PHYSICS 307 HOMEWORK 7
Due Tuesday, 13 November, at 5 PM
Progress report due 6 November before class

This homework assignment is the first of three parts. I encourage you to try to finish this assignment ahead of time and begin the next one before the deadline of this one. This assignment requires you to write some code; the next assignment will involve using that code to study some physics.

Your "report" for this assignment will just consist of keeping us up-to-date on your progress; next Thursday we'd like to see where you are, and will come around and check on you.

Please start early. I am giving you two weeks to do this assignment in order to accommodate anyone who may have some difficulty. This will be a bit more challenging to code than your previous one; please don't put it off until a few days before it's due. We'll be doing a check-in in a week for a grade; you'll be given a grade based on how well your code works at that point. We don't expect it to be perfect, only that you're making good progress.

In this assignment, you will model a stretched string (as in a guitar or violin) using a lattice elasticity model, in which point masses ("nodes") are connected by Hooke's-law springs. For reference, the force law on any given node i (given in more detail in class) is:

$$\dot{v}_{x_i} = \frac{1}{m} \left[k\hat{r}_{i,i-1_x}(r_{i,i-1} - r_0) + k\hat{r}_{i,i+1}(r_{i,i+1} - r_0) \right] \tag{1}$$

$$\dot{v}_{y_i} = \frac{1}{m} \left[k\hat{r}_{i,i-1_y}(r_{i,i-1} - r_0) + k\hat{r}_{i,i+1}(r_{i,i+1} - r_0) \right] \tag{2}$$

where

$\vec{r}_{i,j}$ is a vector pointing from the i'th node to the j'th one, and is thus equal to $(x_j - x_i)\hat{i} + (y_j - y_i)\hat{j}$, k is the spring constant of a single spring, r_0 is its unstretched length, and m is the mass of each node. As before, you can use the "unit vector trick" to write $\hat{r}_{i,j}$ in terms of the dynamical variables in the problem:

$$\hat{r}_{i,j_x} = \frac{x_i - x_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \tag{3}$$

$$\hat{r}_{i,j_y} = \frac{y_i - y_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \tag{4}$$

A lattice simulation of a stretched string has the following parameters:

- The number of nodes in the simulation $N + 1$ (or the number of springs N)

- The linear mass density of the unstretched string μ

- The unstretched length of the string L_0

- The tension T applied to it to stretch it, which gives you the stretched length L

- The stiffness α (Young's modulus times cross-sectional area)

1. Determine values of m, k, and r_0 (the "microscopic" quantities of interest to your simulation) as a function of N, μ, L_0 and α (the "macroscopic" quantities).

2. Write a program that simulates the vibrations of a stretched string of uniform density and elasticity, as we will discuss in class. (Notes will be posted on the website, too.) You should be able to easily change (by setting variables) some crucial parameters of your simulation: N, μ, L_0, and α.

   You should use the leapfrog solver for your simulation, although it might be useful to code it using Euler-Cromer first as a learning exercise.

3. Determine the initial conditions (all of the x_i's; all the y-coordinates and velocities will be zero) for a stretched but unexcited string with tension T applied to it. You will first need to determine the stretched length L', then put your N masses at equidistant points between $x = 0$ and $x = L'$. Implement these initial conditions, and verify that the string doesn't move.

4. Now stretch your string with sufficient tension to stretch it by 25-100% of its unstretched length, and excite it somehow. You can do this by applying some external force to it at some instant in time, or by choosing initial values for the y_i's. Verify that it moves realistically. A simple initial condition that you can use is a Gaussian deformation, in which

$$y(x) = exp \left[A \frac{-(x - x_c)^2}{\sigma^2} \right]$$

   where x_c is the center of the excitation, σ is its width, A is its amplitude, and σ and x_c are chosen so that the Gaussian dies down before you get to the ends of the string. You may use any other initial conditions you like for testing.

5. Play around with your initial conditions, along with N, the stretch amount, μ, L, and so on. Discuss anything interesting that you find. Note that your behavior ought to be relatively independent of N. Your report should contain descriptions of all the stuff you did and what happened.

6. Modify your program to track conservation of energy (recall that the elastic potential energy in each spring is $U = \frac{1}{2}k(r - r_0)^2$) and verify that energy is approximately conserved.