

# Data Warehousing: Data Summarization Technology

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Adapted from Slides of Hector Garcia-Molina, George Kollios, Sudarshan

### Outline

- What is a data warehouse?
- Why a warehouse?
- Data Summarization: Models & operations
- Implementing a warehouse
- Future Outlook

#### What is a Warehouse?

A data warehouse is a

- database that is maintained separately from an operational database
- subject oriented
  - aimed at executive, decision maker
  - often a copy of operational data with value-added data (e.g., summaries, history)
- Integrated
- time-varying
- non-volatile

collection of detailed and summary data used to support the strategic decision making process for the enterprise

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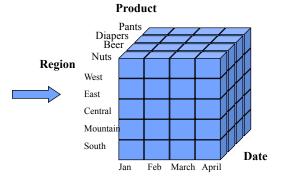
### What is a Warehouse?

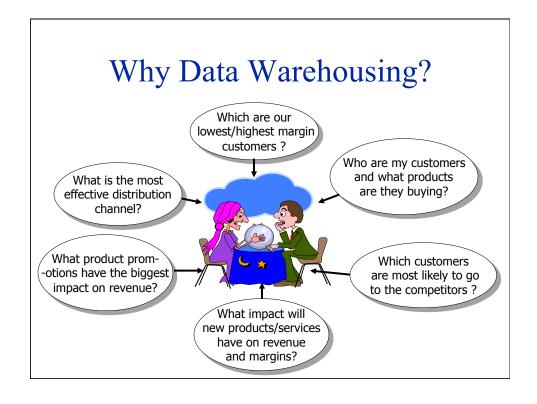
- Collection of tools
  - gathering data
  - cleansing, integrating, ...
  - querying, reporting, analysis
  - data mining
  - monitoring, administering warehouse

# Subject-view

■ Example: regional product monthly sales comparison

Product	store	Date	Sale
acron	Rolla,MO	7/3/99	325.24
budwiser	LA,CA	5/22/99	833.92
large pants	NY,NY	2/12/99	771.24
3' diaper	Cuba,MO	7/30/99	81.99





### Data Warehouse vs. Operational DBMS

- OLTP (on-line transaction processing)
  - Major task of traditional relational DBMS
  - Day-to-day operations: purchasing, inventory, banking, manufacturing, payroll, registration, accounting, etc.
  - Describes processing at operational sites
- OLAP (on-line analytical processing)
  - Major task of data warehouse system
  - Data analysis and decision making
  - Describes processing at warehouse

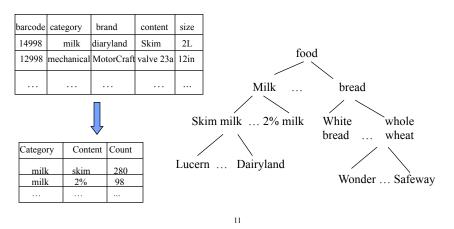
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### Examples of OLAP

- Comparisons (this period v.s. last period)
  - Show me the sales per region for this year and compare it to that of the previous year to identify discrepancies
- Multidimensional ratios (percent to total)
  - Show me the contribution to weekly profit made by all items sold in the northeast stores between may 1 and may 7
- Ranking and statistical profiles (top N/bottom N)
  - Show me sales, profit and average call volume per day for my 10 most profitable salespeople
- Customer consolidation (market segments, ad hoc groups)
  - Show me an abbreviated income statement by quarter for the last four quarters for my northeast region operations

# Summarization Technique: Attribute-Oriented Induction

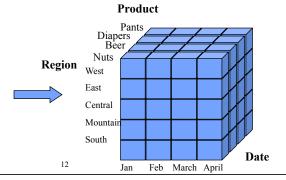
■ Generalization using Concept hierarchy (taxonomy)

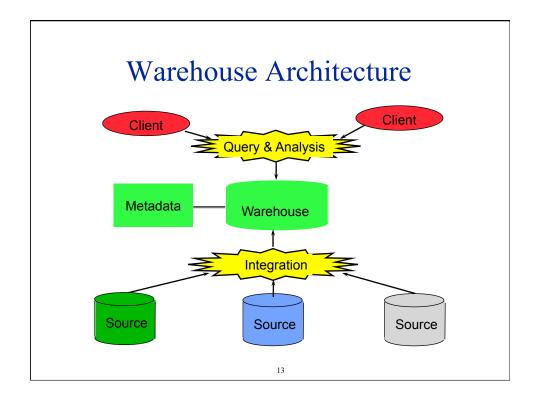


# On-Line Analytical Processing

- Data Cube
  - a multidimensonal array
  - Summariztion/aggregation attributes v.s. generalization attributes
  - each generation attribute represents a dimension
  - variable a special dimension for aggregation

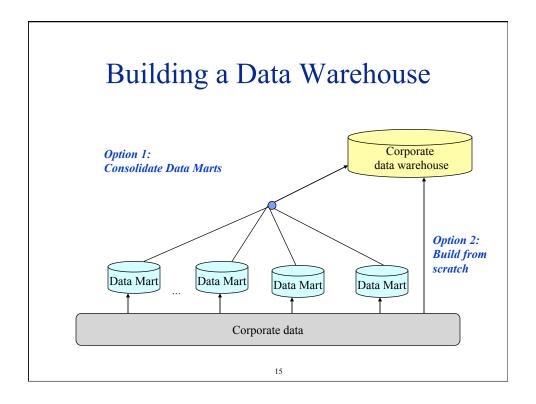
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# Advantages of Warehousing

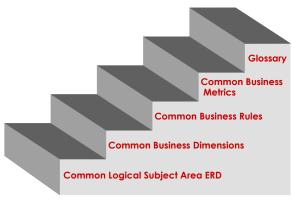
- High query performance
- Queries not visible outside warehouse
- Local processing at sources unaffected
- Can operate when sources unavailable
- Can query data not stored in a DBMS
- Extra information at warehouse
  - Modify, summarize (store aggregates)
  - Add historical information



### **Data Marts**

- Smaller warehouses
- Spans part of organization
  - e.g., marketing (customers, products, sales)
- Do not require enterprise-wide consensus
  - but long term integration problems?

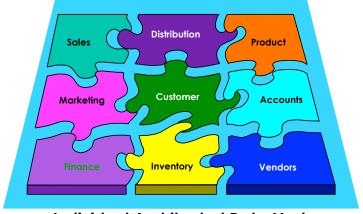




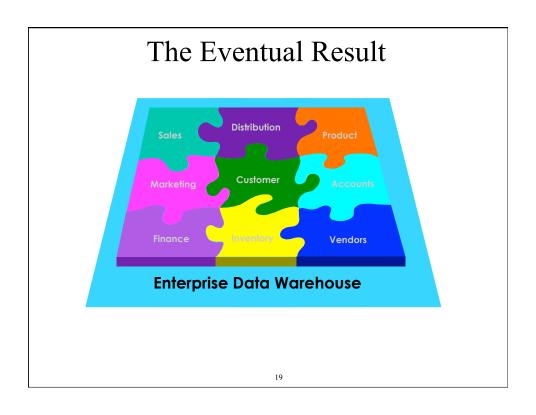
**Individual Architected Data Marts** 

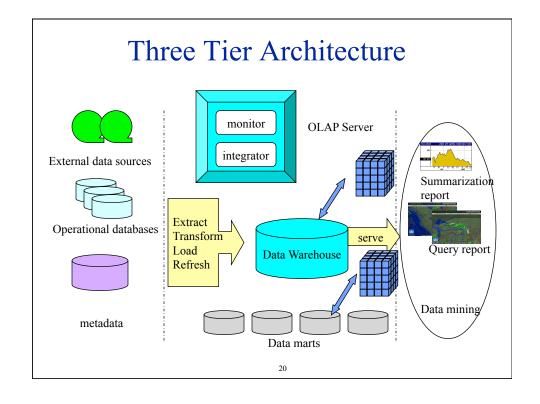
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# An Incremental Approach



**Individual Architected Data Marts** 





# Data Warehouse Design (1)

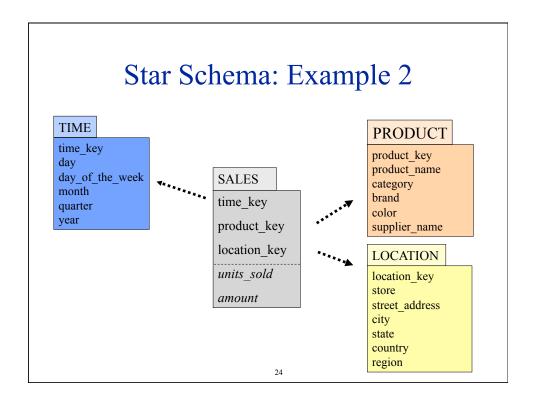
- Most of data warehouses use **a star schema** to represent multi-dimensional data model
- Each dimension is represented by **a dimension table** that provides its multidimensional coordinates
  - LOCATION(location\_key,store,street\_address,c ity,state,country,region)
  - dimension tables are not normalized
- The star fact table stores **measures** for those coordinates

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#### Star Schema: Example 1 Product Date **ProductNo** Sale Fact Table Date **ProdName** ProdDesc Month Categoryu Year Date Product Store Customer Customer unit\_Sales Store CustID dollar Sales CustName StoreID Summarization CustCity City attributes State CustCountry Country Region 22

### Data Warehouse Design (2)

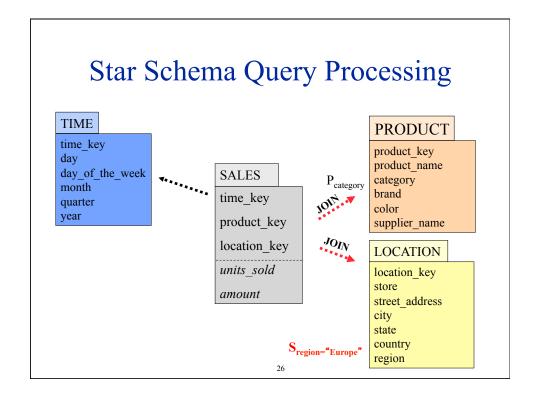
- A fact table
  - connects to all dimension tables with a multiple join. Each tuple in the fact table consists of a pointer to each of the dimensional tables
- Transactions are described through a fact-table
  - each tuple consists of a pointer to each of the dimension-tables (foreign-key) and a list of measures (e.g. sales \$\$\$)
- The links between the fact table and the dimensional tables form a shape like a star



### Advantages of Star Schema

- Facts and dimensions are clearly depicted
  - dimension tables are relatively static, data is loaded (append mostly) into fact table(s)
  - easy to comprehend (and write queries)
- Example

"Find total sales per product-category in our stores in Europe"



# Example

"Find total sales per product-category in our stores in Europe"

**SELECT** PRODUCT.category, SUM(SALES.amount)

FROM SALES, PRODUCT, LOCATION

WHERE SALES.product\_key = PRODUCT.product\_key

AND SALES.location\_key = LOCATION.location\_key

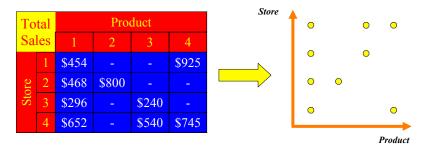
AND LOCATION.region="Europe"

**GROUP BY PRODUCT.category** 

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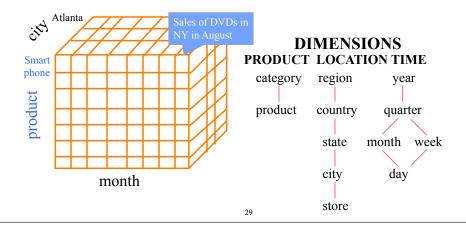
# Multidimensional Modeling

■ Example: compute total *sales* volume per *product* and *store* 



# Dimensions and Hierarchies

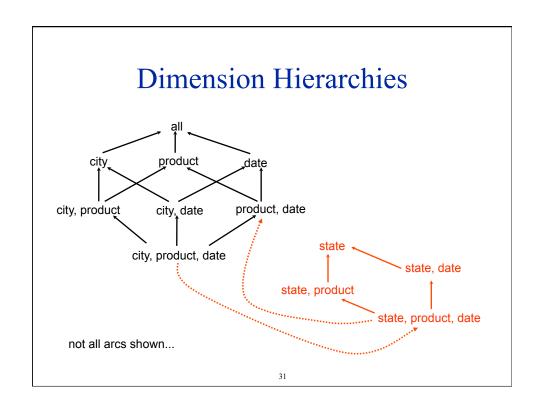
• A cell in the cube may store values (measurements) relative to the combination of the labeled dimensions

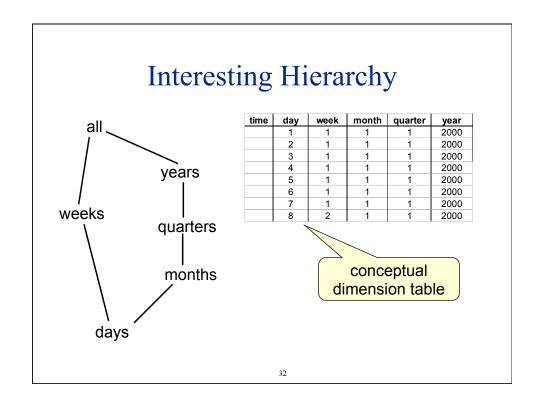


### **Dimension Hierarchies**



cities	city	state
	c1	GA
	c2	NY



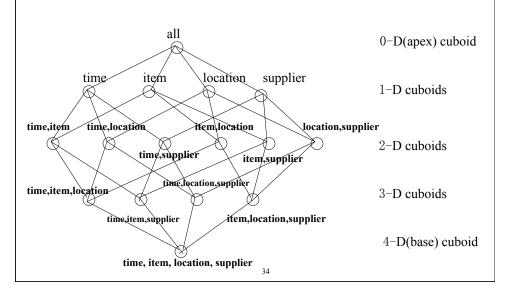


# From Tables and Spreadsheets to Data Cubes

- A data warehouse is based on a multidimensional data model which views data in the form of a data cube
- A data cube, such as sales, allows data to be modeled and viewed in multiple dimensions
  - **Dimension tables**, such as product (name, brand, type), or time(day, week, month, quarter, year), store(name, city, state, country)
  - Fact table contains measures (such as dollars\_sold) and keys to each of the related dimension tables
- In data warehousing literature, an n-D base cube is called a **base cuboid**. The top most 0-D cuboid, which holds the highest-level of summarization, is called the apex cuboid. The lattice of cuboids forms a data cube.

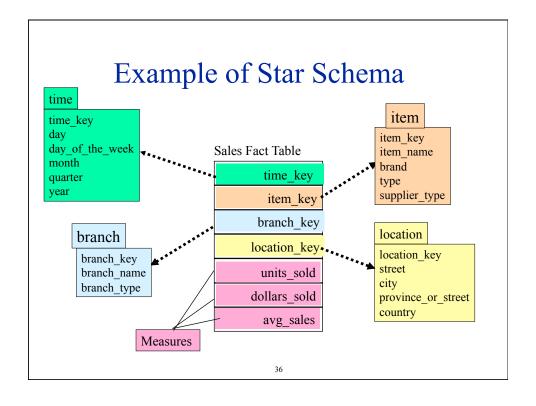
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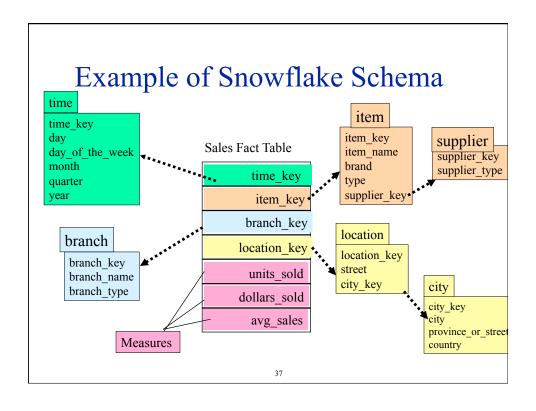
# Cube: A Lattice of Cuboids

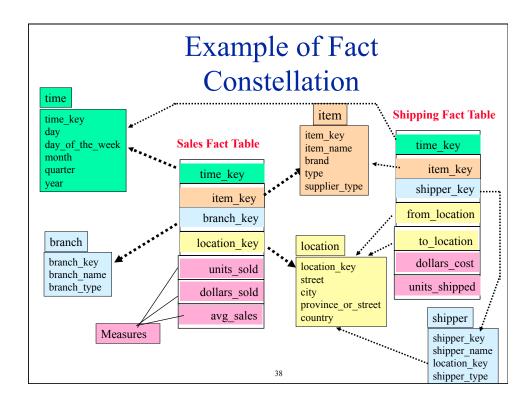


# Conceptual Modeling of Data Warehouses

- Modeling data warehouses: dimensions & measures
  - <u>Star schema</u>: A fact table in the middle connected to a set of dimension tables
  - Snowflake schema: A refinement of star schema where some dimensional hierarchy is normalized into a set of smaller dimension tables, forming a shape similar to snowflake
  - <u>Fact constellations</u>: Multiple fact tables share dimension tables, viewed as a collection of stars, therefore called galaxy schema or fact constellation







# A Data Mining Query Language, DMQL: Language Primitives

- Cube Definition (Fact Table)
  define cube <cube\_name> [<dimension\_list>]:
  <measure list>
- Dimension Definition ( Dimension Table )

  define dimension <dimension\_name> as

  (<attribute or subdimension list>)
- Special Case (Shared Dimension Tables)
  - First time as "cube definition"
  - define dimension < dimension\_name > as
     < dimension\_name\_first\_time > in cube
     < cube name first time >

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### Defining a Star Schema in DMQL

# Defining a Snowflake Schema in DMQL

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# Defining a Fact Constellation in DMQL

```
define cube sales [time, item, branch, location]:

dollars_sold = sum(sales_in_dollars), avg_sales = avg(sales_in_dollars), units_sold = count(*)

define dimension time as (time_key, day, day_of_week, month, quarter, year)

define dimension item as (item_key, item_name, brand, type, supplier_type)

define dimension branch as (branch_key, branch_name, branch_type)

define dimension location as (location key, street, city, province or state, country)

define cube shipping [time, item, shipper, from_location, to_location]:

dollar_cost = sum(cost_in_dollars), unit_shipped = count(*)

define dimension time as time in cube sales

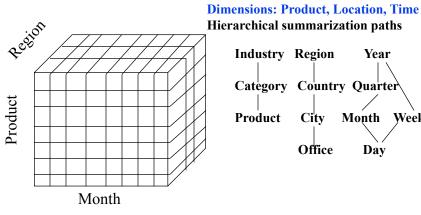
define dimension shipper as (shipper_key, shipper_name, location as location in cube sales, shipper_type)

define dimension from_location as location in cube sales

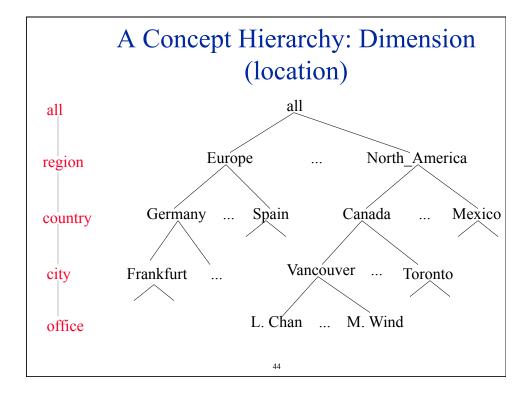
define dimension to_location as location in cube sales
```

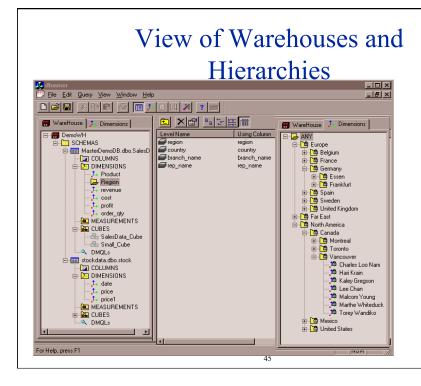
### Multidimensional Data

■ Sales volume as a function of product, month, and region

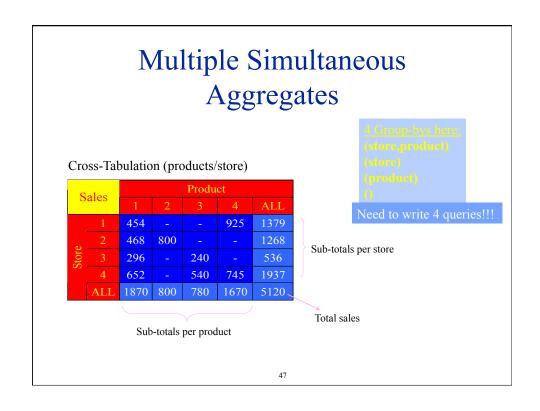


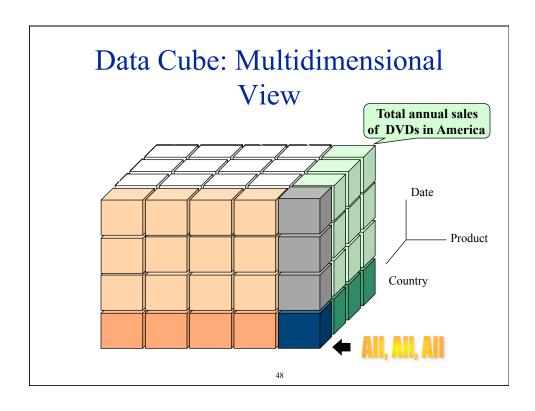
Hierarchical summarization paths **Industry Region** Year Category Country Quarter Product Month Week City Office Day

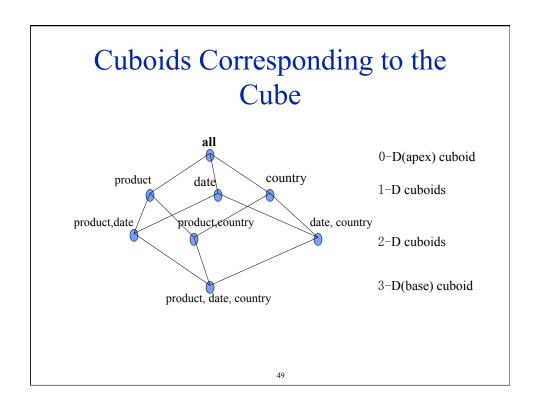


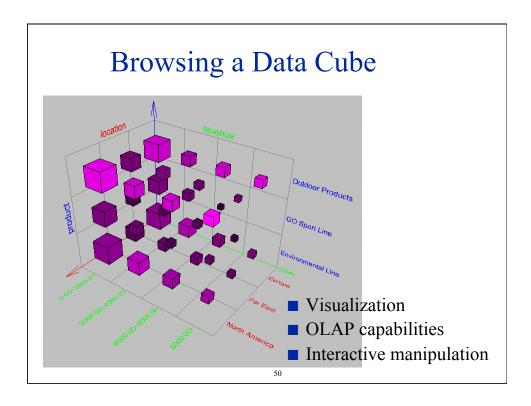


#### Relational View of Data Cube Store Product\_key sum(amout) Product Sales 1870 800 ALL ALL ALL SELECT LOCATION.store, SALES.product\_key, SUM (amount) ALL FROM SALES, LOCATION ALL ALL WHERE SALES.location\_key=LOCATION.location\_key ALL CUBE BY SALES.product\_key, LOCATION.store ALL ALL ALL









### Warehouse Models

- Data Models
  - relations
  - stars & snowflakes
  - cubes

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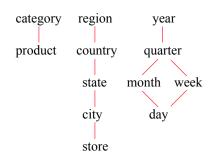
# **Typical OLAP Operations**

- Roll up (drill-up): summarize data
  - by climbing up hierarchy or by dimension reduction
- Drill down (roll down): reverse of roll-up
  - from higher level summary to lower level summary or detailed data, or introducing new dimensions
- Slice and dice:
  - project and select
- Pivot (rotate):
  - reorient the cube, visualization, 3D to series of 2D planes.
- Other operations
  - drill across: involving (across) more than one fact table
  - *drill through*: through the bottom level of the cube to its back-end relational tables (using SQL)

# Common OLAP Operations (1)

- Slice: selection condition set on one of the cube dimensions.
  - Find product sales of atlanta
  - Find 2006 sales of all states
- <u>Dice</u>: selection condition set on all three dimensions
  - Find digital camera sale of 2007 in Atlanta

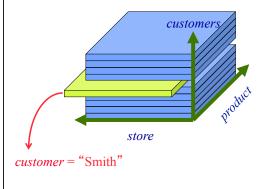
PRODUCT LOCATION TIME

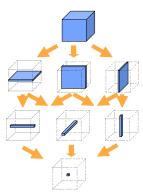


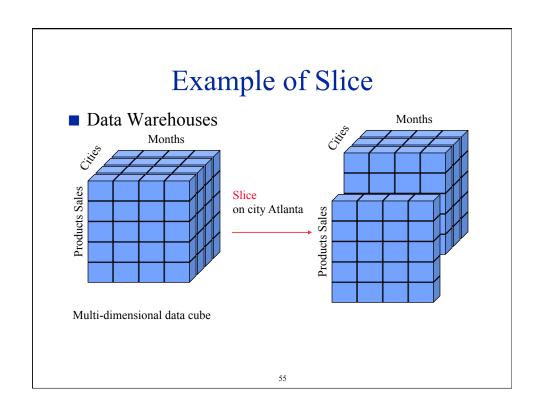
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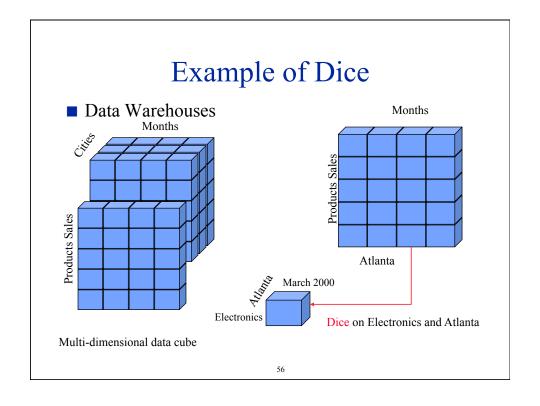
# Slice and Dice Queries

■ <u>Slice and Dice</u>: select and project on one or more dimensions





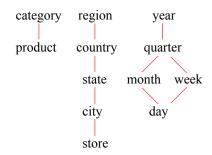


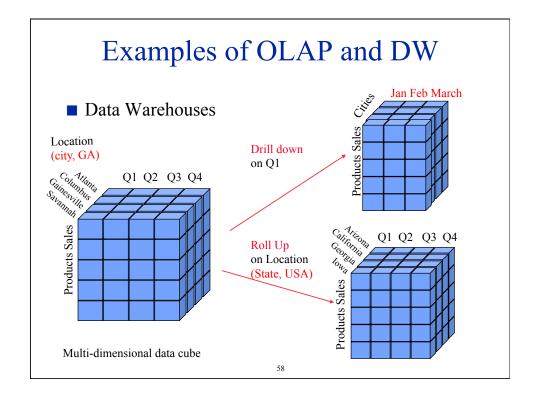


### Common OLAP Operations (2)

- Roll-up: move up the hierarchy
  - given total sales per city, we can roll-up to get sales per state
- <u>Drill-down</u>: move down the hierarchy
  - more fine-grained aggregation
  - lowest level can be the detail records (<u>drill-</u> through)

PRODUCT LOCATION TIME

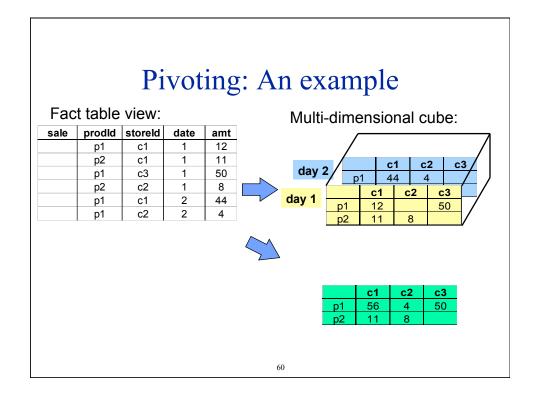




# **Pivoting**

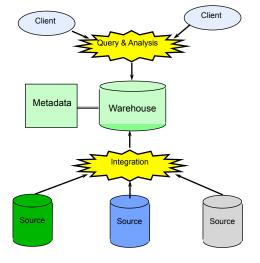
- <u>Pivoting</u>: aggregate on selected dimensions
  - usually 2 dims (cross-tabulation)

Sales		Product				
		1	2	3	4	ALL
Store	1	454			925	1379
	2	468	800			1268
	3	296		240		536
	4	652		540	745	1937
	ALL	1870	800	780	1670	5120





- *Monitoring*:
  - Sending data from sources
- *Integrating*:
  - Loading, cleansing, deriving data ...
- *Processing*:
  - Query processing, indexing, ...
- *Managing*:
  - Metadata, Design, ...



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### Query performance issues

- On Line Transaction Processing.

  Read & write from/to database.
- Data Warehousing, Decision Support System.

  Read mostly environments, with high selectivity factor.

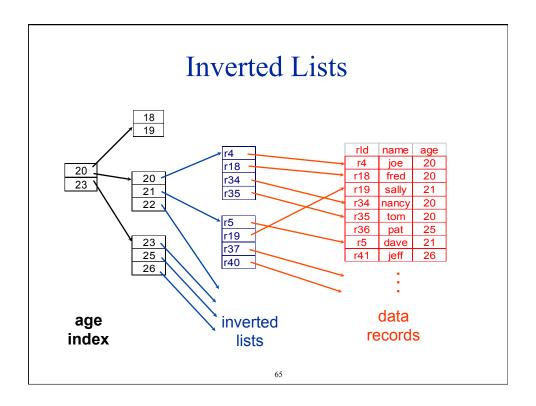
### Overview of Index Data Structures

- One dimensional index data structures:
  - Total order for one-dimension
  - Hash-based:
    - → Optimised for exact match queries, e.g. jetE = 106
  - Tree-based:
    - → Optimised for range queries, e.g. jetE < 106
    - *Most widely used: B+-tree (1972):*
- Multidimensional index data structures
  - No total order for all dimensions
  - Hash-based:
    - → Grid-File, Bang-File, ...
  - Tree based:
    - ▶ R-Trees, Pyramid-Tree, ...
  - Bitmap Indices:
    - → Applied in Data Warehouses for typical read-only environments

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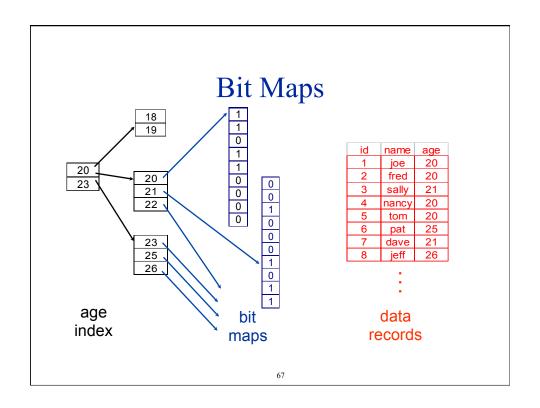
#### **Index Structures**

- Traditional Access Methods
  - B-trees, hash tables, R-trees, grids, ...
- Popular in Warehouses
  - inverted lists
  - bit map indexes
  - join indexes
  - text indexes



# **Using Inverted Lists**

- Query:
  - Get people with age = 20 and name = "fred"
- List for age = 20: r4, r18, r34, r35
- List for name = "fred": r18, r52
- Answer is intersection: r18



# Using Bit Maps

- Query:
  - Get people with age = 20 and name = "fred"
- List for age = 20: 1101100000
- List for name = "fred": 0100000001
- Answer is intersection: 010000000000
- Good if domain cardinality is not too big (small)
- Bit vectors can be compressed

#### Bitmap Index

- A bitmap is simply an array of bits
  - Consists of a collection of bitmap vectors each created to represent a distinct value.
  - More than one conditions in a query can be replied by Boolean operation on the respective bitmaps.
- Bitmap indices are a special type of index designed for efficient querying on multiple keys

#### **Properties:**

- > Suited for low cardinality column.
- Utilizes bitwise operation.
- Easy to build and add new indexed value.
- Whole bitmap segment is locked at index updating.
- Less space for storing indexes. More indexes can be cached in memory.

## Bitmap Indices (Cont.)

- A bitmap is simply an array of bits
- In its simplest form a bitmap index on an attribute has a bitmap for each value of the attribute
  - Bitmap has as many bits as records
  - In a bitmap for value v, the bit for a record is 1 if the record has the value v for the attribute, and is 0 otherwise
  - The cardinalty of the Attribute is the number of bitmap vectors the bitmap index maintains.

record number	name	gender	address	income _level	Bitma m	ps for gender		maps for come_level
0	John	m	Perryridge	L1	f	01101	L1	10100
1	Diana	f	Brooklyn	L2	1	01101	L2	01000
2	Mary	f	Jonestown	L1			L3	00001
3	Peter	m	Brooklyn	L4			L4	00010
4	Kathy	f	Perryridge	L3			L5	00000

### Bitmap Indices (Cont.)

- Bitmap indices are useful for queries on multiple attributes
  - not particularly useful for single attribute queries
- Queries are answered using bitmap operations
  - Intersection (and), Union (or), Complementation (not)
- Each operation takes two bitmaps of the same size and applies the operation on corresponding bits to get the result bitmap
  - E.g. 100110 AND 110011 = 100010 100110 OR 110011 = 110111 NOT 100110 = 011001
  - Males with income level L1: 10010 AND 10100 = 10000
    - Can then retrieve required tuples.
    - Counting number of matching tuples is even faster

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## Bitmap Indices (Cont.)

- Bitmap indices generally very small compared with relation size
  - E.g. if record is 100 bytes, space for a single bitmap is 1/800 of space used by relation.
    - → If number of distinct attribute values is 8, bitmap is only 1% of relation size (8\*1/800)
- Deletion needs to be handled properly
  - Existence bitmap to note if there is a valid record at a record location
  - Needed for complementation
    - → not(A=v): (NOT bitmap-A-v) AND ExistenceBitmap
- Should keep bitmaps for all values, even null value
  - To correctly handle SQL null semantics for NOT(A=v):
    - *→ intersect above result with (NOT* bitmap-A-Null)

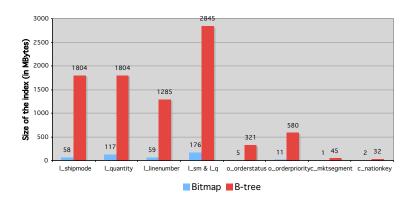
### Advantages

- Compact size.
- Efficient hardware support for bitmap operations (AND, OR, XOR, NOT).
- Fast search.
- Multiple differentiate bitmap indexes for different kind of queries.

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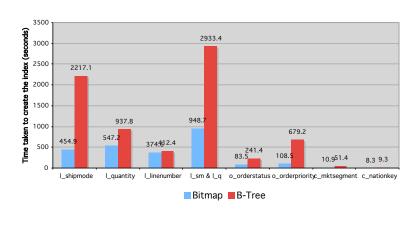
### On-disk Bitmap Index - Index Size Performance

■ Index size - Size of bitmap index is a fraction of B-tree index



# On-disk Bitmap Index - Creation Time Performance

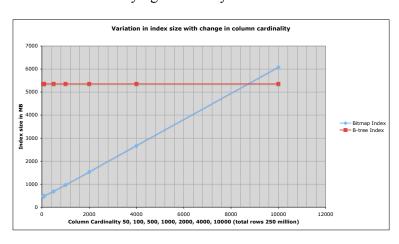
■ Index creation time: Up to 7 times faster index creation



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# On-disk Bitmap Index - Performance with varying cardinality

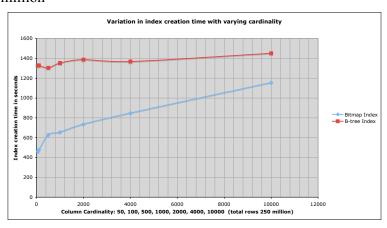
■ Index size with varying cardinality: Total rows 250 million



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# On-disk Bitmap Index - Performance with varying cardinality

■ Index creation time with varying cardinality: Total rows 250 million



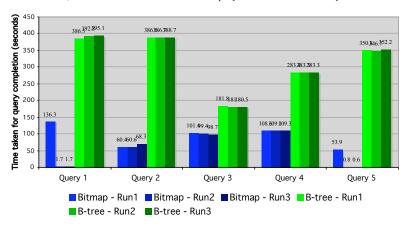
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# On-disk Bitmap Index - Query Performance

### Query 1 Query 2 SELECT sum(lineitem.l\_discount) SELECT avg(lineitem.1 tax) FROM FROM lineitem, orders, customer, nation WHERE nation.n\_name='UNITED STATES' AND orders.o\_orderstatus='F' AND customer.c\_mktsegment='AUTOMOBILE' AND orders.o orderpriority='4-NOT SPECIFIED' AND orders.o orderstatus='P' AND orders.o\_orderpriority='2-HIGH' AND lineitem.l\_linenumber=5 AND lineitem.1\_quantity=5 AND lineitem.l\_shipmode='TRUCK' AND lineitem.l\_shipmode='AIR' AND lineitem.1 quantity=2 AND lineitem.1\_linenumber=5 AND orders.o\_orderkey=lineitem.l\_orderkey; customer.c\_custkey=orders.o\_custkey AND orders.o\_orderkey=lineitem.l\_orderkey AND nation.n\_nationkey=customer.c\_nationkey; Query 3 SELECT count(\*) FROM lineitem WHERE 1\_linenumber=1; Query 4 SELECT count(\*) FROM lineitem WHERE 1\_linenumber in (1,2) AND 1\_shipmode IN ('RAIL','TRUCK'); Query 5 SELECT count(\*) FROM lineitem WHERE 1\_linenumber=5 AND 1\_shipmode='RAIL' AND 1\_quantity=18;

# On-disk Bitmap Index - Query Query Performance: Performance

Run1, Run2 and Run3 indicate that the same query has been run consecutively three-times



# Pros and Cons of Bitmap Indices

### ■ Pros:

- Easy to build and to maintain
- Easy to identify records that satisfy a complex multiattribute predicate (multi-dim. ad-hoc queries)
- Very space efficient for attributes with low cardinality (number of distinct attribute values, e.g. "Yes", "No")

### Cons:

- Space inefficient for attributes with high cardinality
- A possible solution:
  - **→** Bitmap Encoding + Bitmap Compression

# Problems with Bitmap Indexes

- Space inefficient for attributes with high cardinality (sparsity of bitmap vectors)
- Increase the complexity of the software

### Solution:

- 1. Bitmap Encoding
- 2. Bitmap Compression

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# Encoded bitmap indexing

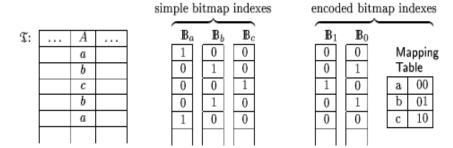


Figure 2: An example of encoded bitmap indexing

## **Encoded Bitmap Indexes**

- •We assume that our attribute domain is given by the table T is  $\{a, b, c\}$ .
- •The encoding schema of EBI is stored in a separate table called mapping table and simply encodes the values from a SBI by means of Huffman encoding (log<sub>2</sub>m for m bitmaps on attribute X).
- Therefore reduces the number of bitmaps vectors. In particular, we use only  $ceil(log_2 3) = 2$  Encoded Bitmap vectors instead of 3 simple bitmap vectors.
- This means that 2 bits are used to encode the domain {a, b, c}.

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# **Encoded Bitmap Indexes**

- We assume that we have a fact table SALES with N tuples and a dimension table PRODUCT with 12,000 different products.
- If we build a simple bitmap index (SBI)on PRODUCT, It will require 12,000 bitmap vectors of N bits in length.
- However, if we use encoded bitmap indexing (EBI) we only need ceil( $\log_2 12,000$ )= 14 bitmap vectors plus a mapping table which is a very significant reduction of the space complexity.

# Applications and variations of encoding indexing

- Hierarchy encoding
- Total order preserving
- Using encoding indexes for range encoding

Mapping Table		
[6,8)	000	
[8,10)	001	
[10,12)	101	
[12,13)	100	
[13,16)	010	
[16,20)	110	
(a)Range	encodir	n

 $\begin{array}{ccccc} 6 \leq A < 10 & : & \mathbf{B}_2' \mathbf{B}_1' \\ 8 \leq A < 12 & : & \mathbf{B}_1' \mathbf{B}_0 \\ 10 \leq A < 13 & : & \mathbf{B}_2' \mathbf{B}_1' \\ 16 \leq A < 20 & : & \mathbf{B}_2' \mathbf{B}_1 \end{array}$ 

(b)Retrieval functions

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# Space time tradeoff of bitmap indexes, for selection queries.

- Space optimal bitmap index.
- Time optimal bitmap index under a given space constraint.
- Bitmap index with optimal space time tradeoff.
- Time optimal bitmap index.

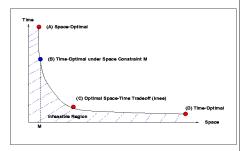
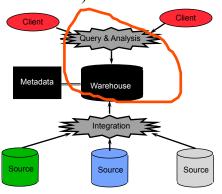


Figure 2: Space-Time Tradeoff Issues

# **Processing**

- ROLAP (Relational OLAP) servers vs. MOLAP servers (Multi-dimensional OLAP)
- Index Structures
- What to Materialize?
- Algorithms



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# Materialized Views

■ Define new warehouse relations using SQL expressions

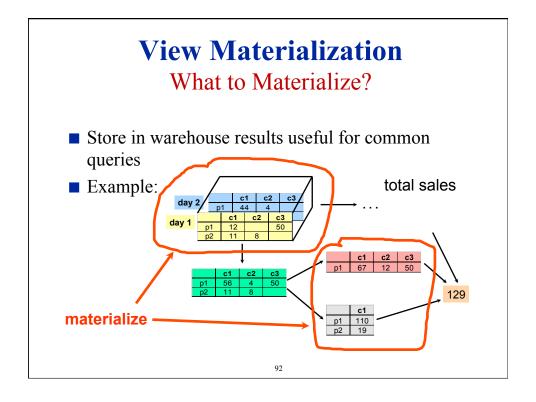
sale	prodld	storeld	date	amt
	p1	c1	1	12
	p2	c1	1	11
	p1	сЗ	1	50
	p2 p1 p2 p1	c2	1	8
	p1	c1	2	44
	p1	c2	2	4

product	id	name pric		
	p1	bolt	10	
	n2	nut	5	

joinTb	prodld	name	price	storeld	date	amt
	p1	bolt	10	c1	1	12
	p2	nut	5	c1	1	11
	p1	bolt	10	c3	1	50
	p2	nut	5	c2	1	8
	p1	bolt	10	c1	2	44
	p1	bolt	10	c2	2	4

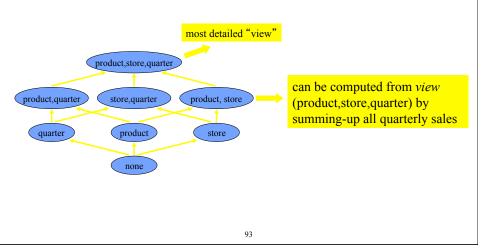


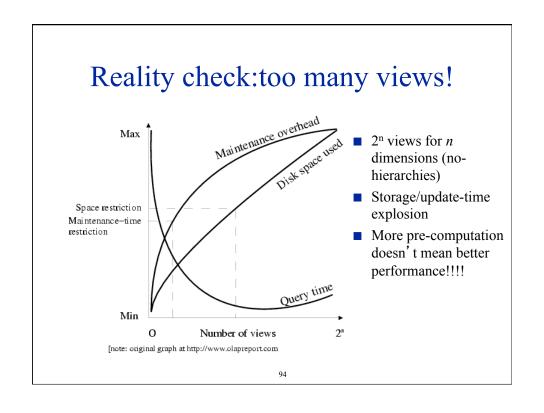
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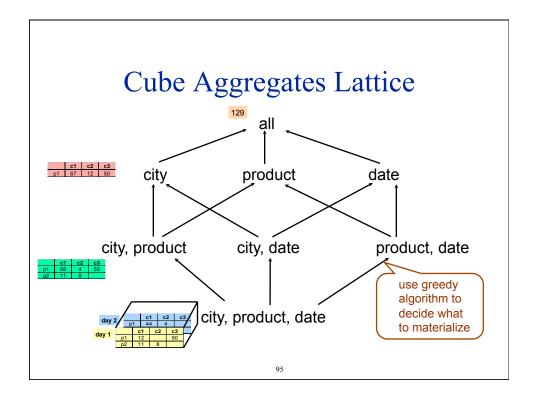


# **Data Cube Computation**

■ Model dependencies among the aggregates:







# **Materialization Factors**

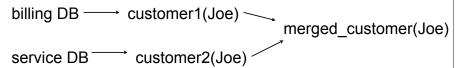
- Type/frequency of queries
- Query response time
- Storage cost
- Update cost

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# Data Cleaning Data Loading Derived Data Metadata Warehouse Source Source Source Source

# Data Cleaning

- Migration (e.g., yen ⇒ dollars)
- Scrubbing: use domain-specific knowledge (e.g., social security numbers)
- Fusion (e.g., mail list, customer merging)

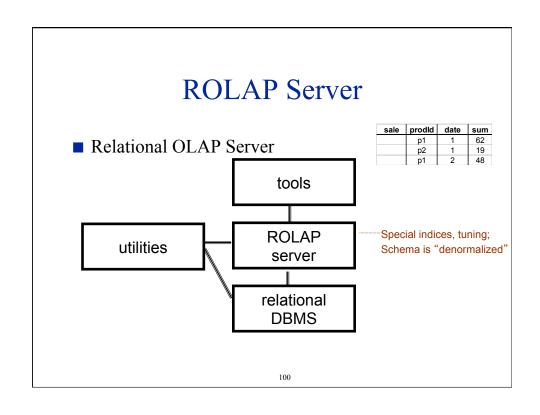


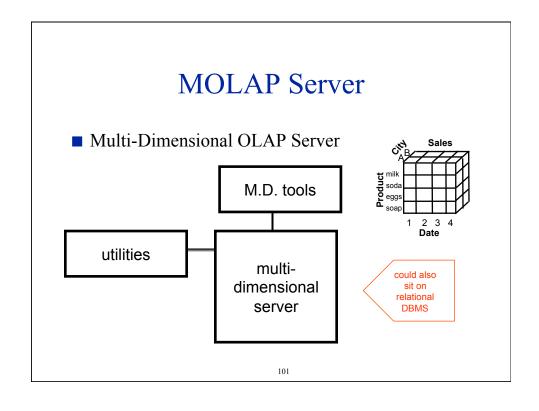
■ Auditing: discover rules & relationships (like data mining)

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# Loading Data

- Incremental vs. refresh
- Off-line vs. on-line
- Frequency of loading
  - At night, 1x a week/month, continuously
- Parallel/Partitioned load





# Data Warehousing

- Growing industry: \$8 billion in 1998, predicted > > \$30~50 billions today
- Range from desktop to huge:
  - Walmart: 900-CPU, 2,700 disk, 23TB Teradata system
- Lots of buzzwords, hype
  - slice & dice, rollup, MOLAP, pivot, ...

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