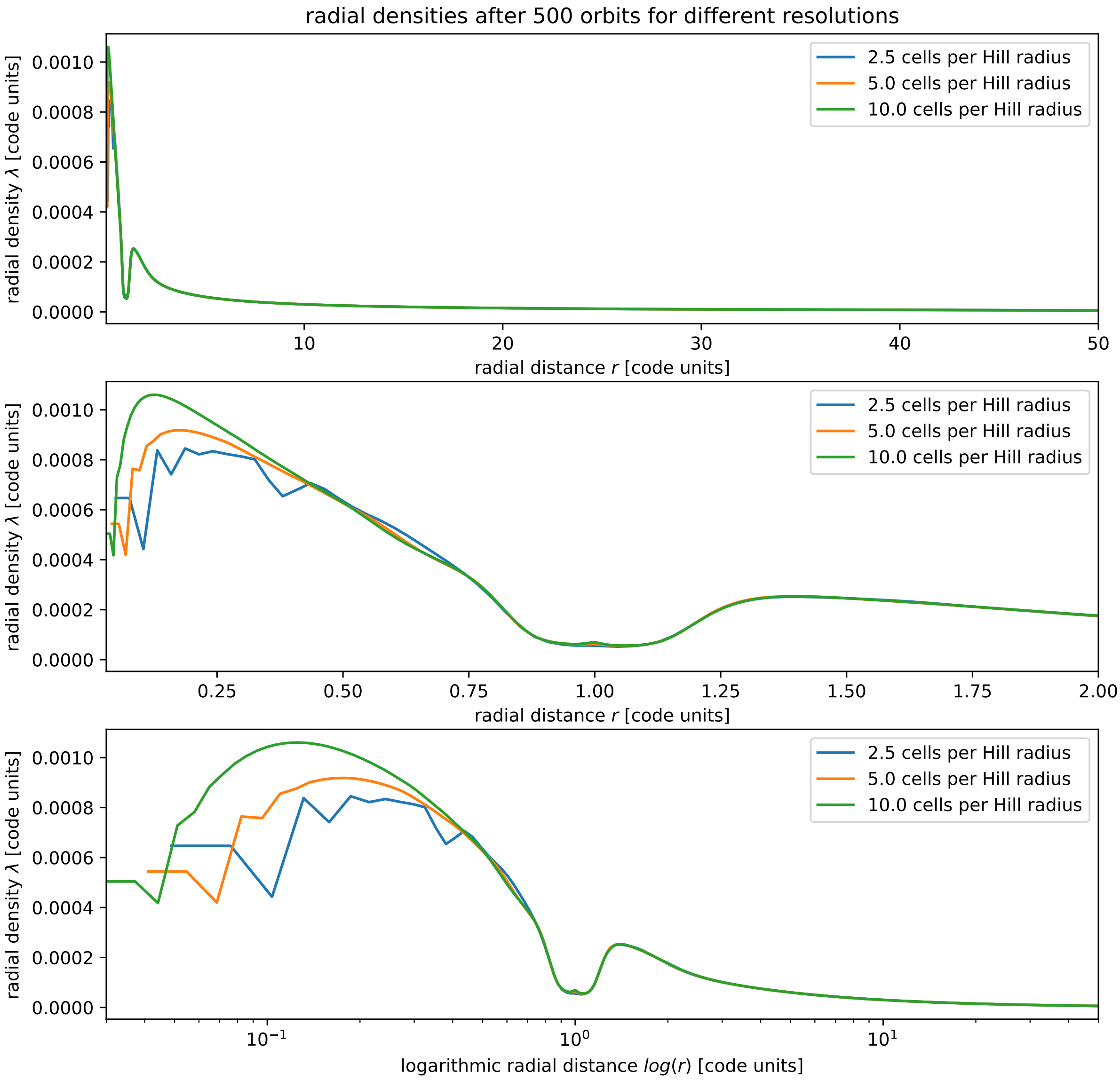

Figure 4.1: radial densities after 500 orbits for different resolutions

Chapter 5

Analysis/Results

5.1 General

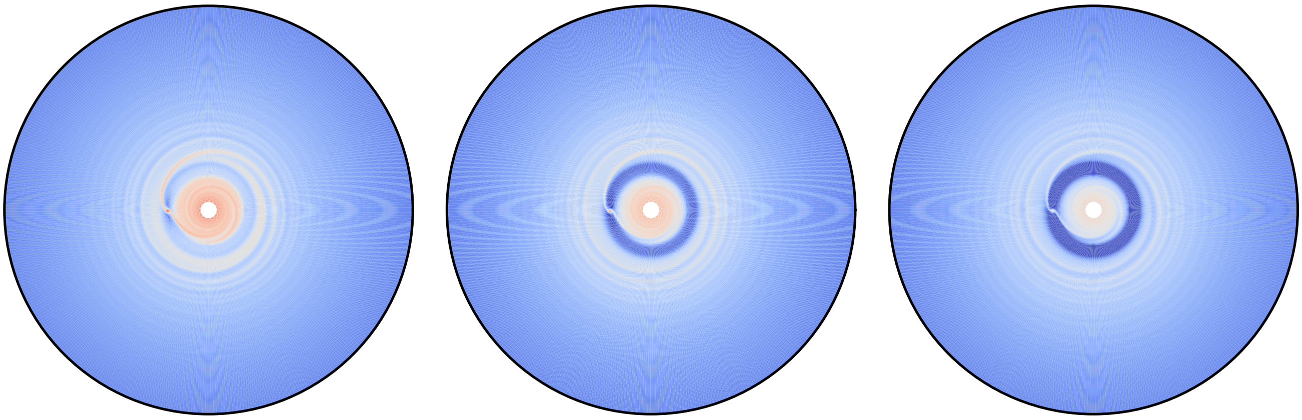

Figure 5.1:

5.2 Variation of the Parameters of the Protoplanetary Disk

5.2.1 Aspect Ratio

5.2.2 Radial Gas Profile

5.2.3 Gas Viscosity Parameter

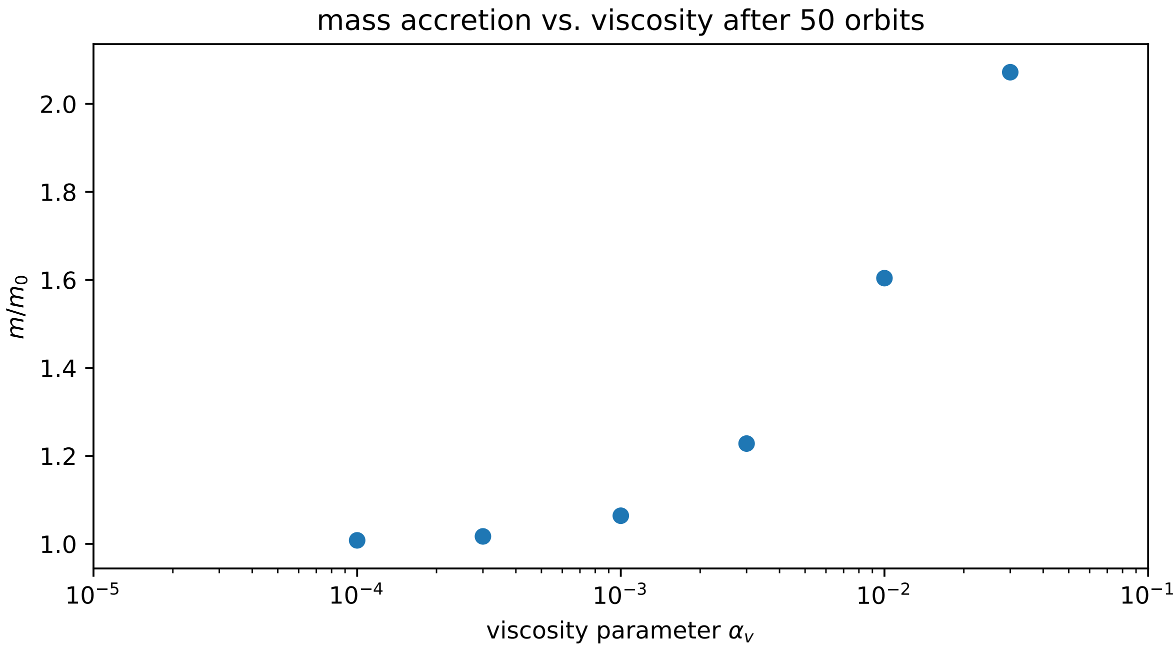

Figure 5.2:

5.3 Variation of the Initial Parameters of the Planet

5.3.1 Initial Planet Mass

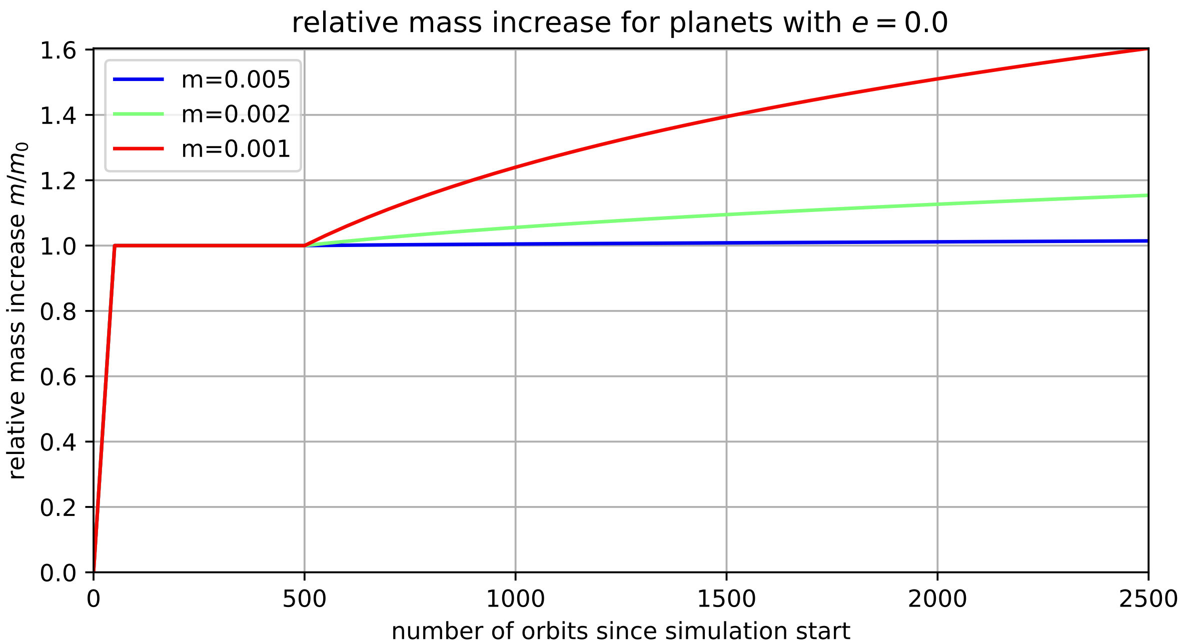

Figure 5.3:

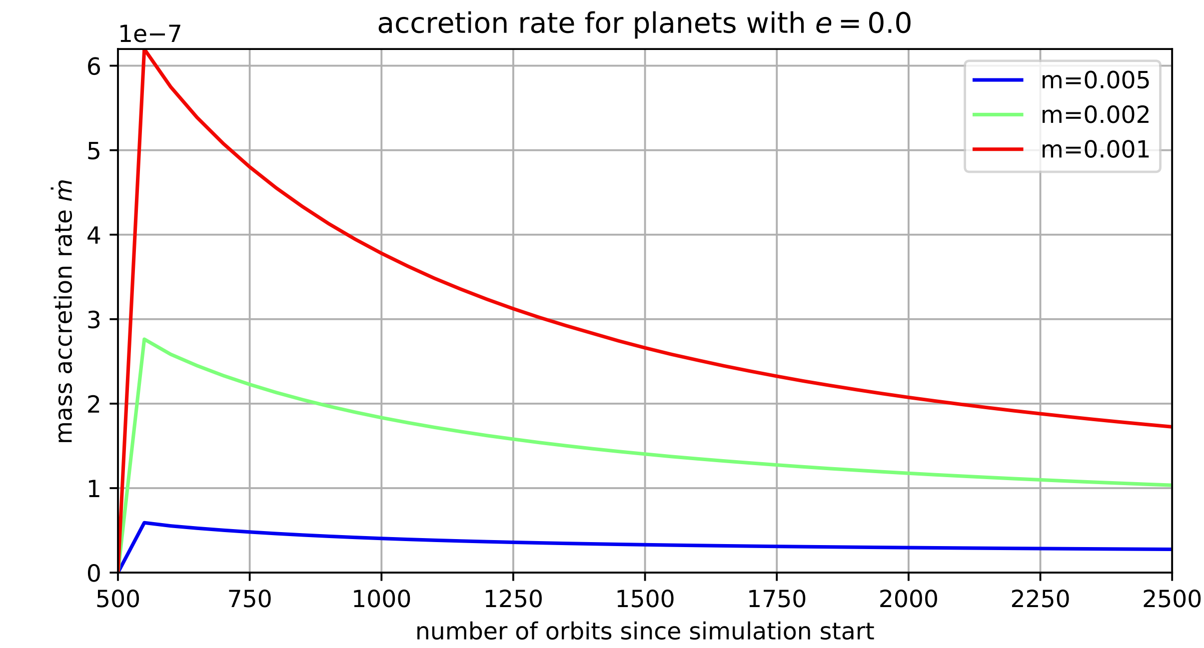

Figure 5.4:

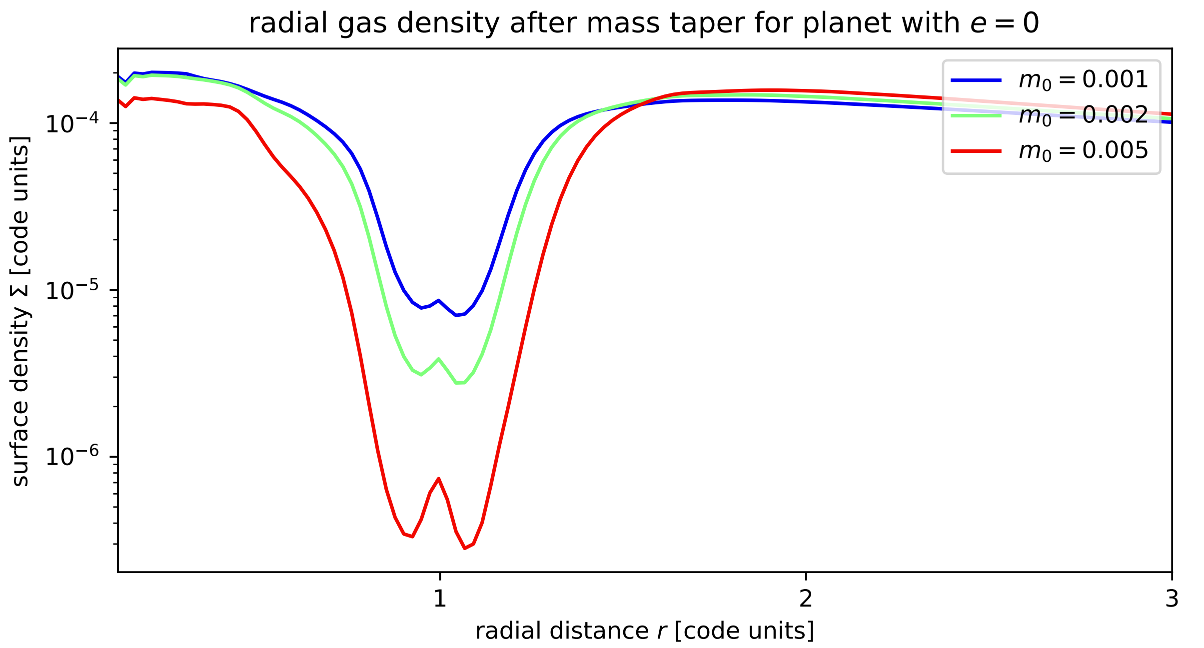

Figure 5.5:

5.3.2 Initial Planet Eccentricity

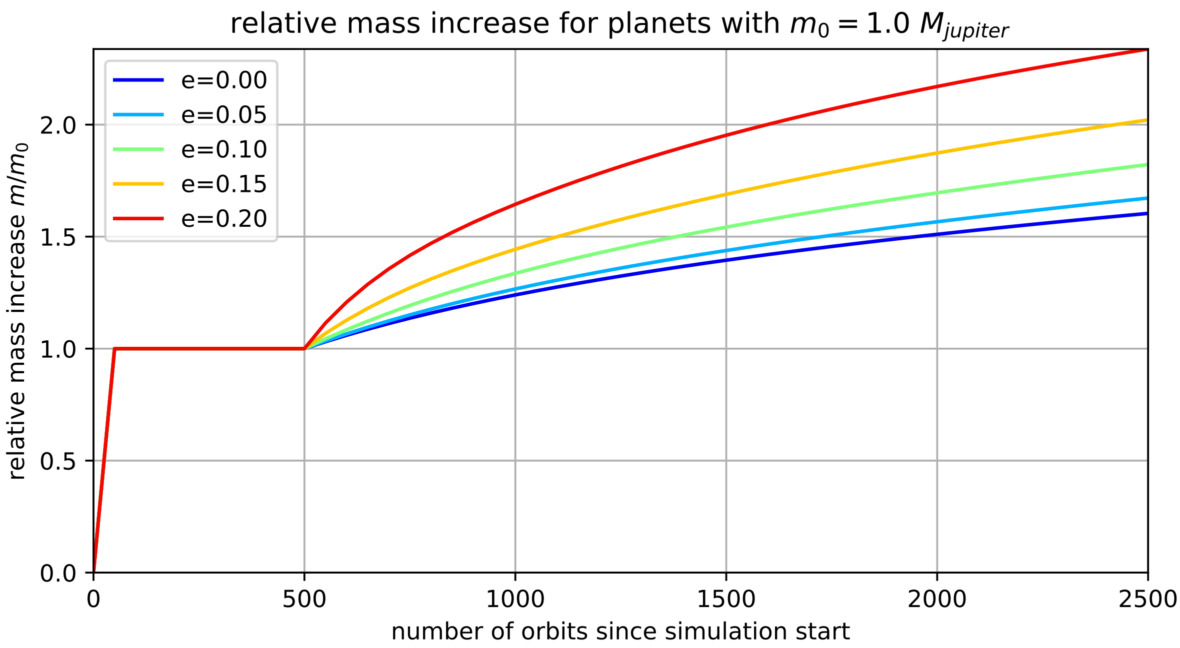

Figure 5.6:

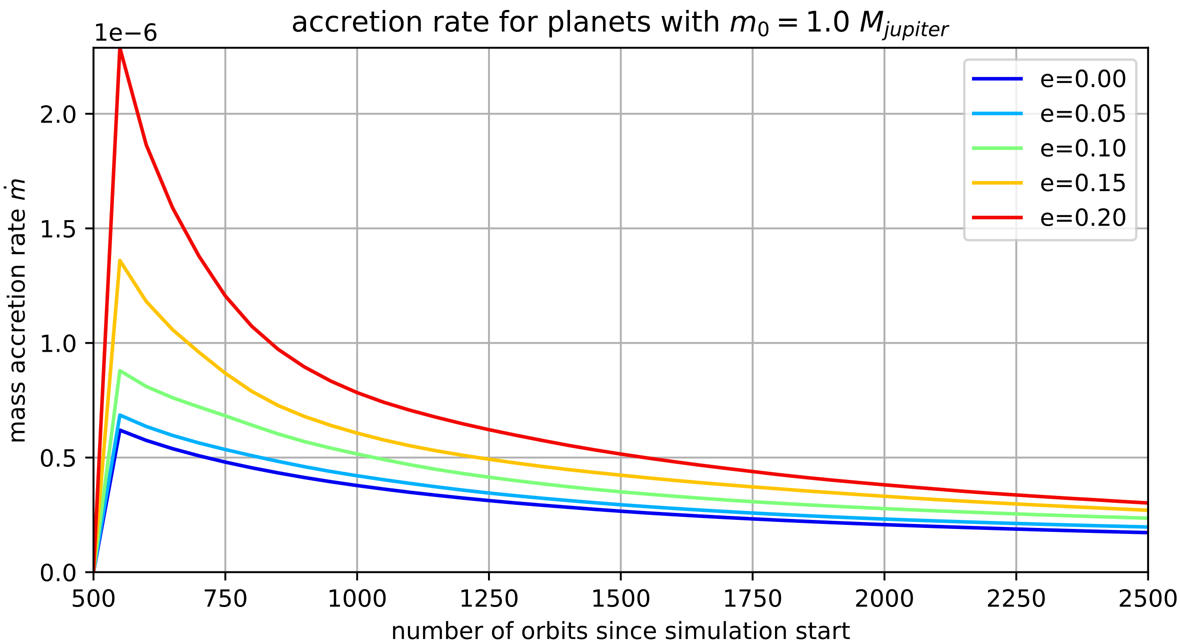

Figure 5.7:

5.3.3 Machida Accretion Factor

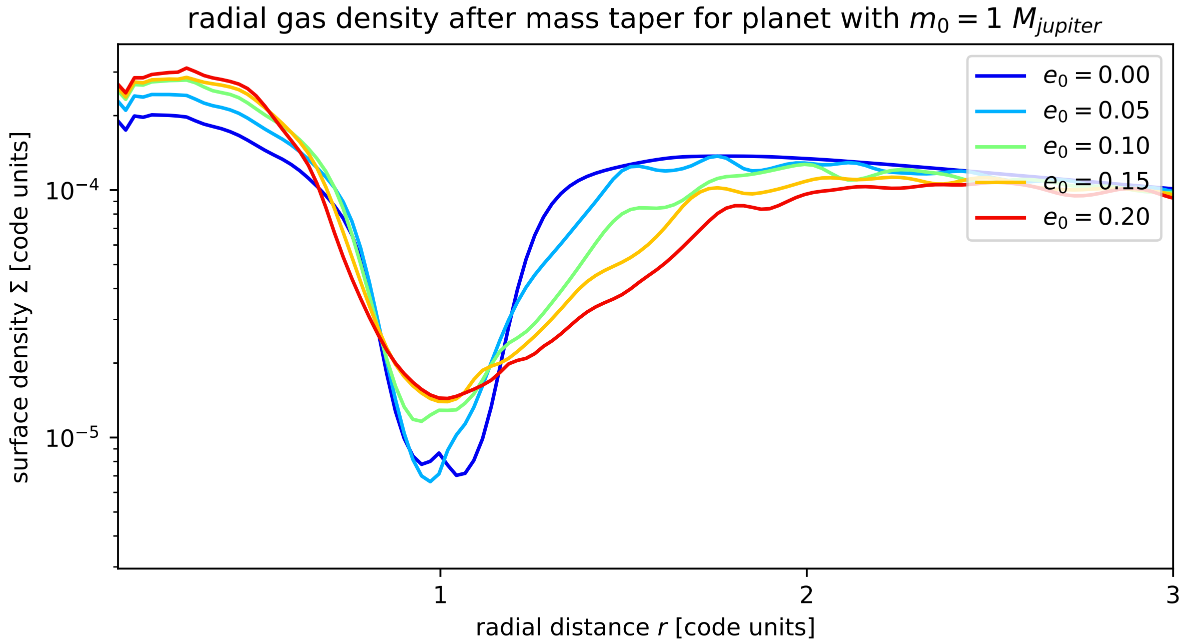

Figure 5.8:

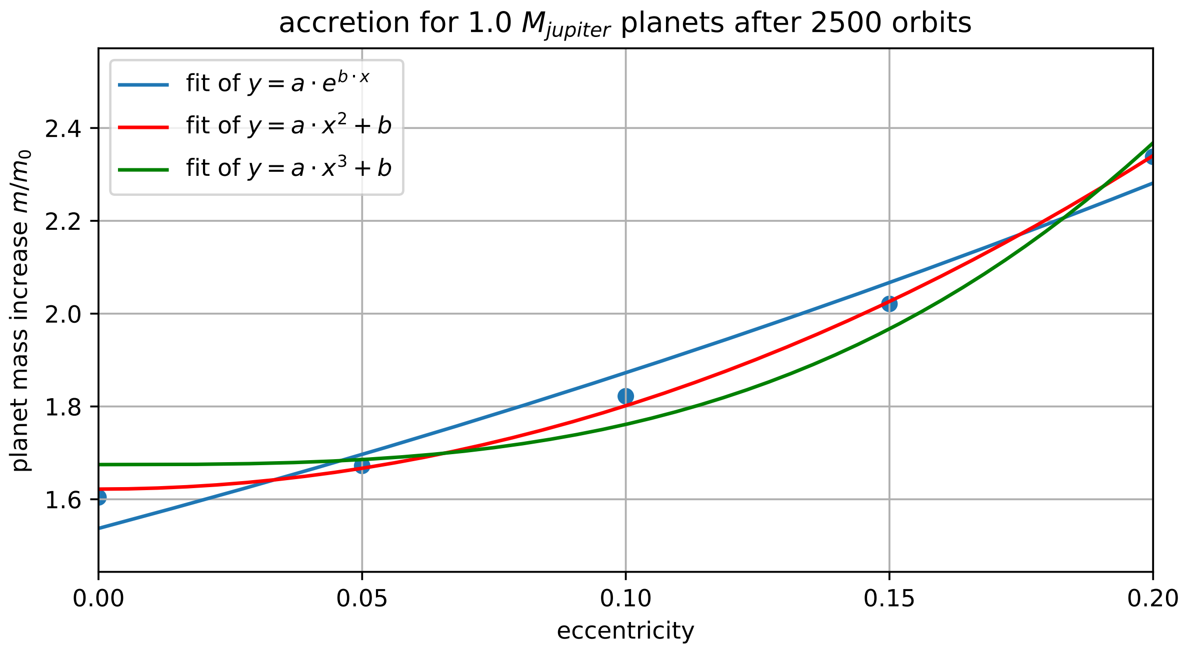

Figure 5.9:

Chapter 6

Summary

Chapter 7

Appendix

7.1 References

[1] Albert Einstein. "Zur Elektrodynamik bewegter Körper. (German) [On the electrodynamics of moving bodies]". In: *Annalen der Physik* 322.10 (1905), pp. 891–921. DOI: `http://dx.doi.org/10.1002/andp.19053221004`.

7.2 Abbreviations

Declaration

Ich versichere, dass ich diese Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Heidelberg, den ...,