Physics 307 Homework 4

Due Tuesday, 18 October, at 11 AM

In this homework assignment, you will simulate the Earth going around the Sun, and then generalize your work to other orbits.

To review the discussion in class: For an orbit in the x-y plane, you have four dynamical variables x, y, v_x, and v_y. The four differential equations that govern them are

$$\dot{x} = v_x \tag{1}$$

$$\dot{y} = v_y \tag{2}$$

$$\dot{v}_x = -\frac{GMx}{r^3} \tag{3}$$

$$\dot{v}_y = -\frac{GMy}{r^3} \tag{4}$$

where M is the mass of the Sun.

1. First we will pretend that the Earth is in an exactly circular orbit; this is approximately the case. If we measure distance in astronomical units and time in years,
   - what initial conditions for the Earth's position and velocity vectors will give a circular orbit with the correct radius, and
   - what is the numerical value of GM?

   (Hint: Remember your freshman physics; what is v^2/r?)

2. Use the Euler method (*not* Euler-Cromer) to simulate the Earth going around the Sun. Animate your simulation using `anim`. (Seeing what's going on visually is a great debugging tool!) Qualitatively, how large of a timestep can you use before your simulation breaks down? What does this breakdown consist of?

3. This is supposed to be a circular orbit; specifically, its radius from the Sun is supposed to be a constant. Is it? For a timestep of $dt = 10^{-3}$ year, plot the Earth's radius vs. time over a period of many years. What happens? Can you make this problem go away by reducing the timestep?

4. The total energy per unit mass ("specific orbital energy") is given by the sum of the potential and kinetic energy

$$E_{\text{spec}} = \frac{1}{2}(v_x^2 + v_y^2) - \frac{GM}{r}. \tag{5}$$

Even without an exact analytic solution for orbits (in general), we can look at departures from conservation of energy as another way to gauge the error in the simulation. Plot the total specific energy over a period of many years. What's happening?

5. Now, modify your code to use the Euler-Cromer method. Reexamine the plots of radius vs. time and energy vs. time. What do you see now?

6. Now, finally, modify your code to use the leapfrog method. *Now* look at the plots of radius vs. time and energy vs. time. We will use the leapfrog method for most of the rest of our work this semester.

7. Modify your initial conditions to simulate a non-circular orbit, changing the initial velocity and position of the Earth. (Note that if the total energy becomes positive, the Earth will escape the Sun's gravity.) What shape do these orbits take? At what part of the planet's orbit does it move faster?

8. Finally, consider what happens if you move the Sun away from the origin. Suppose you put the Sun at coordinates (0.2, 0.3). Change your code to simulate orbits around a star not at the origin.

Finally, finish up HW3 if you haven't already.