A VARIATION OF THE BUFFON'S NEEDLE AND COIN PROBLEMS

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Buffon's Needle Problem is a problem of dropping a needle on a hardwood floor while Buffon's Coin Problem is a problem of dropping a coin on a floor covered with identical square tiles. In both problems, the event of interest is when the needle or coin crosses a crack between floorboards or tiles, and the aim is to find the probability of crossing a crack.

In our project we attempted to apply the Buffon's Needle and Coin Problems to the scenario of a needle or coin falling through a drain cover; we imagine the floorboard and tile to be empty spaces while the cracks are part of the metal grilles. The case of the needle or coin crossing a crack is equivalent to crossing a grille in our application, whereby the needle or coin does not fall through the drain cover if it crosses a grille. As such, the probability of a needle or coin not falling through the drain cover is the probability of the needle or coin crossing a crack.

In real-life, not any form of contact with the grille would stop the needle or coin from falling through the drain cover. Hence in our variation of the Buffon's Needle and Coin Problems we considered the centre of gravity of the needle and the coin; the needle or coin does not fall through if it balances, that is when its centre of gravity acts within its base of support. We derived new formulae for calculating the probability of the needle or coin balancing.

For our variation of the Buffon's Needle Problem,

$$\text{P(needle balancing)} = \begin{cases} \dfrac{2}{\pi}\left[\sqrt{\left(\dfrac{L}{D}\right)^2 - 1} + \sin^{-1}\left(\dfrac{D}{L}\right)\right] - 1, & \text{if } D < L \le 2D \\ 1 + \dfrac{2}{\pi}\left[\sin^{-1}\left(\dfrac{D}{L}\right) - 2\sin^{-1}\left(\dfrac{2D}{L}\right) + \sqrt{\left(\dfrac{L}{D}\right)^2 - 1} - \sqrt{\left(\dfrac{L}{D}\right)^2 - 4}\right], & \text{if } L > 2D \end{cases}$$

where L is the length of the needle and D is the distance between the 2 parallel edges.

For our variation of the Buffon's Coin Problem,

$$\text{P(coin balancing)} = \begin{cases} \pi\left(\dfrac{R}{D}\right)^2, & \text{if } R \le \dfrac{D}{2} \\ \sqrt{4\left(\dfrac{R}{D}\right)^2 - 1} + \left(\dfrac{R}{D}\right)^2\left[\pi - 4\cos^{-1}\left(\dfrac{D}{2R}\right)\right], & \text{if } \dfrac{D}{2} < R < \dfrac{D\sqrt{2}}{2} \\ 1, & \text{if } R \ge \dfrac{D\sqrt{2}}{2} \end{cases}$$

where R is the radius of coin and D is the length of side of the square grid.

We used various programming tools such as Python, NumPy, numexpr, and matplotlib to create computer simulations of needle and coin landing between parallel lines or on a square grid as a means to verify the probabilities obtain using our mathematical formulae. We applied the Chi-Square (χ^2) goodness of fit test to determine if there is any significant difference between our theoretical results and our experimental results. Our calculated χ^2 statistic was consistently less than 3.841 giving $p > 0.05$, and we concluded that there is no significant difference between using our formulae to calculate the probability of needle or coin balancing and using our computer program to obtain the probability.

With the results from our computer simulations and mathematical formulae, we plotted graphs for the original and our variation of the Buffon's Needle and Coin Problems drawn on the same axes for comparisons and observed that the results fall within our expectations.

We used the formulae to find the best combination of coins, worth a total of $1, to carry in the event we drop that amount of coins while standing over a drain cover; and found it to be a combination of one 50-cent, two 20-cent and one 10-cent coins.

In the future we may extend our project to include grids of different shapes such as other regular polygons and see how the probability varies with the shapes of the grids.

Our computer programs are accessible at http://tiny.cc/bce and our source codes are found at https://github.com/wei2912/bce-simulation