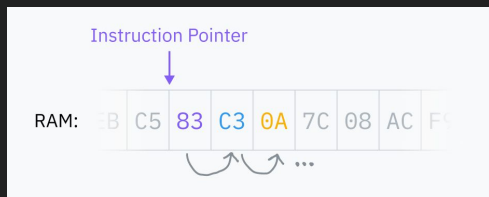


# Learning to Execute Programs with Instruction Pointer Attention Graph Neural Networks

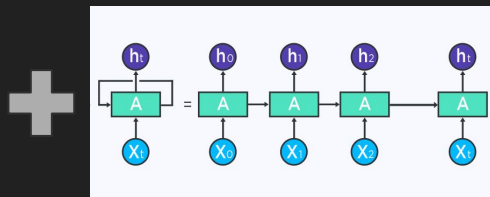
**Authors:** David Bieber, Charles Sutton, Hugo Larochelle, Daniel Tarlow (Google)

**Presenter:** [Xingjian Zhang \(UMich\)](#)

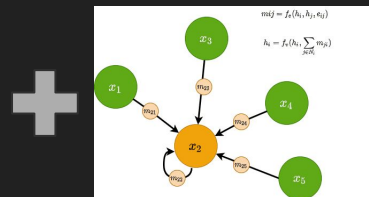
[Link-to-Slides](#)



Instruction Pointer



RNN



MPNN (GNN)

# Outline

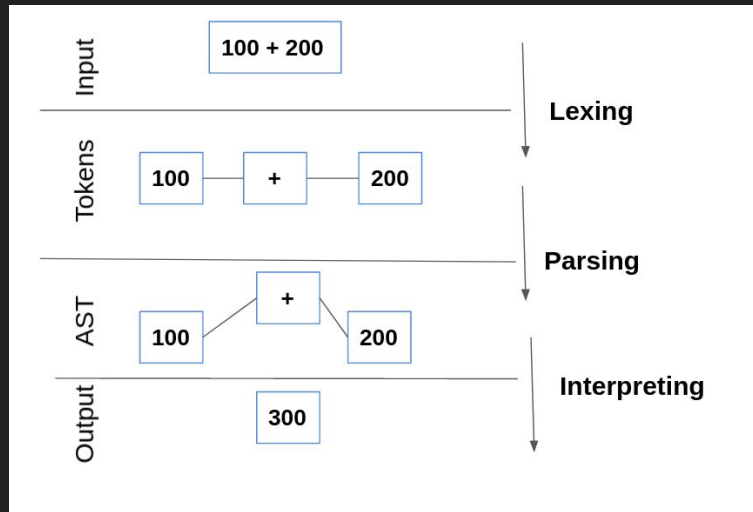
- Background
- Problem Formulation
- Approach
- Experiment
- Conclusion

# Outline

- Background
  - What is “learning to execute program”?
  - Why GNN?
- Problem Formulation
- Approach
- Experiment
- Conclusion

# GNNs are powerful tool for learning SDE tasks

- Many applications
  - Code completion
  - Bug finding
  - Program repair
  - Learning to execute programs
- Inherent graph structure in programs
  - parse trees
  - data flow graphs
  - control flow graphs (see example later)



# Learning to execute programs

*Produce the output of a program, without actually running the program.*

*Introduced by Wojciech Zaremba and Ilya Sutskever. Learning to execute, 2014 (applied RNN to this task)*

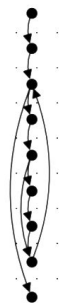
- **Challenges**

- Prediction in bounded time
- Complex structure of programs
  - contains branch created by “if-else”, “while” loop, etc.
- Complex reasoning about program executions
  - hard to predict the discrete branch decisions

- **Non-trivial for vanilla GNN/RNNs**

- GNN - good at complex structure, but not sequential reasoning
- RNN - good at sequential reasoning, but not complex structure

# Example: Control Flow Graph

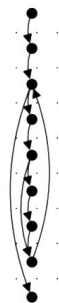
$n$	Source	Tokenization ( $x_n$ )				Control flow graph ( $n \rightarrow n'$ )	$N_{in}(n)$	$N_{out}(n)$
0	<code>v0 = 23</code>	0	=	v0	23		$\emptyset$	$\{1\}$
1	<code>v1 = 6</code>	0	=	v1	6		$\{0\}$	$\{2\}$
2	<code>while v1 &gt; 0:</code>	0	<code>while &gt;</code>	v1	0		$\{1, 7\}$	$\{3, 8\}$
3	<code>  v1 -= 1</code>	1	<code>-=</code>	v1	1		$\{2\}$	$\{4\}$
4	<code>  if v0 % 10 &lt;= 3:</code>	1	<code>if &lt;= %</code>	v0	3		$\{3\}$	$\{5\}$
5	<code>    v0 += 4</code>	2	<code>+=</code>	v0	4		$\{4\}$	$\{6\}$
6	<code>    v0 *= 6</code>	2	<code>*=</code>	v0	6		$\{5\}$	$\{7\}$
7	<code>  v0 -= 1</code>	1	<code>-=</code>	v0	1		$\{4, 6\}$	$\{2\}$
8	<code>&lt;exit&gt;</code>	-	-	-	-		$\{2, 8\}$	$\{8\}$

**Nodes:** individual line statements

**Directed edges:** possible sequences of execution of statements

**Instruction pointer (IP):** indicates the next instruction to execute

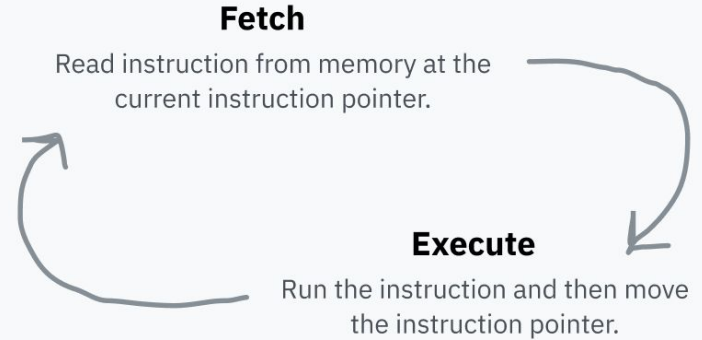
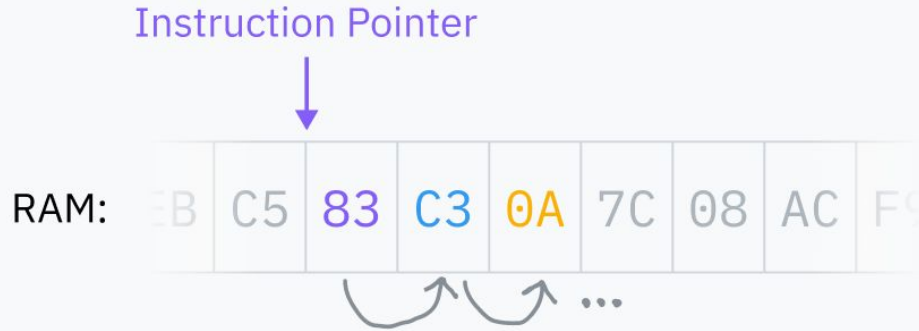
# Example: Control Flow Graph

$n$	Source	Tokenization ( $x_n$ )				Control flow graph ( $n \rightarrow n'$ )	$N_{in}(n)$	$N_{out}(n)$
0	v0 = 23	0	=	v0	23		$\emptyset$	$\{1\}$
1	v1 = 6	0	=	v1	6		$\{0\}$	$\{2\}$
2	while v1 > 0:	0	while	>	v1		$\{1, 7\}$	$\{3, 8\}$
3	v1 -= 1	1	-=	v1	1		$\{2\}$	$\{4\}$
4	if v0 % 10 <= 3:	1	if	<=	%		$\{3\}$	$\{5\}$
5	v0 += 4	2	+=	v0	4		$\{4\}$	$\{6\}$
6	v0 *= 6	2	*=	v0	6		$\{5\}$	$\{7\}$
7	v0 -= 1	1	-=	v0	1		$\{4, 6\}$	$\{2\}$
8	<exit>	-	-	-	-		$\{2, 8\}$	$\{8\}$

Each line of a program is represented by a 4-tuple tokenization containing that line's (indentation level, operation, variable, operand), and is associated with a node in the program's statement-level control flow graph.

Notice that branch decisions are always binary. i.e.  $|N_{out}(n)| \leq 2$ .

# Instruction Pointer



- A concept borrowed from computer architecture.



# Outline

- Background
- Problem Formulation
  - What information are available?
  - What restrictions are imposed?
  - What practical evaluations are considered?
- Approach
- Experiment
- Conclusion


# Two variants

- **Full program execution**

- Input: Full program
- Output: some semantic property of the program (e.g. program output)
- Evaluate the expressiveness and learnability

- **Partial program execution**

- Input: Partially masked program
- Output: Same as above
- Useful in certain downstream applications

<i>n</i>	Source	<i>n</i>	Source
0	v0 = 23	0	v0 = 23
1	v1 = 6	1	
2	while v1 > 0:	2	while v1 > 0:
3	v1 -= 1	3	v1 -= 1
4	if v0 % 10 <= 3:	4	if v0 % 10 <= 3:
5	v0 += 4	5	v0 += 4
6	v0 *= 6	6	v0 *= 6
7	v0 -= 1	7	v0 -= 1
8	<exit>	8	<exit>

# Problem Formulation (ML for Static Analysis)

- **Access to**
  - Textual source of the program
  - Parse tree of the program
  - Any common static analysis results, e.g. control flow graph
- **No access to**
  - Compiler (compile-time)
  - Interpreter (runtime)
  - Dependencies, test suite, etc.
  - Any artifacts not readily available for normal static analysis

Similar to standard static analysis

# Other Specifications

- **Bounded execution**
  - Restrict the model to use fewer steps than are required by the ground truth trace
  - Force short-cuts learning
  - *Motivation: Static analysis requires low latency*
- **Systematic generalization**
  - Train the model with limited complexity
  - Test the model on more complex programs
  - *Motivation: People write programs to do things that have not been done before*
  - *Motivation: Reduce training cost while preserving performance on complex real-world codebase*

Fast prediction & Generalization to long programs

# Formal Specification & Notations

- Given

- D: Dataset consisting of pairs  $(x, y)$ 
  - $x$ : program (code & control flow graph)
    - $x_n$ :  $n$ -th line statement of  $x$
    - $N_{in}(n)$ : the set of statements that can immediately precede  $x_n$
    - $N_{out}(n)$ : the set of statements that can immediately follow  $x_n$
    - $N_{all}(n)$ :  $N_{in}(n) \cup N_{out}(n)$
  - $y$ : a semantic property of the program (*\*integer target is used in paper*)
    - Follow-up work by Google: predict runtime error
- $c(x)$ : Complexity function
- $D_{train}$  contains examples such that  $c(x) \leq C$ ,  $D_{test}$  contains others

Standard supervised learning formulation


# Outline

- Background
- Problem Formulation
- Approach
  - [Baseline] Instruction Pointer RNN Models (IP-RNN)
  - **Instruction Pointer Attention GNN (IPA-GNN)**
  - [Baseline] Standard MPNN (GGNN)
- Experiment
- Conclusion

# Instruction Pointer (IP) Modeling **without** Branch

- Let's start with a special case.



```
v0 = 1
v0 += 2
v1 = v0 + 3
v0 *= 4
```



- Consider a classical interpreter to execute **straight-line** program
  - maintains a state consisting of the values of all variables in the program
  - maintains an instruction pointer indicating the next statement to execute (always pointing to the next statement)
  - updates the instruction pointer to next statement when a statement is executed (causal structure)
- RNN is a natural choice of architecture
  - Reason: same causal structure

# Model 1: Line-by-Line RNN

- At step  $t$  of interpretation,  $h_t = \text{RNN}(h_{t-1}, \text{Embed}(x_{n_{t-1}}))$ 
  - $h_t$  is the hidden state (“state in interpreter”)
  - $n_t = t$  is the model’s instruction pointer
    - At each step, always increment the IP by 1
    - Correct only for straight-line program

$n$	Source	Control flow graph	Line-by-Line RNN
0	<code>v0 = 407</code>		
1	<code>if v0 % 10 &lt; 3:</code>		
2	<code>    v0 += 4</code>		
3	<code>else:</code>		
4	<code>    v0 -= 2</code>		
5	<code>&lt;exit&gt;</code>		





# Instruction Pointer (IP) Modeling **with** Branch

- In general, setting  $n_{\dagger} = \dagger$  is problematic!
  - Most programs are not straight-line.
- One can define different rules for updating the IP.
  - We call these model variants Instruction Pointer RNNs (IP-RNNs)













## Model 2: Trace RNN (Oracle)

- At step  $t$  of interpretation,  $h_t = \text{RNN}(h_{t-1}, \text{Embed}(x_{n_{t-1}}))$ 
  - $n_t$  is the model's ground truth trace of IP
    - At each step, always points to the next exact statement to execute
    - Correct for all programs

$n$	Source	Control flow graph	Trace RNN
0	<code>v0 = 407</code>		
1	<code>if v0 % 10 &lt; 3:</code>		
2	<code>    v0 += 4</code>		
3	<code>else:</code>		
4	<code>    v0 -= 2</code>		
5	<code>&lt;exit&gt;</code>		

























# Model 3: Hard IP-RNN

- At step  $t$  of interpretation,  $h_t = \text{RNN}(h_{t-1}, \text{Embed}(x_{n_{t-1}}))$ 
  - $n_t = N_{\text{out}}(n_{t-1})|_j$  where  $j = \arg \max \text{Dense}(h_t)$ 
    - Not differentiable but might be trained by a supervised fashion\*
    - More precisely, invalidates fully differentiable end-to-end training
  - The authors did not provide implementation or report experimental results for this model

$n$	Source	Control flow graph	Hard IP-RNN
0	<code>v0 = 407</code>		
1	<code>if v0 % 10 &lt; 3:</code>		
2	<code>  v0 += 4</code>		
3	<code>else:</code>		
4	<code>  v0 -= 2</code>		
5	<code>&lt;exit&gt;</code>		

# Summary of IP-RNN

Oracle IP

$n$	Source	Control flow graph	Line-by-Line RNN	Trace RNN	Hard IP-RNN
0	<code>v0 = 407</code>				
1	<code>if v0 % 10 &lt; 3:</code>				
2	<code>    v0 += 4</code>				
3	<code>else:</code>				
4	<code>    v0 -= 2</code>				
5	<code>&lt;exit&gt;</code>				

Sequential IP

Predictive IP w/o gradient

# Continuous Relaxation of Discrete Branch Choice

- Consider soft branch decision
  - A distribution over the possible branches (from a particular statement)
- Replace  $n_t$  with  $p_{t,n}$  (so-called IPA)
  - At step  $t$ ,  $p_{t,n}$  is a distribution over all statements  $x_n$ .
- Replace  $h_t$  with  $h_{t,n}$  (Intuition: state in an interpreter)
  - At step  $t$ ,  $h_{t,n}$  models the representation assuming the program is executing statement  $n$ .
  - Intuition: we want the model to have a different representation of the program state for possible IPs.

# Model 4: IPA-GNN

$n$	Source	Control flow graph	IPA-GNN
0	<code>v0 = 407</code>		
1	<code>if v0 % 10 &lt; 3:</code>		
2	<code>    v0 += 4</code>		
3	<code>else:</code>		
4	<code>    v0 -= 2</code>		
5	<code>&lt;exit&gt;</code>		

Branch prediction

Soft branch decisions

## Model 4: IPA-GNN

- State proposal is produced for **each possible current statement  $n$** 
  - **IPA-GNN:**  $a_{t,n} = \text{RNN}(h_{t-1,n}, \text{Embed}(x_n))$
  - **IP-RNN:**  $h_t = \text{RNN}(h_{t-1}, \text{Embed}(x_{n_{t-1}}))$  only for statement  $n_{t-1}$
- Consider special case
  - For straight-line code, i.e.  $|\text{N}_{\text{out}}(x_n)| = 1, n \rightarrow n'$ 
    - Simply have  $h_{t,n'} = a_{t,n}$

However, in general, a lot of potential traces may lead to current statement

# Model 4: IPA-GNN

- Soft branch determines how much of the state proposals flow to each next statement
  - When branching between  $n_1$  and  $n_2$

$$b_{t,n,n_1}, b_{t,n,n_2} = \text{softmax}(\text{Dense}(a_{t,n}))$$

- Update hidden state from all potential traces

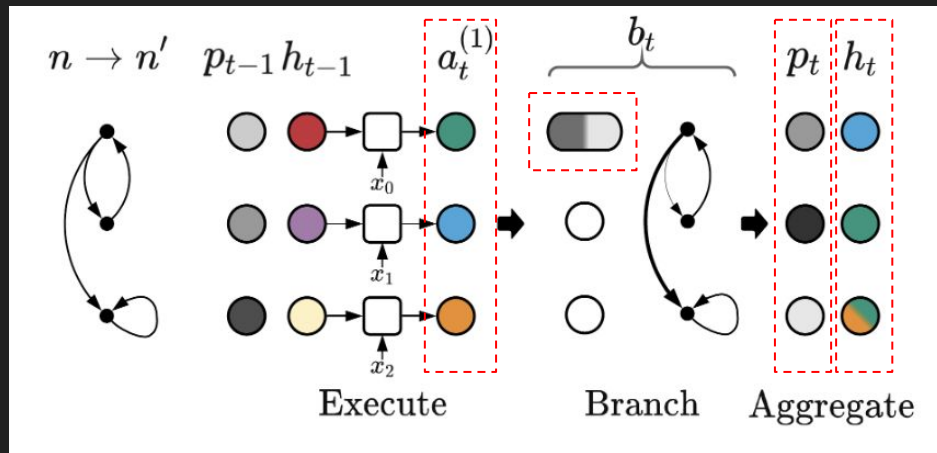
$$h_{t,n} = \sum_{n' \in N_{\text{in}}(n)} p_{t-1,n'} \cdot b_{t,n',n} \cdot a_{t,n}$$

- Update IPA (probability distribution of IP)

$$p_{t,n} = \sum_{n' \in N_{\text{in}}(n)} p_{t-1,n'} \cdot b_{t,n',n}$$



# Intuition of IPA-GNN



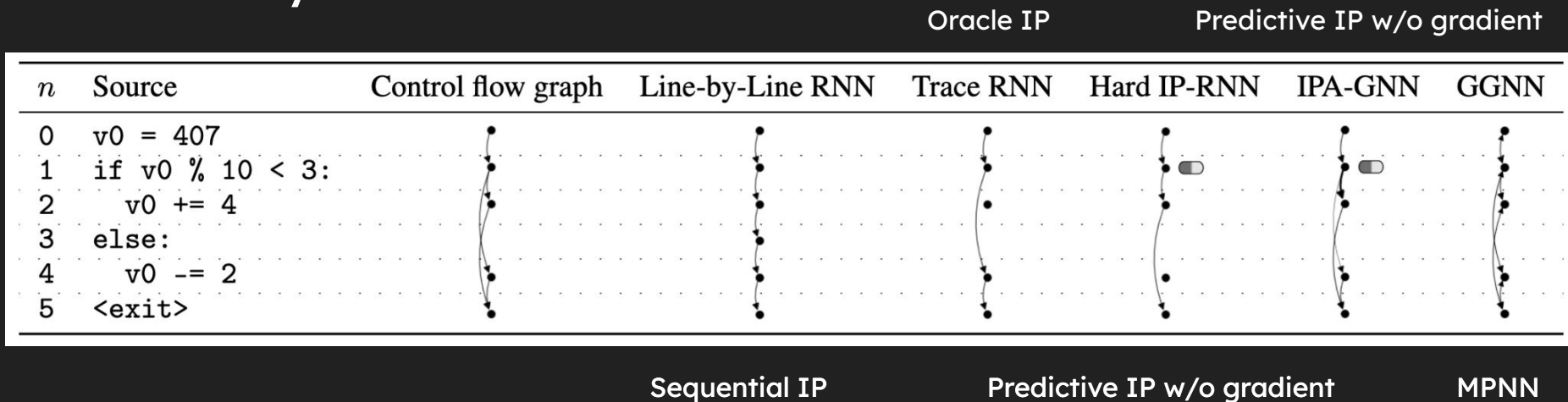
1. Execute: generate state proposal
2. Branch: generate soft branch decision if needed (and update IPA)
3. Aggregate: generate new hidden states by weighted sum from 1 and 2

# Model 5: Gated GNN (GGNN)

	IPA-GNN (Ours)	NoControl	NoExecute	GGNN
$h_{0,n}$	$= 0$	$= 0$	$= \text{Embed}(x_n)$	$= \text{Embed}(x_n)$
$a_{t,n}^{(1)}$	$= \text{RNN}(h_{t-1,n}, \text{Embed}(x_n))$	$= \text{RNN}(h_{t-1,n}, \text{Embed}(x_n))$	$= h_{t-1,n}$	$= h_{t-1,n}$
$a_{t,n,n'}^{(2)}$	$= p_{t-1,n'} \cdot b_{t,n',n} \cdot a_{t,n}^{(1)}$	$= 1 \cdot a_{t,n}^{(1)}$	$= p_{t-1,n'} \cdot b_{t,n',n} \cdot \text{Dense}(a_{t,n}^{(1)})$	$= 1 \cdot \text{Dense}(a_{t,n}^{(1)})$
$\tilde{h}_{t,n}$	$= \sum_{n' \in N_{\text{in}}(n)} a_{t,n,n'}^{(2)}$	$= \sum_{n' \in N_{\text{all}}(n)} a_{t,n,n'}^{(2)}$	$= \sum_{n' \in N_{\text{in}}(n)} a_{t,n,n'}^{(2)}$	$= \sum_{n' \in N_{\text{all}}(n)} a_{t,n,n'}^{(2)}$
$h_{t,n}$	$= \tilde{h}_t$	$= \tilde{h}_t$	$= \text{GRU}(h_{t-1,n}, \tilde{h}_{t,n})$	$= \text{GRU}(h_{t-1,n}, \tilde{h}_{t,n})$

- IPA-GNN shares similar computational structure with message passing neural nets (MPNN) like GGNN.
- IPA-GNN has two extra mechanisms
  - Modeling of execution (RNN)
  - Modeling of controlling (p and b)
- 3 baselines: NoControl, NoExecute, GGNN (NoControlNoExecute)

# Summary



- IPA-GNN is closely related to both IP-RNNs and GGNN (MPNN)
  - Continuous relaxation of IP-RNNs
  - Adaptation of MPNNs on sequential reasoning

# Outline

- Background
- Problem Formulation
- Approach
- Experiment
  - Experiment settings
  - Evaluation Criteria
  - Results
- Conclusion

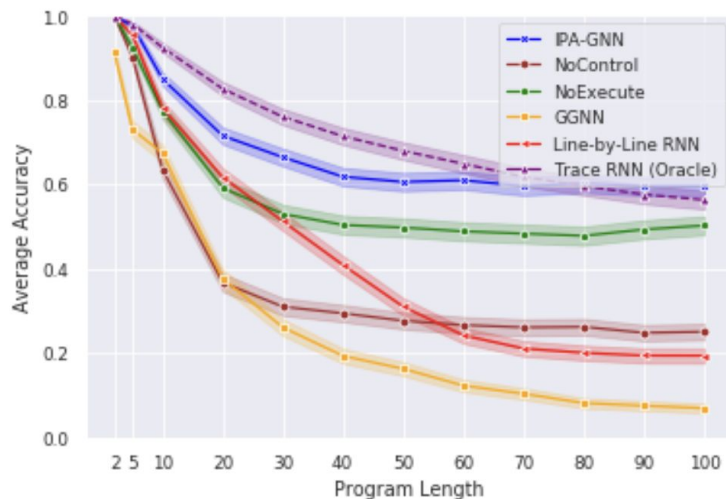
# Dataset

- **x: Python codes** **y: integer target** ( $v_0 \bmod 1000$ )
- **Statements include**
  - variable assignments ( $v_0, \dots, v_9$ )
  - multi-digit arithmetic
  - while loops
  - if-else statements
  - Complex nesting is allowed
- **Use program (lines) length as complexity measure**
  - $c(x) := \text{len}(x)$
- **Train test split forces systematic generalization**
  - Train:  $5M \text{ len}(x) \leq 10$       Test:  $4.5k \text{ len}(x) \in \{20, 30, \dots, 100\}$
- **Partial execution**
  - mask out 1 non-control statement uniformly at random

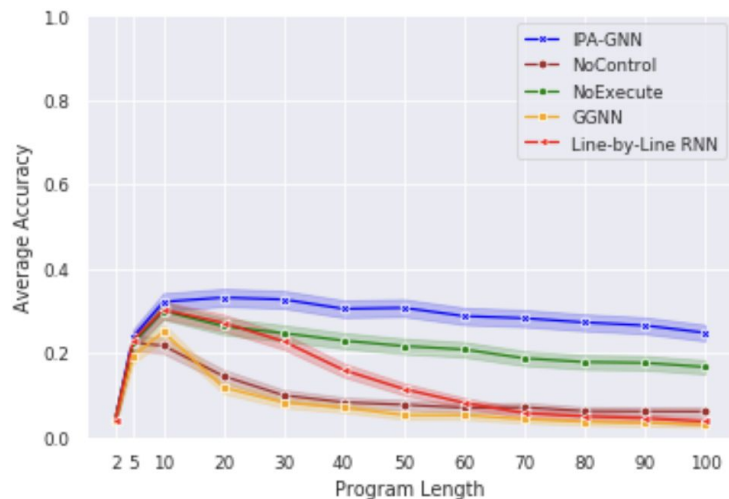
# Other specifications

- Use a two-layer LSTM as the underlying RNN cell for the RNN and IPA-GNN models
- Hparams
  - batch size: 32
  - hidden dimension:  $H \in \{200, 300\}$

# Accuracy vs. Program Length

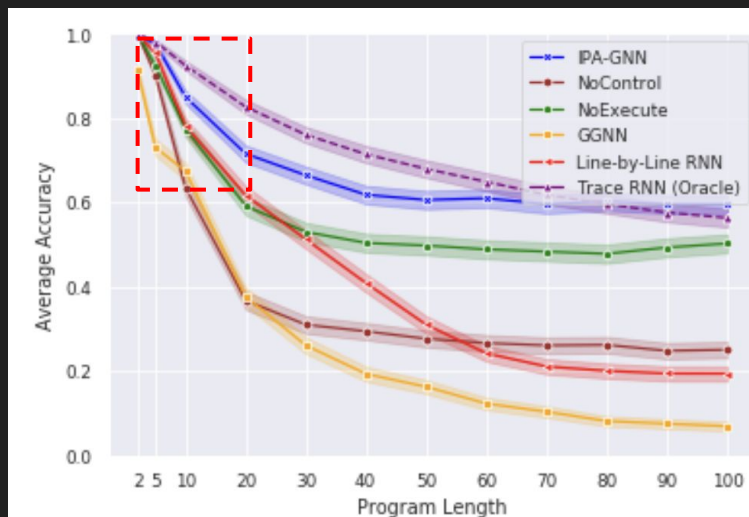


(a) Execution of full programs

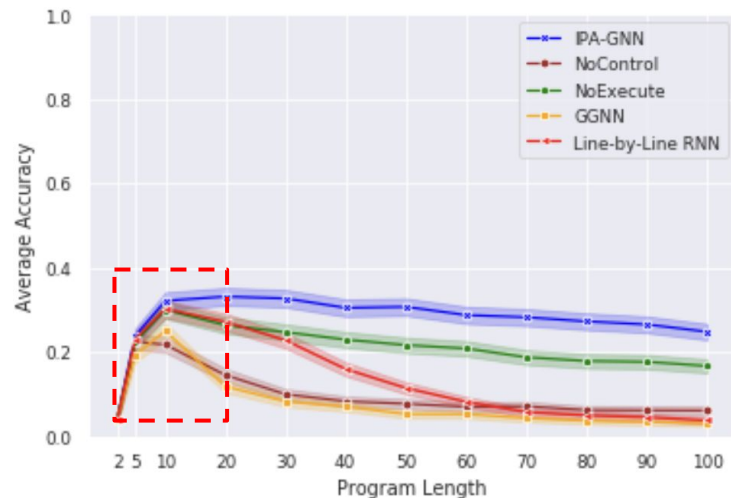


(b) Execution of partial programs

# Accuracy vs. Program Length



(a) Execution of full programs

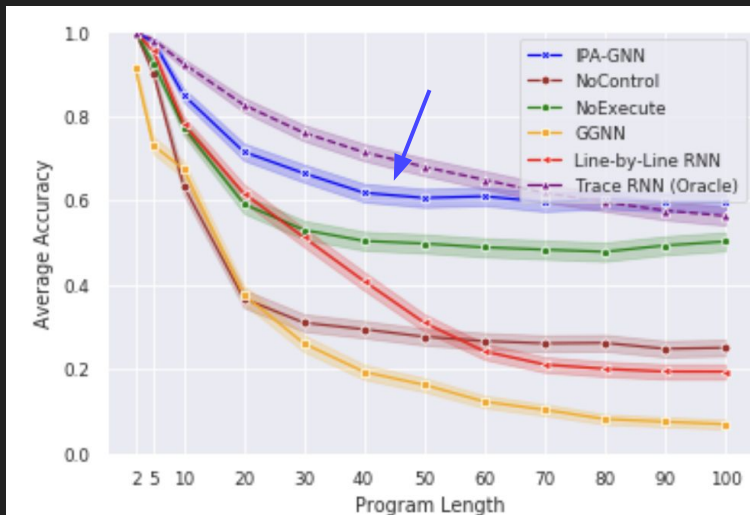


(b) Execution of partial programs

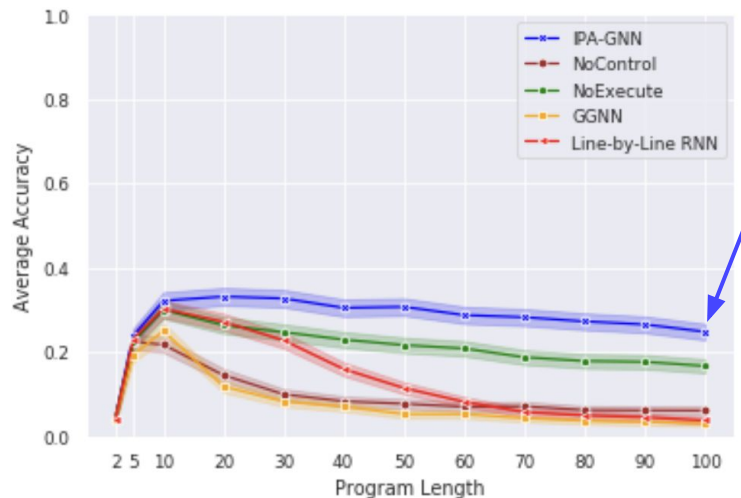
At low complexity, the **Line-by-Line RNN** model performs almost as well as the **IPA-GNN**.



# Accuracy vs. Program Length



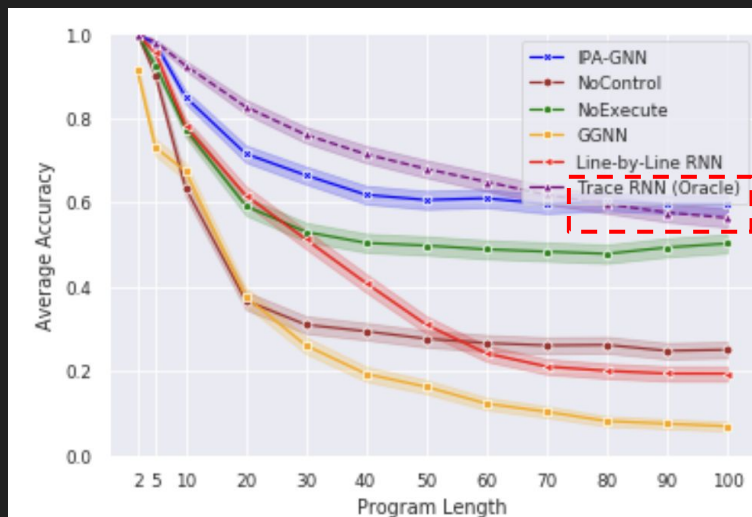
(a) Execution of full programs



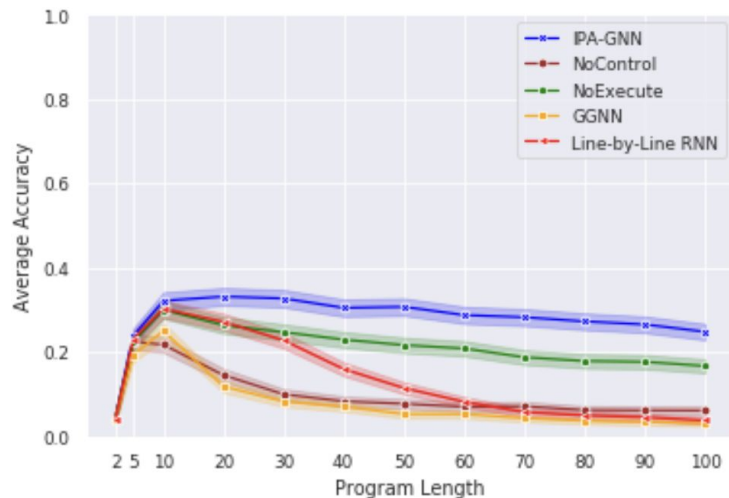
(b) Execution of partial programs

As complexity increases, the performance of all baseline models drops off faster than that of the **IPA-GNN**.

# Accuracy vs. Program Length



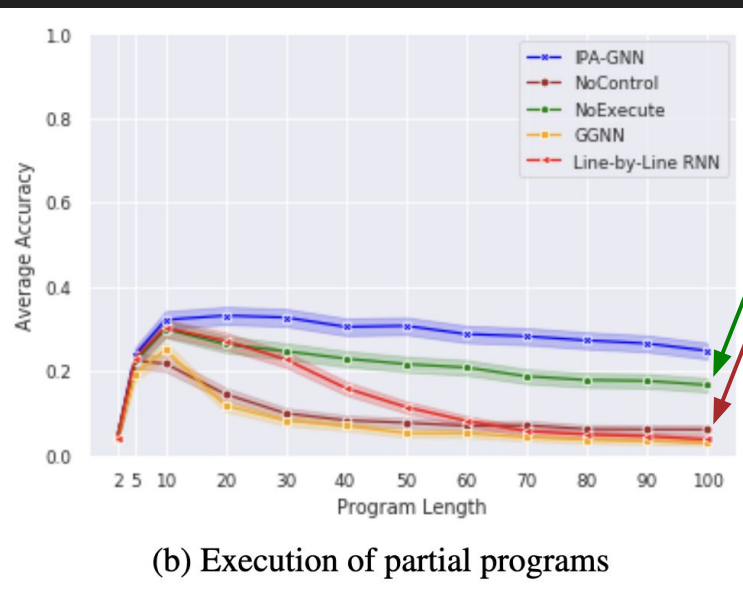
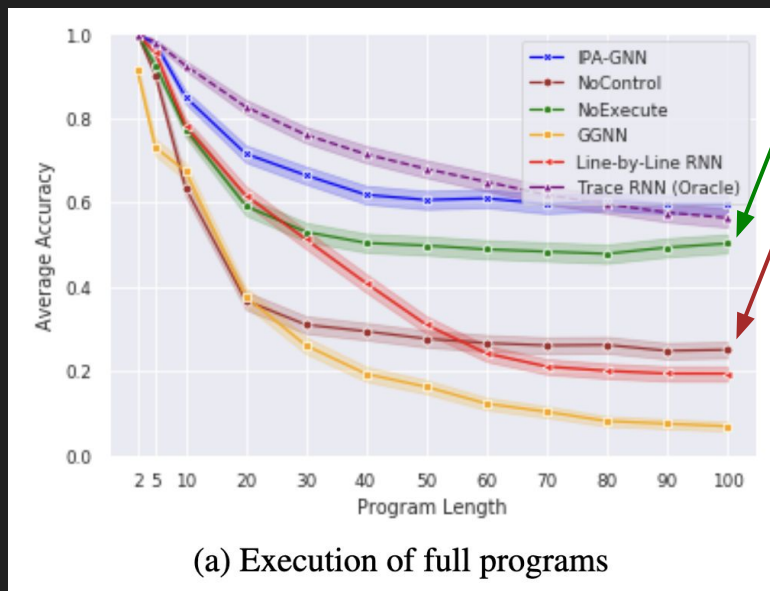
(a) Execution of full programs



(b) Execution of partial programs

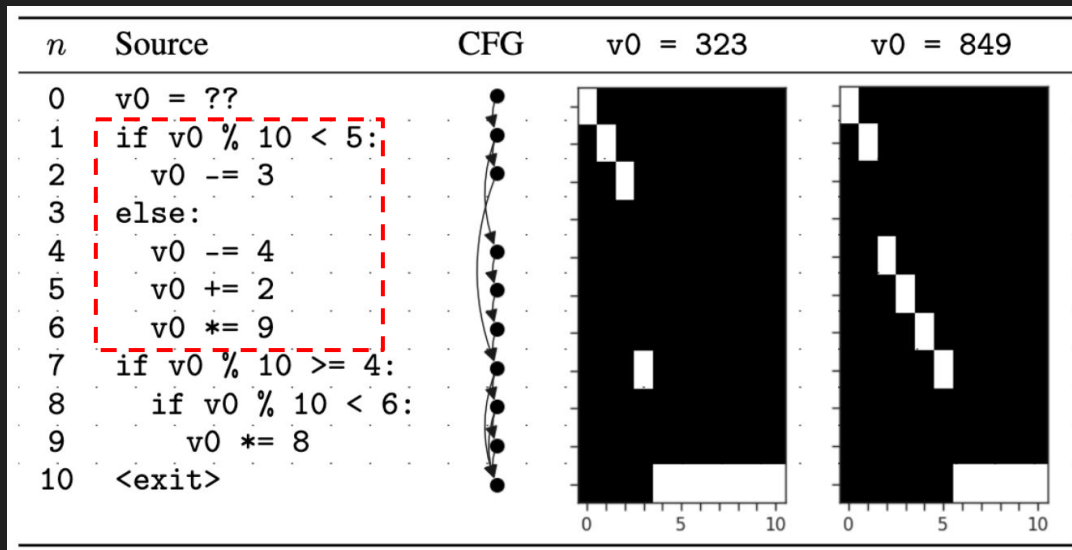
Despite using the ground truth control flow, the **Trace RNN** does not perform as well as the **IPA-GNN** on long programs.

# Accuracy vs. Program Length



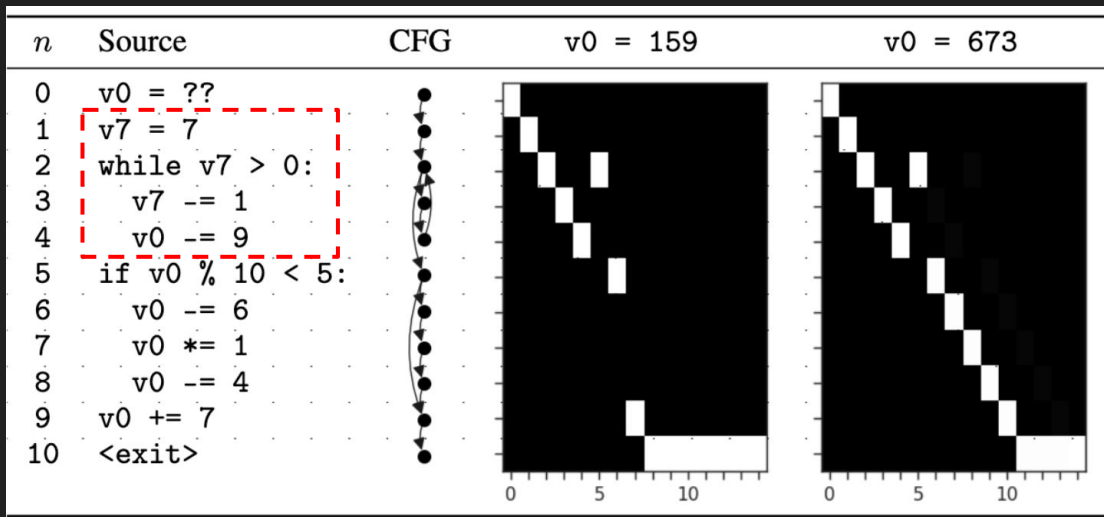
**NoExecute** significantly outperforms **NoControl**, indicating the importance of instruction pointer attention for the IPA-GNN model.

# Visualization of IPA $p_{t,n}$



- Learns to frequently produce discrete branch decisions.
- Learns to short-circuit execution
- Attends only to the path relevant to the program's result.

# Visualization of IPA $p_{t,n}$



- Learns to frequently produce discrete branch decisions.
- Learns to short-circuit execution
- Attends to the while-loop body only once rather than 7 times.

# Outline

- Background
- Problem Formulation
- Approach
- Experiment
- Conclusion

# Takeaways

- Learning-to-execute is a challenging ML task for static analysis.
- The authors proposed a new architecture IPA-GNN for this task, inspired by both RNNs and GNNs.
  - Continuous relaxation of IP-RNNs
  - Adaptation of MPNNs on sequential reasoning
- The key components of IPA-GNN is the modeling of Instruction Pointer Attention.
- The proposed method shows systematic generalization (on longer programs).

*Thank you for listening!*