Companion software for "Volker Ziemann, *Physics and Finance, Springer, 2021*" (https://link.springer.com/book/10.1007/978-3-030-63643-2)

Fitting a straight line with error bars (Section 7.1 and 7.2)

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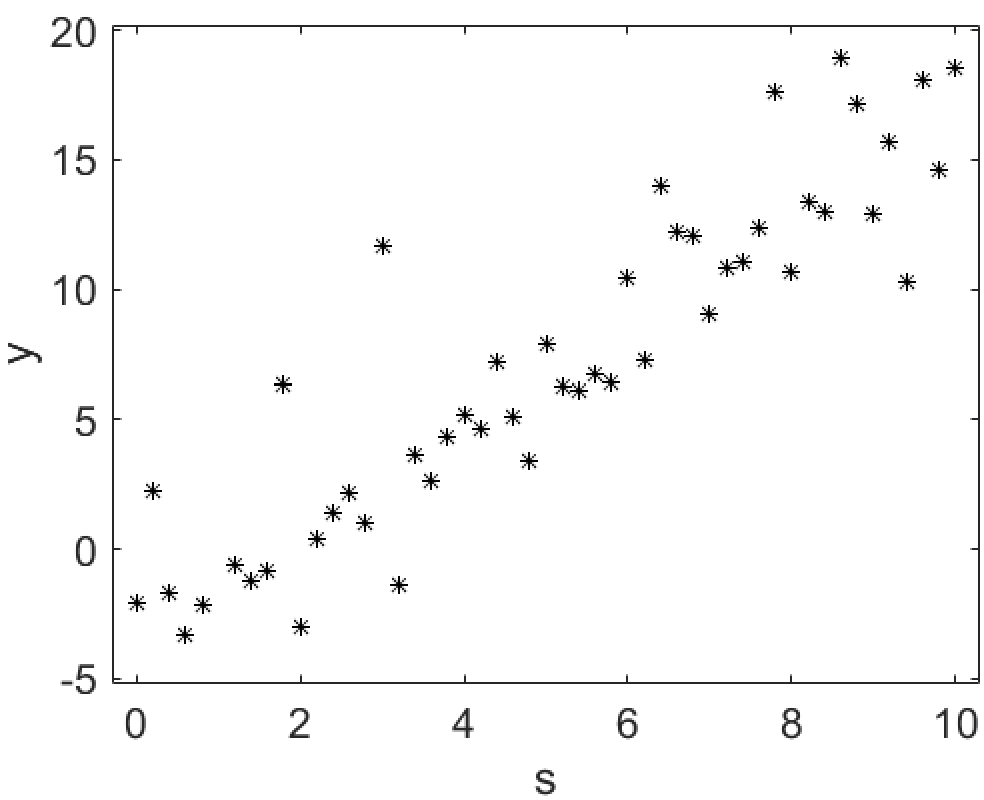
In this example we first generate data points scattered around a straight line and subsequently use a linear fit to recover the parameters of the line and the error bars of those parameters.

Create data points

Let us first generate data points y that are scattered with $\sigma_y = 3$ around $y = as + b = 2s - 3$, plot the data points, and annotate the axes. Note that a is the slope of the line and b is its intercept.

```
clear
sigy=3;                                 % scatter
s=0:0.2:10;
y=2*s-3+sigy*randn(1,length(s));        % straight line with errors
plot(s,y,'k*');                         % data points as black asterisks
xlabel('s'); ylabel('y');               % annotate axes
xlim([-0.3,10.3]); ylim([-5.2,20.2])    % axis limits
set(gca,'FontSize',16)                  % larger fonts
```



Recover the fit parameters from the data points

And now we try to recover the fit parameters a and b from the `N` data points (s, y) by constructing the matrix that appears in Equation 7.1, which relates the slope a and intercept b to the data points.

```
N=length(s);         % number of data points
A=[s',ones(N,1)]     % Matrix in eq. 7.1
```

```
A = 51×2
         0    1.0000
    0.2000    1.0000
    0.4000    1.0000
    0.6000    1.0000
    0.8000    1.0000
    1.0000    1.0000
    1.2000    1.0000
    1.4000    1.0000
    1.6000    1.0000
    1.8000    1.0000
     ⋮
```

Equation 7.1 is an over-determined linear system that, following the argument in Section 7.2, we can solve in the least-squares sense by calculating the Moore-Penrose pseudo-inverse $(A^tA)^{-1}A^t$ and multiplying it with the column vector of y.

```
x=inv(A'*A)*A'*y'   % pseudo-inverse, eq. 7.4
```

```
x = 2×1
    2.0150
   -3.1435
```

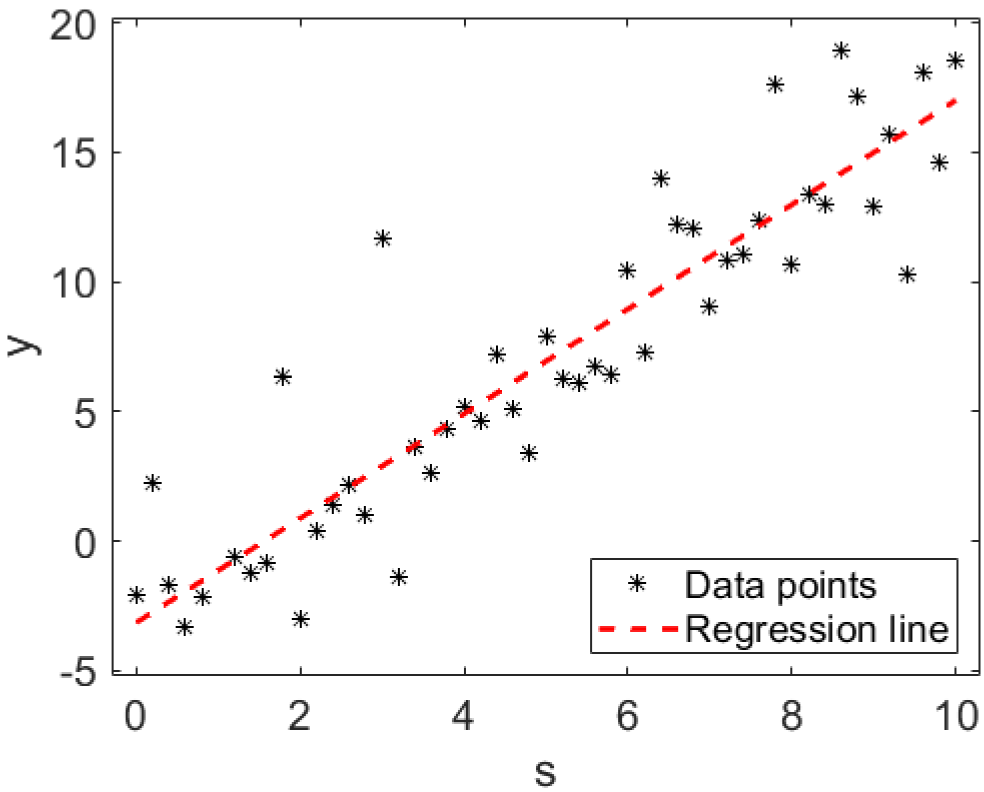
```
% x=pinv(A)*y'
% x=polyfit(s,y,1)
```

Note that Matlab's pinv(A) would also calculate $(A^tA)^{-1}A^t$. Likewise can we use `x=polyfit(s,y,1)` to find the fit parameters `x=(a,b)`. We use the found fit parameters in x to generate the regression line with

```
y_reg=x(1)*s+x(2);
```

and plot it into the same graph as the data points.

```
hold on
plot(s,y_reg,'r--','LineWidth',2)
legend('Data points','Regression line','Location','SouthEast')
```



Running this script multiple times with different random number will produce many different solutions x. We therefore ask ourselves by how much the fit parameters x(1)=a and x(2)=b scatter. In other words, we try to find their error bars.

Error bars of the fit parameters

Of course the error bars of the fit parameters depend on the error bars σ_y of the y. We incorporate them by dividing each row in Equation 7.1 by σ_y, which, in a slightly more general way is equivalent to Equation 7.3 and that leads to Equation 7.5 to take the error bars of the y into account. In that equation we need Λ

```
Lambda=eye(N)/sigy;
```

and find x from Equation 7.5

```
x=inv(A'*Lambda^2*A)*A'*Lambda^2*y'
```

```
x = 2×1
     2.0150
    -3.1435
```

which leads to the same value of x as the previously calculated, because in this particular case all the error bars σ_y are equal and therefore Λ is proportional to the unit matrix.

But by incorporating Λ in the fit we can utilize Equation 7.7 and directly obtain the covariance matrix C of the fit parameters, which contains the error bars on its diagonal.

```
C=inv(A'*Lambda^2*A)
```

```
C = 2×2
    0.0204   -0.1018
   -0.1018    0.6855
```

```
sig_a=sqrt(C(1,1))   % error bar of the slope
```

```
sig_a = 0.1427
```

```
sig_b=sqrt(C(2,2))   % error bar of the intercept
```

```
sig_b = 0.8280
```

We thus find the result for the slope.

```
disp(['a = ',num2str(x(1)),' +/- ',num2str(sqrt(C(1,1)))])  % a was 2
```

```
a = 2.015 +/- 0.1427
```

```
disp(['b = ',num2str(x(2)),' +/- ',num2str(sqrt(C(2,2)))])  % b was -3
```

```
b = -3.1435 +/- 0.82796
```

We add in passing that the error bars σ_y that populate the diagonal of Λ do not need to be equal. We just put the error bar $\sigma_y(i)$ of the $i-$th measurement $y(i)$ on the diagonal to account for individual "measurement errors" of the $y(i)$.