Which Route to Follow in Urban Transportation Network?

An Introduction to Traveler's Route Choice Behavior

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Outline

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   - Source of Traffic Congestion
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Traffic congestion

(a) Cross-Harbor Tunnel in Hong Kong

(b) Heavy traffic in Beijing

Figure: Traffic congestion in modern cities [1]

[1] The images were downloaded respectively from the Wikipedia (http://en.wikipedia.org/wiki/File:HK_Cross_Harbour_Tunnel.jpg, visited on July 9, 2010) and the United Press International (http://www.upi.com/News_Photos/gallery/Daily-Life-in-Beijing/2012/2, visited on July 9, 2010).

Non-decreasing travel time

The travel time of a road depends on the traffic flows on this road:

The more vehicles on a road, the more time needed to pass through it.

Road travel time, $t(x)$, is defined as a non-decreasing function of flows x.

What does *non-decreasing* mean here?

Mathematically, for any $x_1 > x_2$, we have

$$t(x_1) \geqslant t(x_2)$$

Which route to follow?

Example

Figure: Two roads between *rs*

Travel Time

- Free travel time on minor road is 20 minutes.
  - The travel time increases 5 minutes for each additional vehicle.
- Travel time on highway is always 50 minutes.

Flows on a 2-node network

- The travel time functions:
  - Minor road: $c_{min}(x) = 20 + 5 \cdot x$
  - Highway: $c_{high}(x) = 50$
- There are 10 persons traveling between *rs*.
- The total travel time for Pattern 1 is $(3 \times 35 + 7 \times 50) \div 10 = 45.5$
- The total travel time for Pattern 2 is $(5 \times 50 + 4 \times 50) \div 10 = 50$

Example

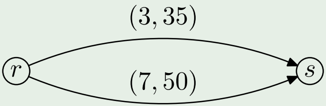

Figure: Traffic Flow Pattern 1

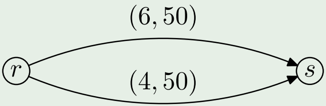

Figure: Traffic Flow Pattern 2

All possible traffic flow patterns

Table: All possible traffic flow patterns

x_{min}	x_{high}	t_{min}	t_{high}	$\bar{t}$
0	10	20	50	50.0
1	9	25	50	47.5
2	8	30	50	46.0
3	7	35	50	**45.5***
4	6	40	50	46.0
5	5	45	50	47.5
6	4	50	50	**50.0**
7	3	55	50	53.5
8	2	60	50	58.0
9	1	65	50	63.5
10	0	70	50	70.0

Note: Average travel time $\bar{t} = (x_{min} \cdot t_{min} + x_{high} \cdot t_{high}) \div (x_{min} + x_{high})$

Some insights

In larger networks, the number of possible traffic flow patterns can be astronomical. And different patterns result in different network performances.

Two questions

1. Which traffic flow pattern is most likely to occur in the real world?
2. What is the source of traffic congestion?

The shortest path problem

Every traveler tries to find the quickest route between r and s.

Day 1

- Initially, there is no traveler on the network.
- The minor road is the quickest route between r and s.

$\Rightarrow$

Day 2

- All the travelers crowd into the minor road.
- The highway is the quickest route between r and s.

$\Rightarrow$

Day n

$\ldots$

Complex interaction

Route Choices & Route Travel Time:

- The travelers attempt to choose the quickest route, and this choice depends on the travel time of each route.
- However, the route travel time in turn depends on the route choices made by all the travelers in this network.

Question

If this process goes to infinity, is there a stationary state or an equilibrium?

Wardrop's Principle

Each traveler non-cooperatively seeks to minimize his travel time. An equilibrium is reached when no traveler may lower his travel time through unilateral action.

Wardrop's Principle

> *"The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route."* [2]

[2] Wardrop, J. G., 1952. Some theoretical aspects of road traffic research, Proceedings, Institute of Civil Engineers, Part II, Vol.1, pp. 325-378.

Find the equilibrium traffic flows

Wardrop's Principle has two-fold meaning.

"The journey times on all the routes actually used are equal."

At equilibrium, the travel time on minor road is equal to that of highway,

$$
\begin{aligned}
t_{min} &= t_{high} \\
20 + 5 \cdot x_{min} &= 50 \\
\therefore x_{min} &= 6 \text{ and } x_{high} = 4
\end{aligned}
$$

"... and less than those on any unused route."

If the number of trips between r and s is 5, all the travelers will choose the minor road, because of

$$
\begin{aligned}
t_{min}(5) &= 45 \\
t_{high}(0) &= 50 \\
\therefore t_{min}(5) &< t_{high}(0)
\end{aligned}
$$

Implication of the equilibrium

Day 1

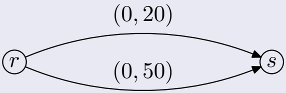

Figure: Initial traffic flows.

$\Rightarrow \cdots \Rightarrow$

Day n

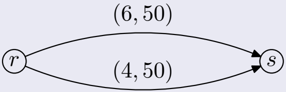

Figure: Equilibrium traffic flows.

Implication of the equilibrium

If we assume that every traveler chooses the quickest route, the traffic flows that satisfy Wardrop's Principle is most likely to occur in real world.

Equilibrium flows vs. Optimal flows

Table: Equilibrium traffic flows are different form optimal flows.

Flows patterns	x_{min}	x_{high}	t_{min}	t_{high}	$\bar{t}$
Optimal flows	3	7	35	50	45.5*
Equilibrium flows	6	4	50	50	50.0

Note: $\bar{t} = (x_{min} \cdot t_{min} + x_{high} \cdot t_{high}) \div (x_{min} + x_{high})$

- Compared to optimal flows, too many travelers crowd into the minor road in equilibrium flows.
- The average travel time of equilibrium flows is 4.5 minutes larger than that of optimal equilibrium.

Source of traffic congestion

Choosing quickest route $\Rightarrow$Crowding effects $\Rightarrow$Traffic congestion

Summary

Investigate the traffic congestion in light of travelers' route choice behaviors:

Individual's behavior The quickest path problem.

Aggregate quantity Equilibrium traffic flow.

Macroscopic phenomenon Traffic congestion.

State-of-the-Art Research

Considerable number of studies have emerged in recent years:

- Road congestion pricing
- Random errors in travelers' choices
- Uncertainties in travel time
- . . .

Network Structure

An abstract model for urban transportation network:

- A set of nodes N, and a set of links A.
- Origin-destination (O-D) pair rs, where $r, s \in N$.
- Number of trips made between each O-D pair f^{rs}.
- For each link $a \in A$, a travel time function $t_a(x_a)$ is given.

Example

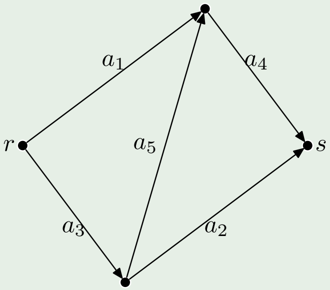

Figure: Braess's network