Lecture 2 :

- Questions on lecture videos ?
  - Stern - Gerlach experiment with 3 steps : thought experiment
    (Stern - Gerlach observed separation in 2 states only)

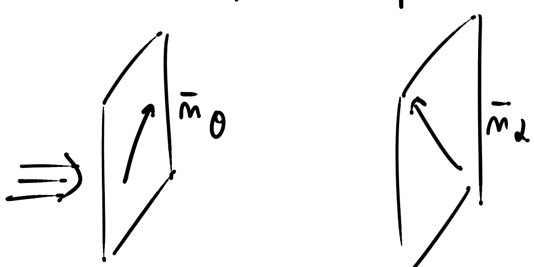
    Technically and full, correct quantum treatment requires entanglement of internal spin state and external position state.
    → this muddies the introduction significantly

- Preparation of a quantum state / quantum system in $\hat{x}$ or $\hat{y}$
  Measurement of a quantum system by passing through analyzer

Polarization of light wave ⟶ polarization of single photon

$N \gg 1$ ⟶ intensity = $|\text{wave amplitude}|^2$

$N = 1$ ⟶ probability = $|\text{probability amplitude}|^2$



polarizer θ analyzer α ⟶ measures wave with amplitude $\bar{m}_\theta \cdot \bar{m}_\alpha$

$$\bar{a}(\theta \to \alpha) = \cos(\theta - \alpha)$$

↓

intensity of $|a(\theta \to \alpha)|^2 = \cos^2(\theta - \alpha)$

in particular if $\theta - \alpha = 45°$: $\frac{1}{2}$

polarizer θ ⟶ prepares wave with

$$|\theta\rangle = \cos\theta\,|x\rangle + \sin\theta\,|y\rangle$$

$$\langle\theta|\theta\rangle = \cos^2\theta + \sin^2\theta = 1$$

circular polarization

$$|R\rangle = -\frac{1}{\sqrt{2}}\left(|x\rangle + i|y\rangle\right)$$

$$|L\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle - i|y\rangle\right)$$

⟹ transition to probability amplitudes, $\langle N\rangle = 0.1$

$a(\theta \to x) = \cos\theta = \langle x|\theta\rangle$ $\qquad a(x \to \alpha) = \cos\alpha$

$a(\theta \to y) = \sin\theta = \langle y|\theta\rangle$ $\qquad a(y \to \alpha) = \sin\alpha$

$a(\theta \to x \to \alpha) = \cos\theta \cos\alpha$

$a(\theta \to y \to \alpha) = \sin\theta \sin\alpha$

$a(\theta \to \alpha) = \cos\theta\cos\alpha + \sin\theta\sin\alpha = \cos(\theta - \alpha) = \langle \alpha | \theta \rangle$

Basis of Hilbert space of polarization states is $\{|x\rangle, |y\rangle\}$ (or $\{|+\frac{1}{2}\rangle, |-\frac{1}{2}\rangle\}$)
any state can be written as

$|\Phi\rangle = \lambda|x\rangle + \mu|y\rangle$ with $|\lambda|^2 + |\mu|^2 = 1$ for normalization
(rays: $e^{i\varphi}|\Phi\rangle$ is identical to $|\Phi\rangle$)

Other basis could be $|\theta\rangle$ and $|\theta_\perp\rangle = -\sin\theta\,|x\rangle + \cos\theta\,|y\rangle$

$\langle\theta|\theta\rangle = 1, \quad \langle\theta|\theta_\perp\rangle = 0, \quad \langle\theta_\perp|\theta_\perp\rangle = 1$

↳ because $|\langle\theta|x\rangle|^2 = \cos^2\theta$ is a probability → no well defined value for polarization of $|x\rangle$ in basis $\{|\theta\rangle, |\theta_\perp\rangle\}$
↓
incompatible bases

Special case of incompatible bases: complementary bases
such that $|\langle\theta|x\rangle|^2 = \frac{1}{N}$ for $N \hat{=}$ dimension of Hilbert space

here: $|\langle\theta|x\rangle|^2 = \frac{1}{2} \to \theta = \pm 45° \to \hat{x}', \hat{y}'$

(also: $\{|L\rangle, |R\rangle\}$: $|\langle L|x\rangle|^2 = \frac{1}{2}$)

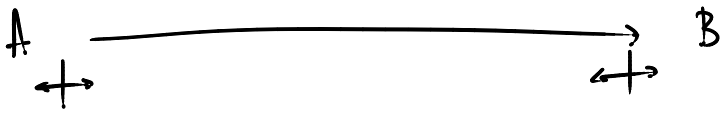
→ measurement has equal probability to return any basis state
→ least amount of information

- Quantum key distribution: $\underline{N} = pq$, $\underline{c}$: no common factor with $(p-1)(q-1)$
↳ factorization $\sim O(\exp[1.9(\ln N)^{1/3}(\ln\ln N)^{2/3}])$

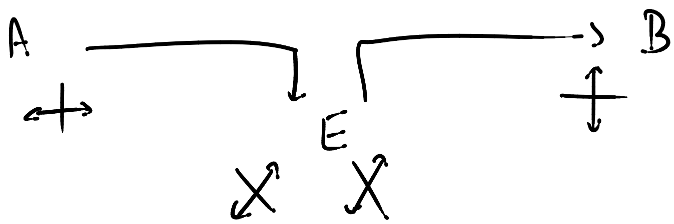
$1 = |x\rangle$ or $|x'\rangle$
$0 = |y\rangle$ or $|y'\rangle$
↳ Pockel's cell

→ BB84 , Bennett, Brassard, 1984
E91 , Ekert, 1991 → entanglement

Compatible basis: 100% probability of measuring correct polarization
Incompatible basis: 50% probability for both options
↓
resending has 50% probability of being right



→ 100% probability of correct measurement



→ mismatched measurement