Companion software for "Volker Ziemann, *Physics and Finance, Springer, 2021*" (https://link.springer.com/book/10.1007/978-3-030-63643-2)

Student's t-distribution and hypothesis testing (Section 7.5 and 7.6)

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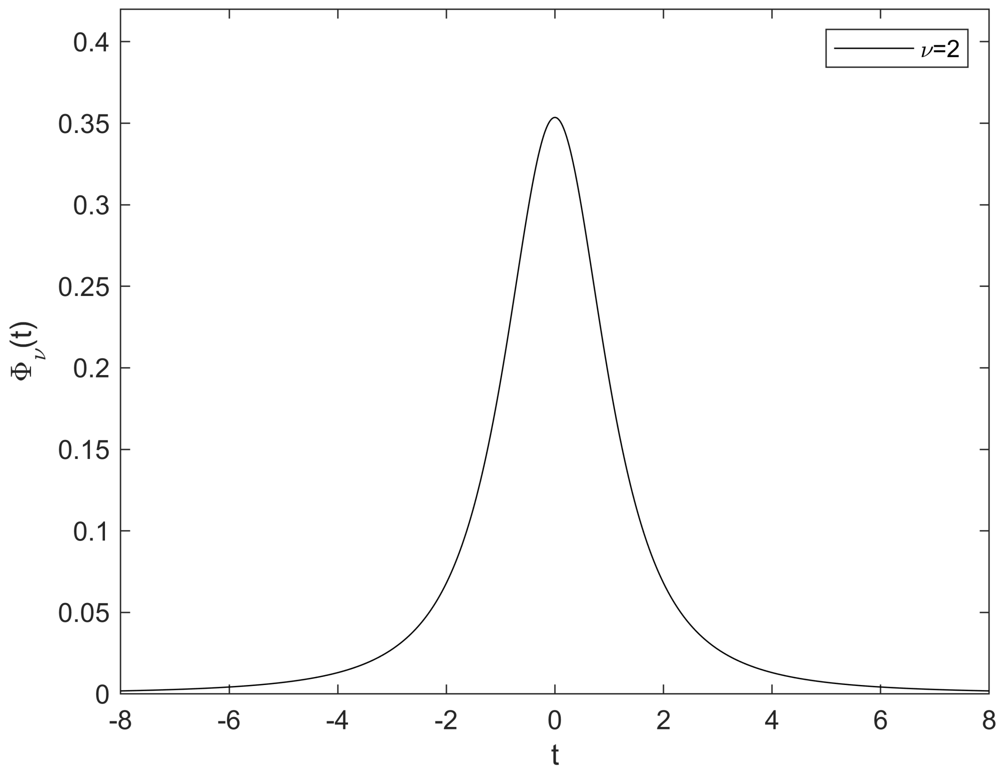
When determining fit-parameters from few data points N, measurement noise can 'conspire' to cause fit-parameters that deviate significantly from their 'true' value, where the latter is the value we would obtain with many data points. Student's t-distribution describes this excess probability of finding outliers. We emphasize that t is the deviation of a fit-parameter from the true value, normalized to the empirically determined standard deviation, as defined in Equation 7.27.

The degree of freedom $\nu = N - q$ of a fit is excess of data points N over the number of fit parameters q and provides the redundancy to provide the increased reliability that a fit parameter is 'true'. After setting the value of ν with a slider, we define an anonymous function `tdist()` to encode the t-distibution from Equation 7.33, plot the t-distribution for ν degrees of freedom, and annotate the axes. Note that MATLAB also provides a built-in function `tpdf()` that can be used as a drop-in replacement for `tdist()`.

```
clear
nu=2; % Slider to set the degrees of freedom nu
t=-8:0.01:8;
tdist=@(t,nu)(gamma(0.5*(nu+1))./(sqrt(pi*nu)*gamma(0.5*nu))) ...
  *(1+(t.^2./nu)).^(-0.5*(nu+1));   % eq. 7.33
plot(t,tdist(t,nu),'k');  % or tpdf(t,nu)
xlabel('t'); ylabel('\Phi_\nu(t)')
legend(['\nu=',num2str(nu)])
xlim([-8,8]); ylim([0,0.42])
```



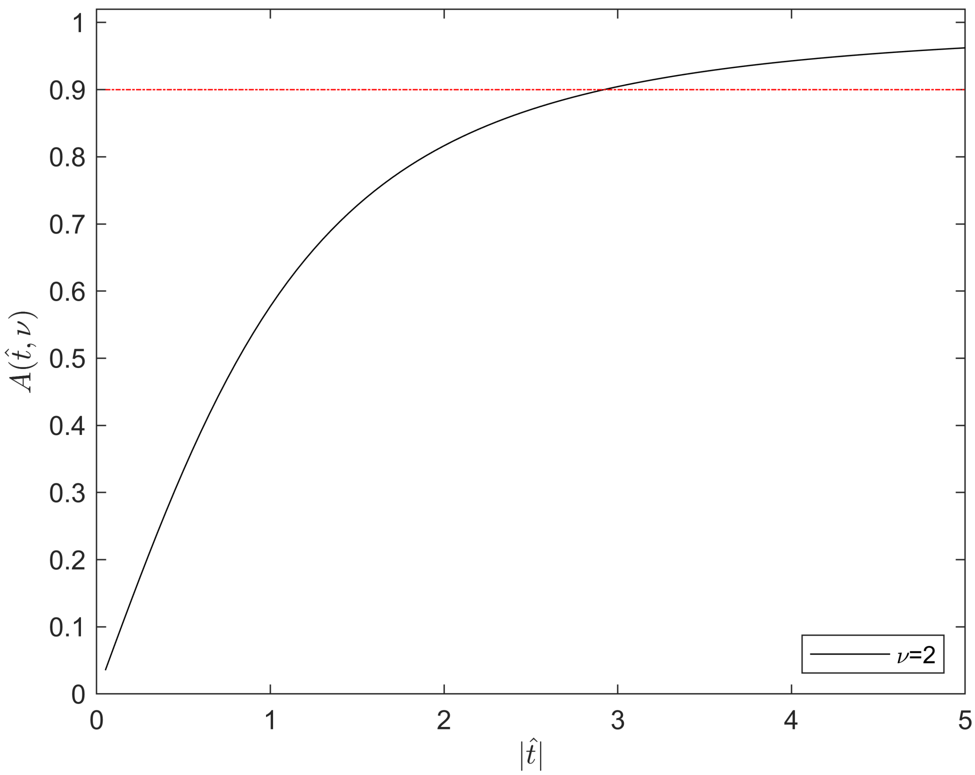
We observe that for small ν the tails of the distribution are much more pronounced compared fto those with larger values of ν.

Confidence level

The interval around the central value (here $t = 0$) that contains a certain percentage, say 90%, of the entire distribution is the 90%-percent confidence level. Here we see that this interval is much larger for smaller ν, because the tails of the distribution are more pronounced. We therefore need to determine the value of $\hat{t}$ such that $\pm\hat{t}$ contains 90% of the distribution.

In the following code segment we use a slider to set the desired confidence level and then use Equation 7.34 to calculate the probability $A(\hat{t},\nu)$ that is contained in the interval of $\pm\hat{t}$. The plot shows $A(\hat{t},\nu)$ as a function of $\hat{t}$ as a black line and the chosen confidence level as a red dashed line. The intersection of the black and red lines determines the value if $\hat{t}$ that defines the confidence interval.

```
confidence_level=0.9; % Slider to set confidence_level
A=@(that,nu)1-betainc(nu./(nu+that.^2),nu/2,0.5); % eq.7.34
that=0.05:0.05:5;
plot(that,A(that,nu),'k',that,confidence_level*ones(size(that)),'r-.')
ylim([0,1.02])
xlabel('$|\hat t|$','interpreter','latex')
ylabel('$A(\hat t,\nu)$','interpreter','latex')
legend(['\nu=',num2str(nu)],'Location','SouthEast')
```

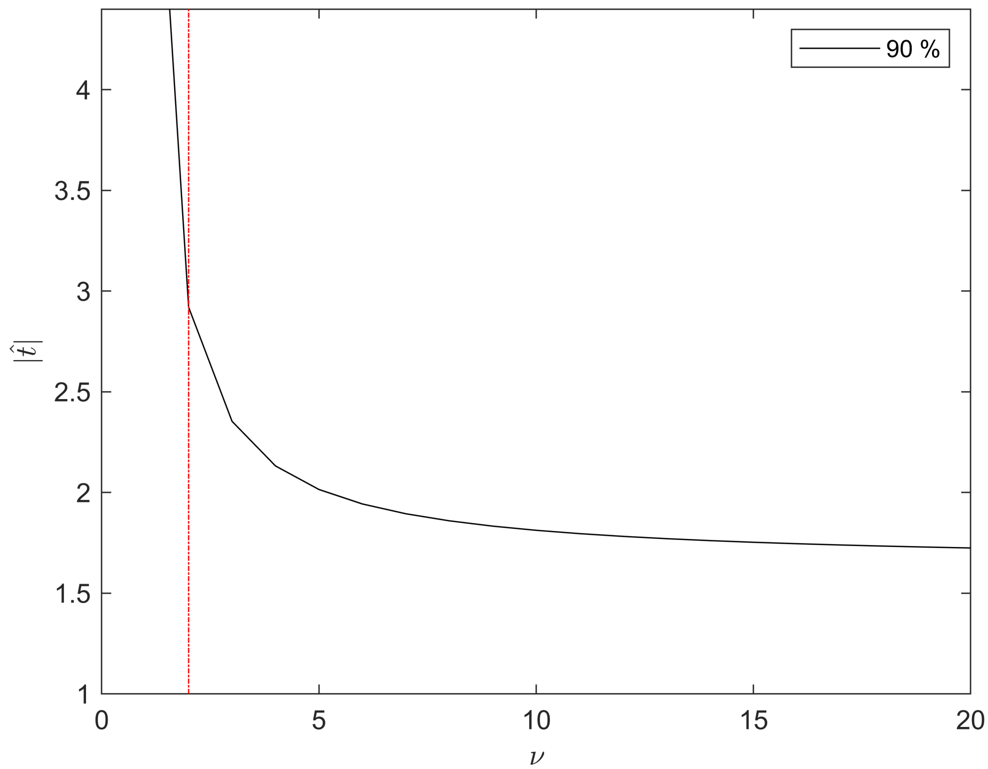


We observe that reducing the confidence level moves the intersection towards smaller values. Thus the confidence interval gets smaller, but the confidence in it is reduced.

Confidence interval

Finally, we work out how the confidence interval that corresponds to a chosen confidence level changes if we have more data points N available which in turn increases the degrees of freedom ν. We therefore find the intersection of $A(\widehat{t},\nu)$ and the confidence level for increasing values of ν and plot $\widehat{t}$ as a function of ν. Note that we also show the value of ν selected with the slider on the top of thus script as a vertical red line. Again, the intersection of the curves occurs at the same value of $\widehat{t}$ as in the previous plot.

```
nuaxis=1:1:20;
data=zeros(length(nuaxis),1);
for k=1:length(nuaxis)
  q=@(t)A(t,nuaxis(k))-confidence_level;
  data(k,1)=fzero(q,[0,19]);
end
figure
plot(nuaxis,data(:,1),'k',[nu,nu],[1,4.4],'r-.')
ylim([1,4.4]); xlim([0,20])
xlabel('\nu'); ylabel('$|\hat t|$','interpreter','latex')
legend([num2str(100*confidence_level),' %'])
```



We observe that $\hat{t}$ decreases with increasing ν and levels off at $\hat{t} = 2$, which is intuitively appealing, because increasing the degrees of freedom causes the t-distribution to approach a Gaussian, which contains 90% probability within two standard deviations. And, as mentioned in the first paragraph, t is the deviation from the true value, measured in units of the standard deviation.

Hypothesis testing and p-value

We can turn the concept of confidence interval upside-down, which help us to decide whether a particular fit-parameter x_j is actually needed in a fit. Conventionally, this is formulated by a *null-hypothesis*, which simply states "we make the hypothesis that x_j is zero." If we then can show that x_j in our particular fit lies outside the 90% confidence interval it's rather probably that the fit parameter is different from zero and we say that the hypothesis is rejected at the 10% level.

Moreover, assume that we calculate a $t-$value for x_j in this particular fit, then the probability p of finding an a value of x_j that causes an even larger value of t is called the p-value.