Multi-Objective Capacitated Vehicle Routing Problem with Time-Dependent Demands (CVRP-TDD) for Medical Waste Collection

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1 Problem Definition

In an urban area, a set of demand nodes (hospitals, clinics, and other medical institutions) produce medical wastes that must be collected by a vehicle with limited load capacity and disposed of in one of the disposal centers. This set of vehicles needs to start and finish their routes in one of the municipal parking lots. Regularly clearing waste from demand nodes and vehicles to avoid risk build-up is crucial. The risk at the demand nodes increases with each passing moment that the waste remains uncollected.

1.1 Mathematical Notation

$G = (P, E)$ is a complete graph that represents the urban area, where $P = \{p_1, p_2, \ldots, p_n\}$ is the set of nodes (containing demand, parking, and disposal nodes), and $E = \{(i, j) : i, j \in P\}$ the set of edges with weights $d_{i,j}$ denoting the euclidean distance between nodes i and j. P is partitioned as $P = P_d \cup P_p \cup P_c$, where P_d is the set of demand nodes, P_d is the set of parking nodes with unlimited availability, and P_c is the set of disposal centers with unlimited processing capability. Each $p_i \in P_d$ generates waste at a variable rate $w_i(t)$, and the exposure risk factor $r_i^e(t)$ escalates if the waste has not been collected. For this case, we will define $w_i(t)$ and $r_i^e(t)$ as:

$$w_i(t) = \alpha_i \cdot t$$
$$e_i^r(t) = \beta_i \cdot W_{i,t}$$

Where α_i and β_i are coefficients associated with the waste production and risk exposure for each processing node i, and $W_{i,t}$ is the amount of uncollected waste at demand node i at time t.

V is the set of vehicles. Each vehicle $v \in V$ has:

- A maximum load capacity c_v.
- A traveling cost cost t_v (per unit of distance).
- A predetermined starting parking node $p_0^v \in P_p$.

- A velocity s_v (measured in units of distance per unit of time). Using s_i, we can transform the distance matrix $d_{j,k}$ into a time matrix for each vehicle $t^v_{j,k}$ that indicates the units of time that it takes for vehicle v to go from node j to node k, by taking $t^v_{j,k} = d_{j,k}/s_v$.

Finally, we consider a discrete time horizon $T = [1, 2, \ldots t_f]$.

2 Mathematical Optimization Model

This section shows the Mixed-Integer Optimization problem for the CVRP-TDD for the Medical Waste Collection Problem.

2.1 Variables

In this section, we show the set of decision variables.

- $x_{i,j,v,t}$: Binary variable indicating if vehicle v departures from node i to node j at time t.
- $W_{i,t}$: Continuous variable denoting the amount of uncollected waste at demand center i at time t.
- $l_{v,t}$: Continuous variable denoting the amount of load at vehicle v in time t.
- $WX_{i,v,t}$: Continuous variable denoting the amount of medical waste that vehicle v would have to load if visited node $i \in P_d$ at time t.

2.2 Objective Functions

The cost objective function is determined by the sum of the traveling costs of every vehicle as:

$$\min : f_1 = \sum_{v \in V} \left[t_v \cdot \sum_{(i,j) \in E} \left(d_{i,j} \cdot \sum_{t \in T} x_{i,j,v,t} \right) \right]$$

The risk objective function is determined by the sum of all factors related to risks, as in:

$$\min : f_2 = \sum_{i \in P_d} \left[\beta_i \cdot \sum_{t \in T} W_{i,t} \right]$$

2.3 Constraints

$$\sum_{i \in P_d} x_{p_v^0, i, v, 1} = 1, \quad \forall v \in V \tag{1}$$

$$\sum_{(i,j) \in E} x_{i,j,v,t} \leq 1, \quad \forall v \in V, \forall t \in T \tag{2}$$

$$x_{i,j,v,t} + \sum_{k \in P} \sum_{t'=t+1}^{t+t_{i,j}-1} x_{j,k,v,t'} \leq 1, \quad \forall (i,j) \in E, \forall v \in V, \forall t \in T \tag{3}$$

$$x_{i,j,v,t} \leq \sum_{k \in P} x_{j,k,v,t+t_{i,j}^v}, \quad \forall i \in P, \forall j \in P_d \cup P_c, \forall v \in V, \forall t \in T \tag{4}$$

$$\sum_{k \in P \setminus \{j\}} x_{j,k,v,t} \leq \sum_{\substack{h \in H \\ H = \{p \in P \setminus \{j\} : t - t_{p,j}^v \geq 1\}}} x_{h,j,v,t-t_{h,j}^v}, \quad \forall j \in P_d \cup P_c, \forall v \in V, \forall t \in T \tag{5}$$

$$\sum_{t \in T} x_{i,i,v,t} = 0, \quad \forall i \in P, \forall v \in V \tag{6}$$

$$\sum_{i \in P} \sum_{v \in V} \sum_{t \in T} x_{i,j,v,t} \geq 1, \quad \forall j \in P_d \tag{7}$$

$$\sum_{p_c \in P_c} \sum_{p_p \in P_p} \sum_{t \in T} x_{p_c, p_p, v, t} = 1, \quad \forall v \in V \tag{8}$$

$$M\left(1 - \sum_{i \in P} x_{i, p_p, v, t'}\right) \geq \sum_{(i,j) \in E} \sum_{t=t'+1}^{t_f} x_{i,j,v,t}, \quad \forall p_p \in P_p, \forall v \in V, \forall t' \in T \tag{9}$$

$$W_{j,1} = w_i(1), \quad \forall j \in P_d \tag{10}$$

$$W_{j,t+1} \geq W_{j,t} + \alpha_j - M \sum_{i \in P} \sum_{v \in V} x_{j,i,v,t+1}, \quad \forall j \in P_d, \forall t \in T \tag{11}$$

$$l_{v,1} = 0, \quad \forall v \in V \tag{12}$$

$$l_{v,t} - M\left(\sum_{p_c \in P_c} \sum_{i \in P} x_{p_c, i, v, t+1}\right) + \sum_{p_d \in P_d} WX_{p_d, v, t+1} \leq l_{v,t+1}, \quad \forall v \in V, \forall t \in T \tag{13}$$

$$l_{v,t} \leq c_v, \quad \forall v \in V, \forall t \in T \tag{14}$$

$$WX_{i,v,t} \geq W_{i,t-1} + \alpha_i - M\left(1 - \sum_{j \in P} x_{i,j,v,t}\right), \quad \forall i \in P_d, \forall v \in V, \forall t \in T \tag{15}$$

Constraint (1) states that every vehicle $v \in V$ must start its route in the predetermined starting node p_v^0. Constraint (2) states that no vehicle can traverse more than one edge simultaneously. Constraint (3) states the traveling times between nodes i and j. Constraint (4) states that every vehicle should leave a visited node as soon as it arrives (except for parking nodes). Constraint (5) states that a vehicle can only depart from a node j if it has arrived at that node previously. Constraint (6) states that a vehicle cannot remain stationary at the same node. Constraint (7) states that every demand node must be visited at least once. Constraint (8) states that every vehicle must visit one parking node immediately after visiting a disposal center to end its route to avoid any vehicle ending the route while still carrying medical waste. Constraint (9) states that each vehicle's route ends once it has visited a parking node. Constraint (10) indicates the initial amount of waste for every demand node. Constraint (11) indicates the amount of waste for each time $t \in T$. Constraint (11) manages the load adjustment in each vehicle when it visits either a disposal center or a demand node: it resets the load of a vehicle to zero when it visits a disposal center or adds to the vehicle's load the waste collected from demand nodes. Constraint (12) ensures that each vehicle starts its route without load. Constraint (14) ensures that the load in any vehicle does not exceed its maximum capacity. Constraint (15) determines the amount of waste $WX_{i,v,t}$ that vehicle v would collect from demand node i at time t.

Remark* 1*: In constraints (11) and (15), the term α_j (or α_i) is the increase in uncollected waste at demand node j per time step, assuming a linear waste generation function. However, this constraint can be adapted to any form of the waste generation function by replacing α_j with the forward difference of the chosen waste production function $w_i(t)$, defined as:

$$\Delta w_i(t) = w_i(t+1) - w_i(t)$$

3 Experiment and Results

This section displays the results, instance parameters, and data of the small experiment used to run the optimization model.

3.1 Experiment and Problem Instance Description

Tables 1, 2, and 3 display the demand, parking, and processing node coordinates. Specifically, there are five demand nodes, two parking nodes, and two processing nodes.

X Coordinate	Y Coordinate
10	10
20	15
30	25
15	30
5	20

Table 1: Demand Node Coordinates

X Coordinate	Y Coordinate
5	5
35	30

Table 2: Parking Node Coordinates

The fleet comprises two vehicles with speeds of 3 and 4 units per time step. Each vehicle starts from one of the parking nodes. The vehicle capacities range randomly between 20 and 30 units

X Coordinate	Y Coordinate
25	20
10	15

Table 3: Processing Node Coordinates

(obtaining the value of 22 for both of them), and the traveling costs vary between 1 and 3 per unit distance (obtaining the values of 3 and 1).

The time horizon for this experiment, denoted as t_f, is calculated as:

$$t_f = \left\lceil \frac{\text{Average Distance Between All Nodes}}{\text{Average Speed}} \times \text{Number of Nodes} \right\rceil$$

For this instance, we got a value of $t_f = 41$.

The values used for $\boldsymbol{\alpha}$ were randomly generated between 0 and 1, obtaining values of 0.77, 0.02, 0.63, 0.74, and 0.49. The values for the exposure risk factor in each production center, $\boldsymbol{\beta}$, are values of 0.1, 0.2, 0.15, 0.25, and 0.3.

This experiment does not utilize a multi-objective approach. Instead, the objective was to minimize the sum of the two objective functions.

3.2 Results

The model got a optimal objective function value of 494.79. Tables 4 and 5 shows the routes of each vehicle:

Departure Time	Departure Node	Destination Node
1	6	1
3	1	2
7	2	3
12	3	4
17	4	5
22	5	9
24	9	6

Table 4: Route for Vehicle 1

Departure Time	Departure Node	Destination Node
1	7	3
3	3	8
5	8	4
9	4	5
13	5	9
15	9	1
16	1	2
19	2	8
21	8	3
23	3	4
27	4	5
31	5	9
33	9	6

Table 5: Route for Vehicle 2

Finally, Figure 1 displays the uncollected wastes in the demand nodes over time.

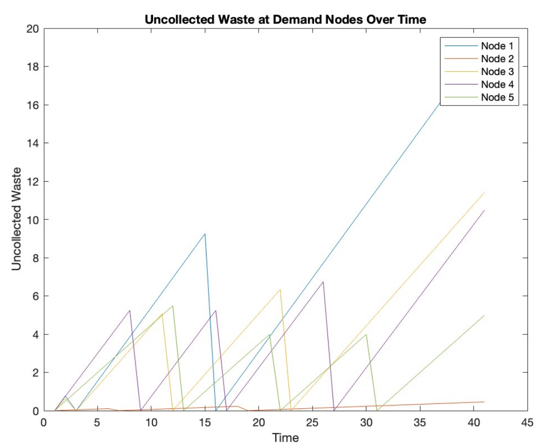

Figure 1: Uncollected Wastes in the Demand Nodes Over Time