Companion software for "Volker Ziemann, *Physics and Finance, Springer, 2021*" (https://link.springer.com/book/10.1007/978-3-030-63643-2)

Donkey's solution (Section 11.5)

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In Section 11.5 we solve the **optimal-control problem** to find the best speed profile for the donkey to pull a mass for some distance `L` in a given time `T` across rough ground with friction constant α, while minimizing the expended power, given by the pulling force squared.

In the first part of the simulation we select the friction constant α, the distance `L` and the given time `T`.

```
alpha=1.1   % friction constant
```

```
alpha = 1.1000
```

```
L=100;       % distance to cover in [m]
T=20;        % time for the travel in [s]
aT=alpha*T;
```

The difficult part of some optimal-control problems is that both initial and final boundary conditions are specified, which often requires solving implicit equations. But in linear problems solving is possible by carfeully solving a succession of linear equations. This derivation is explicitely done in Section 11.5, and here we immediately jump to Equation 11.49, which give us intergration constants `C3` and `C4` that we use to find `C1` and `C2` from the equations given in the text just before Equation 11.49.

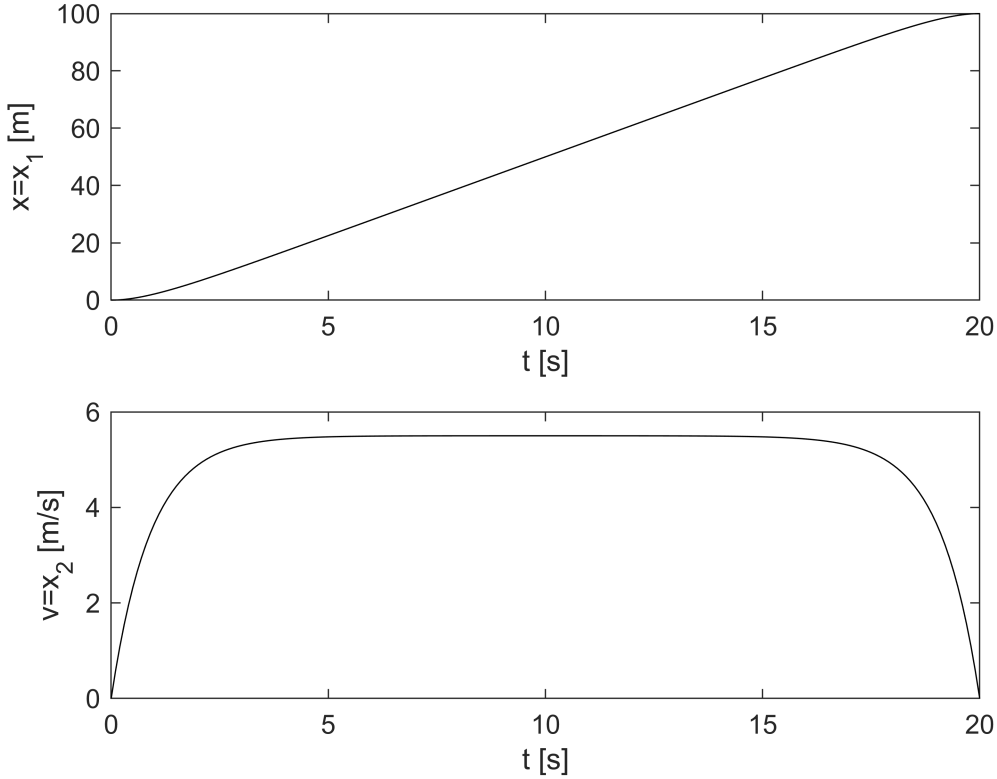
```
y=[L;0];
A=[(exp(aT)-exp(-aT)-2*aT)/alpha , 1+aT-exp(aT);
  (exp(-aT)+exp(aT)-2)/alpha , 1-exp(aT)];
x=A\y; C3=x(1); C4=x(2);
C1=2*alpha^2*C3-alpha^3*C4;
C2=alpha^2*C4;
```

With the integration constants available we can define anonymous functions to describe the position x_1 of the donkey with Equation 11.47 and its speed x_2 by Equation 11.46, which are directly coded here.

```
x1=@(t)C4-(C3/alpha)*exp(-alpha*t)-(C1/alpha^2)*t ...
    -(1/(2*alpha^2))*(C2-C1/alpha)*exp(alpha*t);
x2=@(t)C3*exp(-alpha*t)-C1/alpha^2-(1/(2*alpha))*(C2-C1/alpha)*exp(alpha*t);
```

Finally we plot both the donkey's position and speed for the duration of the travel.

```
t=0:0.1:T;
subplot(2,1,1); plot(t,x1(t),'k')
xlabel('t [s]'); ylabel('x=x_1 [m]');
subplot(2,1,2); plot(t,x2(t),'k')
xlabel('t [s]'); ylabel('v=x_2 [m/s]');
```



Now play with the slider for the friction α and observe how the profile changes. If α is small, say $\alpha = 0.1$, the donkey prefers a somewhat higher speed and a long acceleration to the half-way point. With larger friction, say $\alpha = 1$, the donkey early accelerates to its "cruising speed" and stays there until close to its arrival time.