# CCEVI of Skew Copula

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## 1 Skew Copulas

Bivariate copulas can be transformed to have a skew, adding permutation asymmetry to the copula. A skew transformation to a copula $C$ for parameter $\alpha \in [0, 1]$ is

$$C_{\mathbf{Skew}}(u, v) = C(u, v^{1-\alpha})v^{\alpha} \tag{1}$$

for $(u, v) \in [0, 1]^2$, where larger values of $\alpha$ correspond to a larger skew.

**Theorem 1.** *Suppose a bivariate copula $C$ and its reflection $\widehat{C}$ have CCEVI's $\xi_C > 0$ and $\xi_{\widehat{C}} > 0$, respectively. Then the skew copula defined in* (1) *for $\alpha \in [0, 1]$ has a CCEVI of 1 (or $\xi_C$ if $\alpha = 0$), and the reflection skew copula has CCEVI*

$$\frac{1}{\alpha + (1-\alpha)(1/\xi_{\widehat{C}})}. \tag{2}$$

See Appendix 1.1 for a proof.

Notice that the reciprocal CCEVI of the skew reflection copula in (2) is an interpolation of 1 and the reciprocal CCEVI of the original skew copula.

### 1.1 Proof of CCEVI of Skew Copula Class

Here, we present a proof of Proposition 1, first for the (non-reflected) skew copula, then for the reflected.

First, recall the expansion

$$(1-t^{-1})^{\alpha} = 1 - \alpha t^{-1} + \frac{\alpha(\alpha-1)}{2!}t^{-2} + O(t^{-3}) = 1 - t^{-1}O(1), \tag{3}$$

valid for $t$ near infinity. Now, differentiate the skew copula with respect to the first argument to obtain the copula conditional distribution. Then substitute $v = 1 - 1/t$ and send $t \to \infty$:

$$\begin{aligned}
1 - C_{\mathbf{Skew},2|1}(v|u) &= 1 - C_{2|1}(v^{1-\alpha}|u)v^{\alpha} && (4)\\
&= 1 - C_{2|1}(1 - t^{-1}O(1)|u)(1 - t^{-1}O(1)) && (5)\\
&\sim 1 - (1 - \ell(t)t^{-1/\xi_C})(1 - t^{-1}) && (6)\\
&= t^{-1} + O(t^{-1}), && (7)
\end{aligned}$$

where $\ell$ is slowly varying at infinity. The CCEVI of the skew copula is therefore 1.

The reflected skew copula is

$$\widehat{C}_{\mathbf{Skew}}(u,v) = u + v - 1 + C(1-u,(1-v)^{1-\alpha})(1-v)^{\alpha}. \tag{8}$$

The trick is to write the copula conditional distribution in terms of the reflected original copula, again substituting $v = 1 - 1/t$ and sending $t \to \infty$:

$$\begin{aligned}
1 - \widehat{C}_{\mathbf{Skew},2|1}(v|u) &= (1-v)^{\alpha} C_{2|1}((1-v)^{1-\alpha}|1-u) && (9)\\
&= t^{-\alpha} C_{2|1}(t^{-(1-\alpha)}|1-u) && (10)\\
&= t^{-\alpha}\left(1 - \widehat{C}_{2|1}(1 - t^{-(1-\alpha)}|u)\right) && (11)\\
&\sim t^{-\alpha}\hat{\ell}(t)\left(t^{1-\alpha}\right)^{-1/\xi_{\widehat{C}}(u)} && (12)\\
&= \hat{\ell}(t) t^{-1/\xi_{\widehat{\mathbf{Skew}}}(u)}, && (13)
\end{aligned}$$

where $\hat{\ell}$ is slowly varying at infinity, and

$$\xi_{\widehat{\mathbf{Skew}}}(u) = \frac{1}{\alpha + (1-\alpha)(1/\xi_{\widehat{C}}(u))} \tag{14}$$

is the CCEVI of the reflected skew copula.