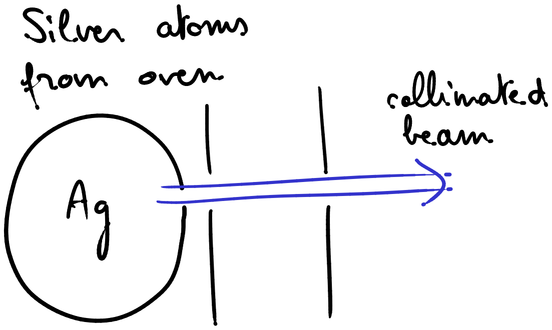
Stern - Gerlach Experiment : demonstration of effect that can only be explained with quantum mechanics

Otto Stern 1921
+ Walther Gerlach 1922



Ag has 47 electrons in $\underbrace{2+8+18+18}_{\text{closed shells}} + \underset{\uparrow}{1}$ configuration

closed shells + $5s^1$ shell : magnetic moment $\bar{\mu}$ determined by spin of single electron

Magnetic moment in a magnetic field (classically)

energy $E = -\bar{\mu} \cdot \bar{B}$ (potential energy)

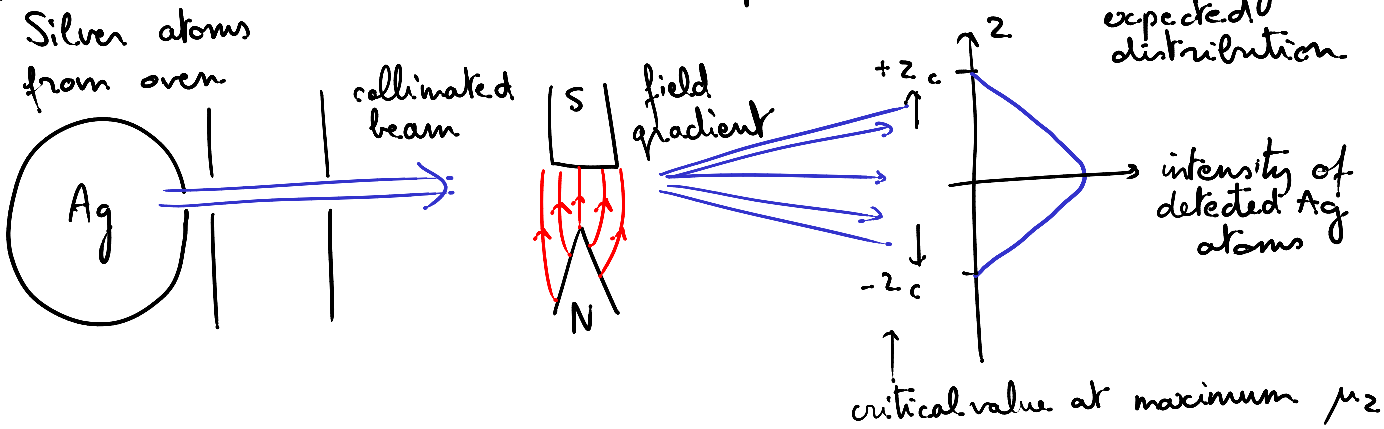
gradient in energy = force $\rightarrow$ $\bar{F} = -\bar{\nabla} E = \bar{\nabla}(\bar{\mu} \cdot \bar{B}) = \mu_z \frac{\partial B_z}{\partial z} \hat{z}$

if field only in z direction

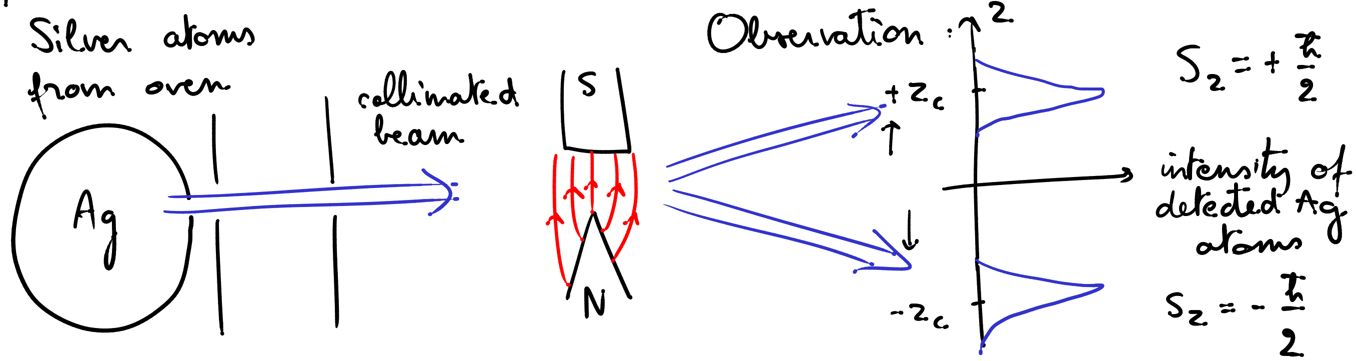
If $\bar{\mu}$ is randomly oriented $\rightarrow$ F_z will vary continuously

$\swarrow \; \nwarrow \; \uparrow \; \downarrow \; \rightarrow$

Experiment : Expectation : classically expected distribution



Experiment:

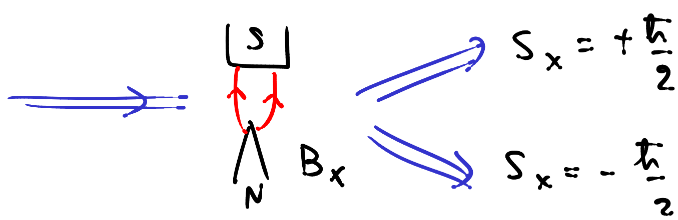


Interpretation:

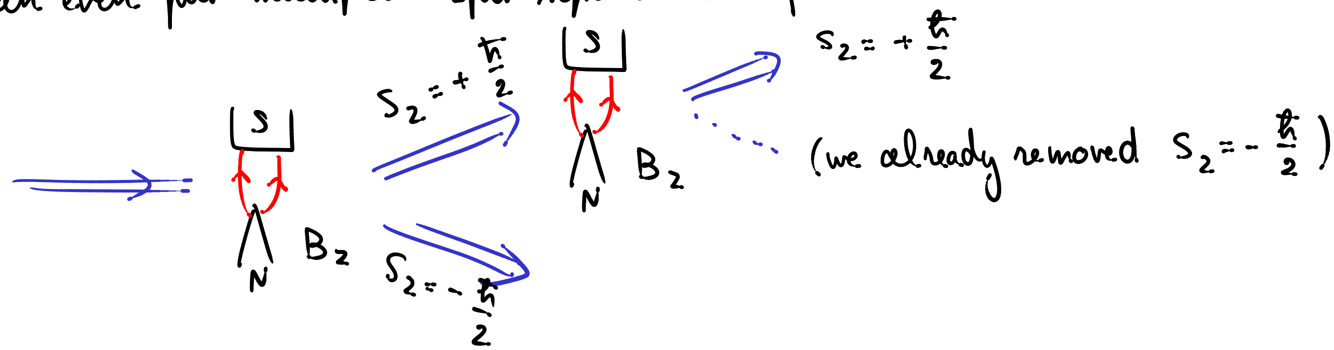
Since μ_z is determined by the 1 outer shell electron in Ag, the intrinsic angular momentum of that electron must be quantized : $\mu_z \propto S_z = \pm \frac{\hbar}{2}$

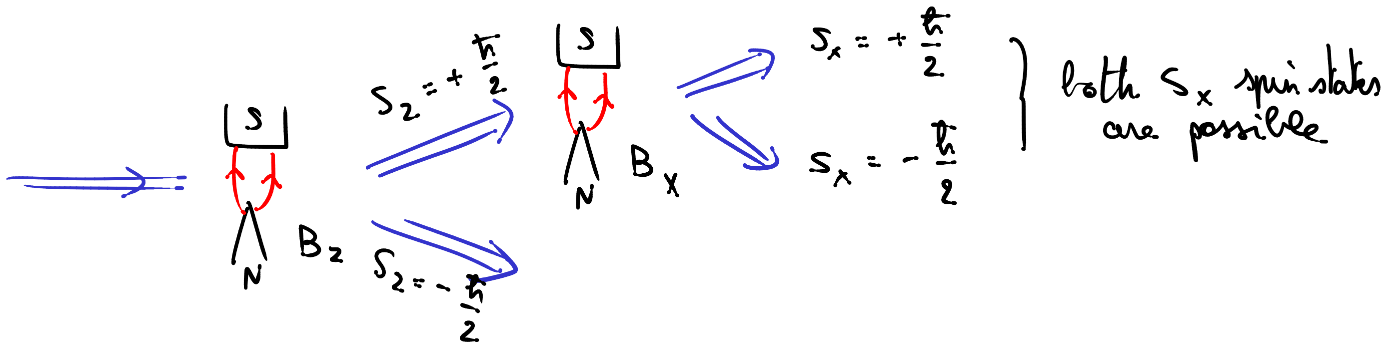
$\Rightarrow$ Quantization of electron spin

Similarly: magnetic field gradients in the x and y direction can be used to demonstrate quantization of the spin in $S_x = \pm \frac{\hbar}{2}$ and $S_y = \pm \frac{\hbar}{2}$

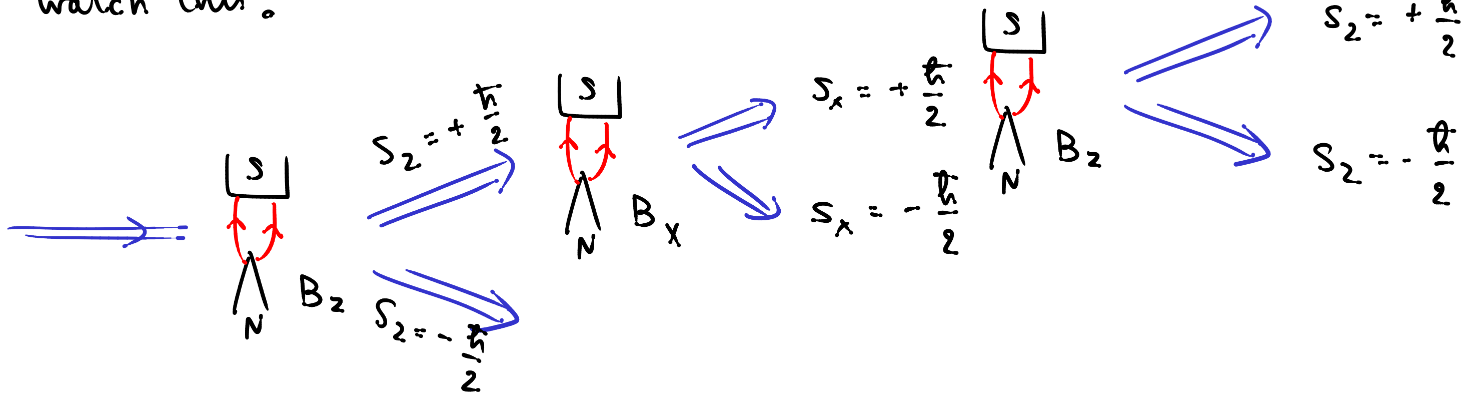


We can even put multiple "spin separators" after each other:





But watch this!



A measurement of S_x destroys any knowledge of S_z!

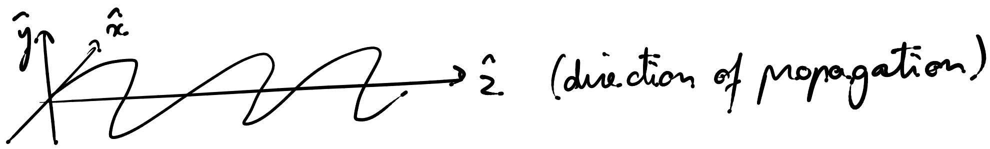
We cannot simultaneously know S_x and S_z!

This is not an experimental short-coming, but a fundamental restriction due to the quantum mechanical nature of the process.

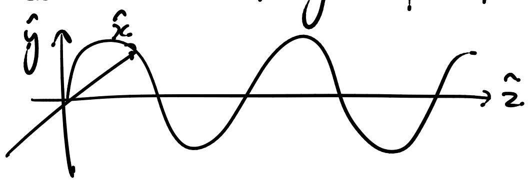
Polarized Light and Quantum Mechanics

Consider linearly polarized light, classically described as

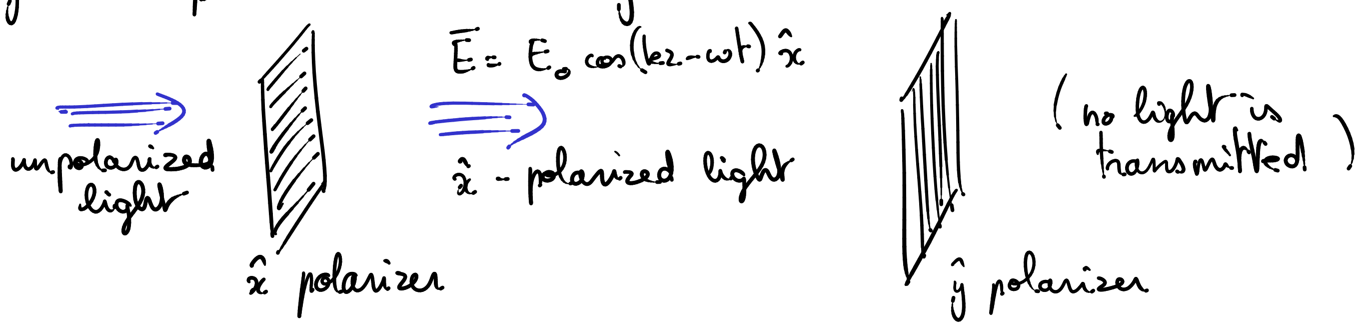
$$\bar{E} = E_0 \cos(kz - \omega t)\ \hat{x} \quad \text{for polarization in the } \hat{x} \text{ direction}$$



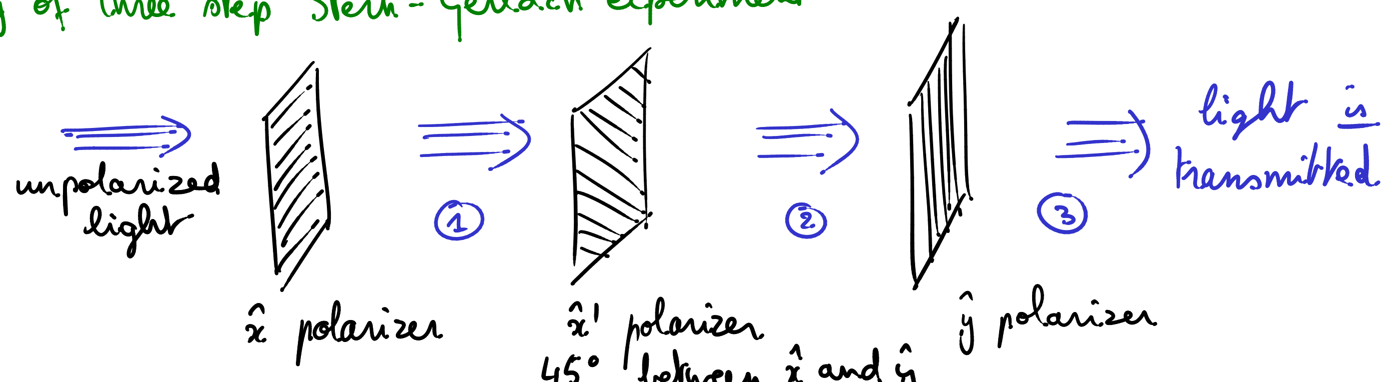
$$\bar{E} = E_0 \cos(kz - \omega t)\ \hat{y} \quad \text{for polarization in the } \hat{y} \text{ direction}$$



Polarized filters can be used to select only the components in the $\hat{x}$ or $\hat{y}$ directions



Analogy of three step Stern-Gerlach experiment



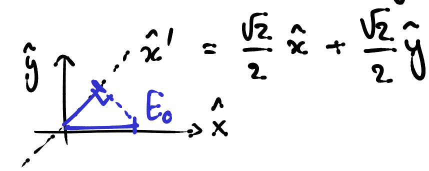
Consider the three regions :

$$E_1 = E_0 \cos(kz - \omega t)\ \hat{x}$$

$$E_2 = E_0 \frac{\sqrt{2}}{2} \cos(kz - \omega t)\ \hat{x}'$$

$$= E_0 \frac{\sqrt{2}}{2}^2 \left(\frac{\sqrt{2}}{2} \hat{x} + \frac{\sqrt{2}}{2} \hat{y} \right) \cos(kz - \omega t)$$

$$E_3 = E_0 \frac{1}{2}\ \hat{y} \cos(kz - \omega t) \neq 0$$



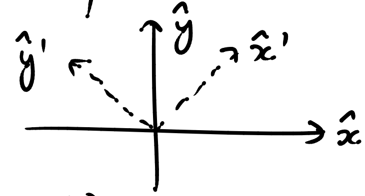
$\longrightarrow$ reappearance of $\hat{y}$ component because $\hat{x}'$ is not orthogonal to $\hat{x}$ or $\hat{y}$ $\to$ incompatible measurement

Further development of analogy :

45° polarizer acts as measurement of S_x in Stern-Gerlach

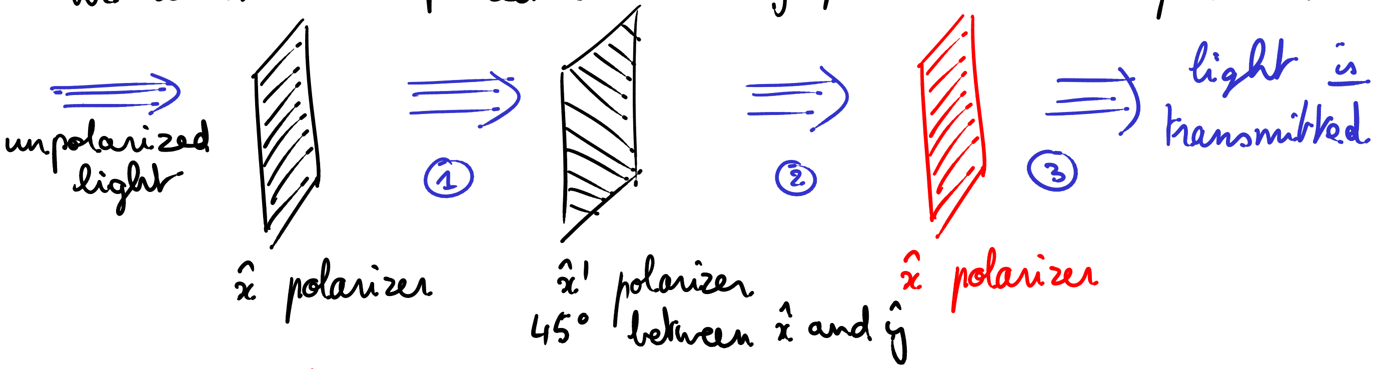
$\hat{x}'$ polarizer $\longleftrightarrow$ $S_x = +\frac{\hbar}{2}$

$\hat{y}'$ polarizer $\longleftrightarrow$ $S_x = -\frac{\hbar}{2}$



$\hookrightarrow$ $E = E_0 \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2}\hat{x} + \frac{\sqrt{2}}{2}\hat{y}\right)\cos(kz - \omega t)$

We could have replaced the third $\hat{y}$ polarizer with $\hat{x}$ polarizer :



$E_3 = E_0 \frac{1}{2} \hat{x} \cos(kz - \omega t)$

$\longrightarrow$ intensity of transmitted light is identical, just as $S_z = +\frac{\hbar}{2}$ and $S_z = -\frac{\hbar}{2}$ had equal intensity in Stern-Gerlach.

Intermediate table of correspondence :

linear polarization in $\hat{x}$ or $\hat{y}$ $\longleftrightarrow$ $S_z = \pm\frac{\hbar}{2}$, up or down

linear polarization in $\hat{x}'$ or $\hat{y}'$ $\longleftrightarrow$ $S_x = \pm\frac{\hbar}{2}$, up or down

$\rightarrow$ if $\hat{x}'$ and $\hat{y}'$ can be written in terms of $\hat{x}$ and $\hat{y}$

as $\begin{cases} \hat{x}' = \frac{\sqrt{2}}{2}\hat{x} + \frac{\sqrt{2}}{2}\hat{y} \\ \hat{y}' = -\frac{\sqrt{2}}{2}\hat{x} + \frac{\sqrt{2}}{2}\hat{y} \end{cases}$

can we now write $S_x = \pm\frac{\hbar}{2}$ in terms of $S_z = \pm\frac{\hbar}{2}$?

$$\left|S_x = +\frac{\hbar}{2}\right\rangle = \frac{\sqrt{2}}{2}\left|S_z = +\frac{\hbar}{2}\right\rangle + \frac{\sqrt{2}}{2}\left|S_z = -\frac{\hbar}{2}\right\rangle$$

$$\left|S_x = -\frac{\hbar}{2}\right\rangle = -\frac{\sqrt{2}}{2}\left|S_z = +\frac{\hbar}{2}\right\rangle + \frac{\sqrt{2}}{2}\left|S_z = -\frac{\hbar}{2}\right\rangle$$

What about the analogy with S_y in the case of polarization?

S_x, S_y, S_z should be (and are) equivalent due to rotational symmetry

$\rightarrow$ what acts as S_y in the case of polarization?

But wait: there is another linearly independent polarization mode:

circular polarization!

$$E_{\pm} = E_0\left[\frac{1}{\sqrt{2}}\hat{x}\cos(kz-\omega t) + \frac{1}{\sqrt{2}}\hat{y}\cos(kz-\omega t \underbrace{\pm \frac{\pi}{2}})\right]$$

phase delay, $e^{\pm i\pi/2} = \pm i$

$$\text{or } E_{\pm} = E_0\left[\frac{1}{\sqrt{2}}\hat{x} \pm \frac{i}{\sqrt{2}}\hat{y}\right]\cos(kz-\omega t)$$

$$\rightarrow \left|S_y = \pm\frac{\hbar}{2}\right\rangle = \frac{1}{\sqrt{2}}\left|S_z = +\frac{\hbar}{2}\right\rangle \pm \frac{i}{\sqrt{2}}\left|S_z = -\frac{\hbar}{2}\right\rangle$$

Conclusions of Stern-Gerlach and light polarization:

① Not all properties of a quantum mechanical system can be known <u>simultaneously</u>. A measurement of an "incompatible" (TBD) property can "destroy" knowledge of other properties.

② We can describe the states of a physical system as vectors in a vector space, with arithmetic relations between them.

$\Rightarrow$ We will adopt similar statements as <u>fundamental principles</u>.

(Similar to: $c =$ constant for relativity)