

# Wamu: A Protocol for Computation of Threshold Signatures by Multiple Decentralized Identities

## Technical Specification

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## 1. Introduction

This document describes the Wamu protocol which augments a state-of-the-art non-interactive threshold signature scheme (e.g. CGGMP20 [1]) by cryptographically associating each signing party with a decentralized identity. This is achieved by:

- Splitting the secret share for each party between the party and the output of a signing operation by its associated decentralized identity thus making the signing operation a requirement for reconstructing the party's secret share.
- Adding peer-to-peer decentralized identity authentication to the key generation and signing protocols (and optionally to the key refresh protocol) of the threshold signature scheme.
- Defining protocols for identity rotation, share addition and removal, threshold modification and share recovery that build on top of the above 2 augmentations.

Wamu is designed to operate in a decentralized, trust-minimized and asynchronous setting with:

- no centralized or trust-based identity infrastructure.
- signing parties being mainstream consumer devices communicating asynchronously.

**NOTE:** For interoperability with existing wallet solutions, the only requirement for decentralized identity providers is the ability to compute cryptographic signatures for any arbitrary message in such a way that the output signature is 1) deterministic and 2) can be verified in a non-interactive manner.

## 2. Preliminaries

The rest of this document describes the Wamu protocol in technical detail. For these descriptions, we'll use the following notation:

- $P$  denotes a party.
- $I$  denotes a decentralized identity.
- $pk$  denotes the address (or public key) of a decentralized identity.
- $sk$  denotes the secret key of a decentralized identity.
- $KeyGen$  denotes a key generation algorithm.
- $Sig$  denotes a signing algorithm.
- $Ver$  denotes a signature verification algorithm.
- $S$  denotes the set of verified decentralized identities for all parties.
- $q$  denotes the prime order of the cyclic group of the elliptic curve.
- $A$  denotes a predefined prefix chosen to ensure that signatures computed for identity authentication cannot be valid transaction signatures.
- $\parallel$  denotes concatenation.

**NOTE:** While the augmenting protocols in this document are described in

relation to the current (circa. 2023) state-of-the-art CGGMP20 [1] non-interactive threshold signature scheme for ECDSA signatures, Wamu is a generic protocol that can be adapted to any non-interactive threshold signature scheme (e.g. GG20 [2] and CMP20 [3]) that allows for asynchronous communication between signing parties.

### 3. Share Splitting and Reconstruction

Given a secret share  $x$  for a party  $P$  with an associated decentralized identity  $I$ , the share splitting and reconstruction protocol describes how to split  $x$  between  $P$  and the output of a signing operation  $Sig$  by  $I$  so that the output of  $Sig$  is required to reconstruct the secret share  $x$ .

This is achieved by generating a message  $m$  (we'll refer to this message as the "signing share") and computing a "sub-share"  $\beta$  (i.e a share of the secret share  $x$ ) in such a way that  $m$  needs to be signed by  $I$  using  $Sig$  to produce another "sub-share"  $\alpha$ , such that  $\alpha$  and  $\beta$  are shares of  $x$  under Shamir's secret-sharing scheme [4].

**NOTE:** Share splitting and reconstruction is a single-party localized concern that happens after (and is not related to) the distributed key generation (DKG) protocol of the threshold signature scheme.

#### 3.1. Share splitting

Given a secret share  $x$  as input and access to the decentralized identity  $I$  with secret key  $sk$ , the share splitting protocol proceeds as follows:

1. Sample a random message  $m$  (i.e. the signing share).
2. Compute a signature  $(r, s) = Sig(sk, m)$ .
3. Compute the first sub-share of  $x$  as the point  $\alpha = (r, s) \pmod{q}$ .
4. Generate a line  $L$  (i.e a polynomial of degree 1) such that  $\alpha$  is a point on the line and  $x$  is the constant term (i.e. Polynomial Interpolation [5])
5. Compute another point  $\beta$  from  $L$  such that  $\beta \neq \alpha$ ,  $\beta$  becomes the second sub-share of  $x$ .
6. Erase both  $\alpha$  and  $L$  from memory.
7. Return the signing share  $m$  and the sub-share  $\beta$ .

#### 3.2. Share reconstruction

Given a signing share  $m$  and a sub-share  $\beta$  as input (i.e. the outputs of the share splitting protocol in section 3.1) and access to the decentralized identity  $I$  with secret key  $sk$ , the share reconstruction protocol proceeds as follows:

1. Compute a signature  $(r, s) = Sig(sk, m)$ .
2. Compute a sub-share  $\alpha$  as the point  $\alpha = (r, s) \pmod{q}$ .
3. Generate the line  $L$  by performing Polynomial Interpolation [5] using  $\alpha$  and  $\beta$  as inputs.

4. Compute  $x$  as the constant term of  $L$ .
5. Erase both  $\alpha$  and  $L$  from memory.
6. Return  $x$  as the secret share.

**NOTE:** For ECDSA signatures, the parameters  $r$  and  $s$  in  $(r, s) = \text{Sig}(sk, m)$  are already computed modulo  $q$ . We use the notation  $\alpha = (r, s) \pmod{q}$  for the sub-share to make it clear (at a glance) that the sub-shares are computed using finite field arithmetic.

## 4. Key Generation

Follow the key generation protocol described in section 3.1 and figure 5 of CGGMP20 [1] to generate ECDSA secret shares with the following modifications:

1. At the end of Round 1, broadcast 2 additional parameters for each  $P_i$  associated with the decentralized identity  $I_i$  with address  $pk_i$  and secret key  $sk_i$  as follows:
  - The decentralized identity address  $pk_i$ .
  - The signature  $\varphi_i = \text{Sig}(sk_i, A \parallel V_i)$ .
2. At the beginning of Round 2, for each  $P_i$ , verify  $\varphi_j$  from all  $P_j$  where  $j \neq i$  by checking that the output of  $\text{Ver}(pk_j, A \parallel V_j, \varphi_j)$  is valid or report the culprit and halt.
3. After the Output phase, follow the share splitting protocol in section 3.1 to split secret share  $x_i$  into a signing share  $m_i$  and a sub-share  $\beta_i$  for each party  $P_i$ .
4. Modify Stored State for each  $P_i$  as follows:
  - Don't store  $x_i$ .
  - Add  $pk_i$ ,  $m_i$ ,  $\beta_i$  and  $S_i = \{pk_j : j \neq i\}$  (i.e the set of verified decentralized identities for all other parties).

## 5. Key Refresh

Follow the key refresh protocol described in section 3.2 and figure 6 of CGGMP20 [1] to generate new ECDSA secret shares with the following modifications:

1. At the end of Round 1, broadcast 2 additional parameters for each  $P_i$  associated with the decentralized identity  $I_i$  with address  $pk_i$  and secret key  $sk_i$  as follows:
  - The decentralized identity address  $pk_i$ .
  - The signature  $\varphi_i = \text{Sig}(sk_i, A \parallel V_i)$ .
2. At the beginning of Round 2, for each  $P_i$ , verify  $\varphi_j$  from all  $P_j$  where  $j \neq i$  as follows:
  - Verify that  $pk_i \in S_j$  or report the culprit and halt.
  - Verify  $\varphi_i$  by checking that the output of  $\text{Ver}(pk_j, A \parallel V_j, \varphi_j)$  is valid or report the culprit and halt.
3. After the Output phase, follow the share splitting protocol in section 3.1 to split the new secret share  $x_i^*$  into a new signing share  $m_i^*$  and a new

sub-share  $\beta_i^*$  for each party  $P_i$ .

4. Modify Stored State for each  $P_i$  as follows:
  - Don't store  $x_i^*$ .
  - Replace  $m_i$  with  $m_i^*$  and  $\beta_i$  with  $\beta_i^*$ .

## 6. Signing

Follow the signing protocol described in sections 4.2 and 4.3 and figure 8 of CGGMP20 [1] to generate an ECDSA signature with the following modifications:

1. Before Round 1, for each party  $P_i$ , follow the share reconstruction protocol in section 3.2 to reconstruct secret share  $x_i$ .
2. At the end of Round 1, for each  $P_i$  associated with the decentralized identity  $I_i$  with address  $pk_i$  and secret key  $sk_i$ , send 2 additional parameters to all  $P_j$  where  $j \neq i$  as follows:
  - The decentralized identity address  $pk_i$ .
  - The signature  $\varphi_i = \text{Sig}(sk_i, A||m)$ .
3. At the beginning of the Output phase, verify  $\varphi_j$  from all  $P_j$  where  $j \neq i$  as follows:
  - Verify that  $pk_i \in S_j$  or report the culprit and halt.
  - Verify  $\varphi_i$  by checking that the output of  $\text{Ver}(pk_i, A||m, \varphi_i)$  is valid or report the culprit and halt.

## 7. Identity Authenticated Request Initiation and Verification

Decentralized identity authenticated requests allow parties to perform or request actions based on their associated decentralized identity.

### 7.1. Identity Authenticated Request Initiation

To initiate an identity authenticated request with a command  $C$  from a party  $P_i$  associated with decentralized identity  $I_i$  with address  $pk_i$  and secret key  $sk_i$ :

1. Read the current UTC timestamp  $t$ .
2. Compute the signature  $\varphi = \text{Sig}(sk_i, A||C||t)$ .
3. Broadcast  $C$ ,  $pk_i$ ,  $t$  and  $\varphi$ .

### 7.2. Identity Authenticated Request Verification

To verify an identity authenticated request with a command  $C$  from a party  $P_i$  given its associated decentralized identity address  $pk_i$ , a timestamp  $t$ , a signature  $\varphi$  and a set of verified decentralized identities for all other parties  $S_j$  as input:

1. Verify that  $pk_i \in S_j$  or report the culprit and halt.
2. Verify that  $t$  is within the current epoch for identity authenticated requests or report the culprit and halt.

3. Verify  $\varphi$  by checking that the output of  $Ver(pk_i, A\|C\|t, \varphi)$  is valid or report the culprit and halt.

## 8. Identity Challenge

Identity challenges are used to verify that a party controls a decentralized identity.

**8.1. Identity Challenge Initiation** To issue an identity challenge to a party  $P_i$  from all verifying parties  $P_j$  where  $j \neq i$ : 1. Sample a random  $v_j$ . 2. Broadcast  $v_j$  to all parties, such that all parties can compute  $v = \parallel_{j \neq i} v_j$ .

**8.2. Identity Challenge Response** For a party  $P_i$  with associated decentralized identity secret key  $sk_i$ , to respond to an identity challenge given  $v_j$  from all parties  $P_j$  where  $j \neq i$ :

1. Compute  $v = \parallel_{j \neq i} v_j$ .
2. Compute the signature  $\psi = Sig(sk_i, A\|v)$ .
3. Broadcast  $\psi$  to all verifying parties  $P_j$ .

**8.3. Identity Challenge Response Verification** To verify an identity challenge response from a party  $P_i$  given its associated decentralized identity address  $pk_i$ , a signature  $\psi$  and  $v_j$  from all verifying parties  $P_j$  where  $j \neq i$  as input:

1. Compute  $v = \parallel_{j \neq i} v_j$ .
2. Verify  $\psi$  by checking that the output of  $Ver(pk_i, A\|v, \psi)$  is valid or report the culprit and halt.

## 9. Identity Rotation

Identity rotation allows any party to change the decentralized identity associated with its secret share.

Identity rotation for a party  $P_i$  from a decentralized identity  $I_i$  with address  $pk_i$  and secret key  $sk_i$  to a decentralized identity  $I_i^*$  with address  $pk_i^*$  and secret key  $sk_i^*$  proceeds as follows:

1. For  $P_i$ , initiate an “identity-rotation” request by following the protocol in section 7.1.
2. For all  $P_j$  where  $j \neq i$ :
  - Verify the “identity-rotation” request by following the protocol in section 7.2.
  - Initiate an identity challenge for  $P_i$  by following the protocol in section 8.1.
3. For  $P_i$ , respond to the identity challenge by following the protocol in section 8.2 with the following augmentations:

- Compute an additional signature  $\psi_i^* = \text{Sig}(sk_i^*, A \| v)$ .
  - Add  $pk_i^*$  and  $\psi_i^*$  to the broadcast parameters.
4. For all  $P_j$  where  $j \neq i$ :
    - Verify the identity challenge response from  $P_i$  by following the protocol in section 8.3.
    - Verify that  $P_i$  controls the new decentralized identity address  $pk_i^*$  as follows:
      - Compute  $v = \parallel_{j \neq i} v_j$ :
      - Verify  $\psi^*$  by checking that the output of  $\text{Ver}(pk_i^*, A \| v, \psi^*)$  is valid or report the culprit and halt.
    - Modify Stored State as follows:
      - Create  $S_i^*$  by replacing  $pk_i$  with  $pk_i^*$  in  $S_i$ .
      - Replace  $S_i$  with  $S_i^*$ .
    - Send confirmation of successful rotation of the identity to  $P_i$ .
  5. For  $P_i$ , upon receiving confirmation of successful rotation from a quorum of  $P_j$ :
    - Compute the new signing share  $m_i^*$  and sub-share  $\beta_i^*$  based on the new decentralized identity  $I_i^*$  as follows:
      - Compute the secret share  $x_i$  by following the share reconstruction protocol in section 3.2.
      - Follow the share splitting protocol in section 3.1 to split  $x_i$  into a new signing share  $m_i^*$  and a new sub-share  $\beta_i^*$  based on the new decentralized identity  $I_i^*$ .
    - Modify Stored State as follows:
      - Replace  $pk_i$  with  $pk_i^*$ .
      - Replace  $m_i$  with  $m_i^*$ .
      - Replace  $\beta_i$  with  $\beta_i^*$ .

## 10. Quorum Approved Request Initiation and Verification

Quorum approved requests allow any verified party to initiate actions that require explicit approval from a quorum of verified parties before execution (e.g. share addition and removal, and threshold modification).

A quorum approved request with a command  $C$  from a party  $P_i$  associated with decentralized identity  $I_i$  with address  $pk_i$  and secret key  $sk_i$  proceeds as follows:

1. For  $P_i$ , initiate an identity authenticated request by following the protocol in section 7.1.
2. For all  $P_j$  where  $j \neq i$  that approve the requested action:
  - Verify the identity authenticated request by following the protocol in section 7.2.
  - Initiate an identity challenge for  $P_i$  by following the protocol in section 8.1 with the following augmentations:
    - Compute a signature  $\phi_j = \text{Sig}(sk_j, A \| v_j \| C \| t)$ .
    - Add  $pk_j$  and  $\phi_j$  to the broadcast parameters.

3. For  $P_i$ , upon receiving an augmented identity challenge from a quorum  $S_c$  such that  $S_c \subset S_j$  and  $j \neq i$ , respond to the identity challenge by following the protocol in section 8.2 with the following modifications:
  - At the beginning of the identity challenge response protocol, verify that approvals have been received from a quorum  $S_c$  by checking that  $\exists S_c \subset S_j$  such that  $\forall pk_c \in S_c$  where  $c \neq i$ , the output of  $Ver(pk_c, A||v_c||C||t, \phi_c)$  is valid or halt.
  - Compute  $v$  as  $v = \parallel_{c \neq i} v_c$ .
  - Add  $S_c$  to the broadcast parameters.
4. For all  $P_j$  where  $j \neq i$ :
  - Verify the augmented identity challenge response from  $P_i$  by following the protocol in section 8.3 with the following modifications:
    - Compute  $v$  as  $v = \parallel_{c \neq i} v_c$ .
  - Verify that a valid quorum  $S_c$  such that  $S_c \subset S_j$  and  $j \neq i$  has approved the request as follows:
    - Verify that  $\forall pk_c \in S_c$ ,  $pk_c \in S_j$  where  $j \neq i$  and  $c \neq i$  or report the culprit and halt:
    - Verify that  $\forall pk_c \in S_c$  where  $c \neq i$ , the output of  $Ver(pk_c, A||v_c||C||t, \phi_c)$  is valid or report the culprit and halt.

## 11. Share Addition and Removal

Share addition and removal allows a quorum of verified parties to either issue a secret share to a new party and its associated decentralized identity, or revoke the secret share of any party respectively.

### 11.1. Share Addition

Share addition for a new party  $P_i$  with associated decentralized identity  $I_i$  proceeds as follows:

1. Initiate a quorum approved “share-addition” request by following the protocol in section 10.
2. Follow the key refresh protocol described in section 5 with  $P_i$  included as a participant if the quorum approved request above succeeded.

### 11.2. Share Removal

Share removal for a party  $P_i$  with associated decentralized identity  $I_i$  proceeds as follows:

1. Initiate a quorum approved “share-removal” request by following the protocol in section 10.
2. Follow the key refresh protocol described in section 5 without  $P_i$  if the quorum approved request above succeeded.



## 12. Threshold Modification

Threshold modification allows a quorum of verified parties to change the threshold (i.e. change the size of the quorum).

While threshold modification (or more generally  $t$ -out-of- $n$  sharing, and specifically the case where  $n > t + 1$ ) is not formally specified in CGGMP20 [1], it can be derived in a relatively straightforward manner based on GG18 [6] (and GG20 [2]) which CGGMP20 [1] builds upon (see sections 1.2.8, 1.2.1 and 1.2.2 of CGGMP20 [1]). In general, CGGMP20 [1] can be seen as a combination of CMP20 [3] and GG20 [2], and a direct improvement on GG18 [6].

Therefore, threshold modification can be achieved by following the key refresh protocol described in section 3.2 and figure 6 of CGGMP20 [1] and section 5 of this document, with some modifications based on the key generation protocols described in GG18 [6] and GG20 [2], and following the instructions in section 1.2.8 of CGGMP20 [1].

In particular, this entails performing a  $t$ -out-of- $n$  Feldman’s VSS [7] sharing of the values  $x_i^k$  (as defined in section 3.2 of CGGMP20 [1]), with the new threshold  $t$  used as the threshold parameter (similarly defined as  $t$ ) for Feldman’s VSS [7] protocol as described in section 2.8 and phase 2 of section 3.1 in GG20 [2] (and similarly in section 2.6 and phase 2 of section 4.1 in GG18 [6]).

Threshold modification would then proceed as follows:

1. Initiate a quorum approved “threshold-modification” request by following the protocol in section 10.
2. Follow the key refresh protocol described in section 5 with the modifications described above if the quorum approved request succeeds.

**NOTE:** Similar modifications can be applied to the key generation protocol described in section 3.1 and figure 5 of CGGMP20 [1] and section 6 of this document to achieve a  $t$ -out-of- $n$  sharing of the secret key for  $n \geq t + 1$ . In particular, this entails performing a  $t$ -out-of- $n$  Feldman’s VSS [7] sharing of the value  $x_i$  (as defined in section 3.1 of CGGMP20 [1]), based on the same modifications from GG20 [2] and GG18 [6] described above, and following the instructions in section 1.2.8 of CGGMP20 [1].

## 13. Share Recovery

Share recovery is only possible if the user’s decentralized identity either survived or can be recovered after the disastrous event. In either case, there are two options for share recovery depending on:

- A quorum of honest parties surviving the disastrous event.
- A backup (preferably encrypted) of a signing share  $m$  and sub-share  $\beta$  pair on user-controlled secondary or device-independent storage.

### 13.1. Share recovery with a surviving quorum of honest parties

If a quorum of honest parties survives the disastrous event, share recovery can be accomplished based on peer-to-peer decentralized identity authentication.

Share recovery for a party  $P_i$  with associated decentralized identity  $I_i$  with address  $pk_i$  and secret key  $sk_i$  proceeds as follows:

1. For  $P_i$ , Initiate a “share-recovery” request by following the protocol in section 7.1.
2. For all  $P_j$  where  $j \neq i$ :
  - Verify the “share-recovery” request by following the protocol in section 7.2.
  - Initiate an identity challenge for  $P_i$  by following the protocol in section 8.1.
3. For  $P_i$ , respond to the identity challenge by following the protocol in section 8.2.
4. For all  $P_j$  where  $j \neq i$ , verify the identity challenge response from  $P_i$  by following the protocol in section 8.3.
5. Follow the key refresh protocol described in section 5 if all verifications above pass.

### 13.2. Share recovery with a backup on user-controlled secondary or device-independent storage

**13.2.1. Overview of share recovery with a backup** From the share splitting and reconstruction protocol in section 3, we note that for any party  $P$ , the combination of a signing share  $m$  and a sub-share  $\beta$  alone is insufficient to reconstruct the secret share  $x$ . This is because a signature of  $m$  from the decentralized identity  $I$  is required to compute the sub-share  $\alpha$ , so that  $\alpha$  and  $\beta$  can then be used to reconstruct  $L$  and compute the secret share  $x$  as the constant term of  $L$ .

Therefore, a signing share  $m$  and sub-share  $\beta$  pair can be safely backed up to user-controlled secondary (e.g. a secondary device or a flash drive) or device-independent storage (e.g. Apple iCloud <sup>1</sup>, Google Drive <sup>2</sup>, Microsoft OneDrive <sup>3</sup>, Dropbox <sup>4</sup> e.t.c) without exposing the secret share  $x$ .

**13.2.2. Generating an encrypted backup for share recovery** For increased security, a signature of a standardized phrase can be used as entropy for generating an encryption secret which can then be used to encrypt the signing share  $m$  and the sub-share  $\beta$  using a symmetric encryption algorithm before saving them to back up storage.

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<sup>1</sup>Apple iCloud. <https://www.icloud.com>.

<sup>2</sup>Google Drive. <https://drive.google.com>.

<sup>3</sup>Microsoft OneDrive. <https://www.microsoft.com/en-us/microsoft-365/onedrive/online-cloud-storage>.

<sup>4</sup>Dropbox. <https://www.dropbox.com>.

Given a standardized phase  $k$ , a key derivation function  $H$ , a symmetric encryption algorithm  $E$ , this proceeds as follows:

1. Compute the signature  $\phi = \text{Sig}(sk, k)$ .
2. Generate the encryption secret  $\varepsilon = H(\phi)$ .
3. Compute the ciphertext for the signing share  $m$  as  $m_c = E_{enc}(m, \varepsilon)$ .
4. Compute the ciphertext for the sub-share  $\beta$  as  $\beta_c = E_{enc}(\beta, \varepsilon)$ .
5. Erase both  $\phi$  and  $\varepsilon$  from memory.
6. Save  $m_c$  and  $\beta_c$  to backup storage.

**13.2.3. Decrypting an encrypted backup** Share recovery would then start by signing this standardized phrase, using the signature to recreate the encryption secret and then decrypting the encrypted backup to retrieve the signing share  $m$  and the sub-share  $\beta$ .

Given a standardized phase  $k$ , a key derivation function  $H$ , a symmetric encryption algorithm  $E$ , the ciphertext for the signing share  $m_c$  and the ciphertext for the sub-share  $\beta_c$ , this proceeds as follows:

1. Compute the signature  $\phi = \text{Sig}(sk, k)$ .
2. Generate the encryption secret  $\varepsilon = H(\phi)$ .
3. Compute the signing share  $m = E_{dec}(m_c, \varepsilon)$ .
4. Compute the sub-share  $\beta = E_{dec}(\beta_c, \varepsilon)$ .
5. Erase both  $\phi$  and  $\varepsilon$  from memory.
6. Return the signing share  $m$  and the sub-share  $\beta$ .

**13.2.4. Further security and usability considerations for share recovery with a backup** For further improved security and usability, the signing share  $m$  can be prefixed with a custom message that alerts the user to the purpose of the signature. This can help reduce the effectiveness of an adversary that gains access to the backup and tries to trick the user into signing  $m$ .

Additionally, it's possible to rerun the share splitting protocol to generate a new pair of a signing share  $m^*$  and a sub-share  $\beta^*$  such that  $m^* \neq m$ ,  $\beta^* \neq \beta$  and  $L^* \neq L$  to be specifically used for backup and recovery. This gives us the option to have separate signing shares for backup and recovery with customized prefixes that make it clear to the user that they're signing a backup signing share.

Lastly, the “backup” signing share  $m^*$  can be generated based on user input (e.g. a passphrase or security questions) removing the need for it to be backed up together with a sub-share  $\beta^*$  but instead relying on the user to provide this input during recovery as a security-usability tradeoff.

## 14. Acknowledgements

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<sup>5</sup>Ethereum Foundation: Ecosystem Support Program. <https://esp.ethereum.foundation>.

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