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Summary

It has attracted worldwide attention to improve the stability and equity of the food system in recent years. Due to the lack of equity and sustainability, the unstable food system is threatening the life of food insecure people living in every country. As a result, a robust food system model optimized for equity and sustainability counts. The **Food Allocation Model** proposed in this paper provides an equitable and sustainable solution, which is applicable to food systems of different scale and regions.

To begin with, we introduce food allocation utility function to measure the equity of the food system according to **Marginal Utility Theory**. The **Food Allocation Model** is established based on the data of 110 countries，which considers multiple aspects including profitability, equity, sustainability and climatic conditions. In addition, the **Dynamic Evolution Model** is established to predict the time to complete the evolution of a food system, referring to the **Lotka-Volterra Model**.

Next, we apply the model to the global food system and discuss the benefits and costs of changing the priorities. The food allocation of the optimized system is compared with the original system visually on the world map. It is predicted to reach the critical level in 7.5 years and reach stability in 30 years, which are visualized in the direction field and phase trajectory cluster. We take Spain and Iraq as examples to support our research. The model is verified by the correlation between **food insecurity rate** and **food flow rate**.

Then, the model is applied in the BRICS to verify the scalability and adaptability. It is found that the model is still adaptive and the results are highly matched with the actual situation. It is predicted to reach the critical level in 6.5 years, which is less than that for the whole world.

Finally, the sensitivity of the model is tested and **extreme events** such as extreme weather and pest invasion are taken into consideration to improve the model. In conclusion, the model proposed in this paper is robust enough to solve the problem of unstable food system, which is expected to guide the improvement of the current various food systems.

Keywords: food system; food insecurity; Food Allocation Model; Marginal Utility Theory; Dynamic Evolution Model; extreme events

Contents

I. Introduction

The world today has a global food system with large yields and high efficiency.

In terms of food production, per capita caloric availability and food diversity (the variety of food groups in a diet) have increased since 1960s[1], and food production continues to grow. In terms of trade, thanks to the development of globalization and technology, the volume of trade in food and agriculture has actually more than doubled since 1995[2].

Nevertheless, recent events have shown us that our global food system is unstable and vulnerable to external factors. In 2020, due to the impact of COVID-19 and the locust plague, many countries have announced a food export ban, which has caused the international food market prices to rise all the way. And the economic recession triggered by this series of crises has caused economic slowdown and chaos, which made the situation worse. The Food and Agriculture Organization of the United Nations (FAO) has issued the warning: The epidemic has left 690 million people in hunger, 25 countries are facing severe hunger risks, and the world may usher in the worst food crisis in 50 years[3]. The global food system is in extreme instability!

While issuing the warning, FAO also admitted: there is sufficient food produced to feed every person in the world[1]. In fact, the fundamental reason for this contradiction is that the current global food system always prioritizes efficiency and profitability, while ignoring equity and sustainability. Although the current food system generally served well in the most parts of the world, if these two points are ignored for a long time, it will adversely affect efficiency and profitability. On the one hand, ignoring equity means more hungry people. Victoria and others pointed out: Hunger is also a cause of poverty in a cyclical relationship[4]. Hunger can lead to even greater poverty by reducing people's ability to work and learn by causing health, small body size, low levels of energy and reductions in poor mental functioning, which affects efficiency and profitability in many aspects such as trade and production. In addition, for humanitarian reasons, equity should also be an important point we should consider. On the other hand, ignoring sustainability will result in abnormal climate and frequent extreme weather. According to FAO, every 1 degree Celsius increase in temperature will reduce grain production by 10%[5]. The reduction in grain production caused by extreme weather has reached 50 million tons per year in China alone, which is equivalent to US$4.46 billion[6]. Therefore, it is reasonable and urgent to incorporate equity and sustainability into the food system.

II. Assumptions and Notations

2.1 Assumptions

By adequate analysis of the problem, we make the following well-justified assumptions to simplify our model.

- **The data sources used in the model are real and reliable.** Our conclusions are valid only when the data entered into the model can truly reflect local conditions.
- **In the short-term we considered, the total global food production is approximately unchanged,** which means there will be no significant growth. This is because there has been sufficient food produced to feed everyone all over the world[7]. Considering the population growth and the progress of technology, coupled with the constraints of different degrees of natural disasters, we believe the increase in total food production is limited before the emergence of revolutionary technologies in planting and feeding.
- **A singel country is the basic unit to study the food insecure problem around the world,** which means it is believed that there would be an equitable allocation within a country. When a country's food is allocated enough, its food insecure problem will be solved. Based on this assumption, there is no need to discuss about how to distribute food within the country.
- **Food resources flow evenly from food secure countries to food insecure countries.** Only in this way, can all countries get equitable food allocation all over the world.

2.2 Notations

Table 1. Notations used in our model

Intensity of Value	Interpretation
x_i	Optimized food allocation for country i
$U(x)$	Utility of food allocation
x_{imin}	Minimum food demand for country i
x_{imax}	Maximum food allocation value for country i
m_i	Maximum regulation factor
s_i	The total pollutant emission per unit agricultural land area
W	Total amount of food allocation
W_s	Sustainable development allowed total pollutant emissions
$\boldsymbol{P}$	The set of food secure countries
$\boldsymbol{Q}$	The set of food insecure countries

III. Model Specification

3.1 Food Allocation Model

In order to promote the equity and sustainable development of food allocation, a food allocation model with the objective of food allocation utility is established.

3.1.1 Model Preparation

1) Food allocation utility function

Food allocative utility refers to the contribution of a certain amount of food consumption to the sustainable development and equity of regional food system. Marginal utility is the increase of utility caused by the consumption of food. According to the Austrian economist Wiesel, marginal utility has the characteristics of diminishing[8], that is, with the consumption of food, the utility of food allocation increases, but the marginal utility decreases[9].

$$\frac{dU(x)}{dx} \geq 0, \quad \frac{d^2U(x)}{d^2x} \leq 0 \tag{3.1}$$

Where, x is the food allocation amount (unit: \$ 1 billion), and $U(x)$ is the utility of food allocation amount to the sustainable development and equity of the regional food system.

Exponential distribution probability density function is selected as the marginal utility function, for the reason that the relationship between utility function and marginal utility function is similar to the relationship between the continuous distribution function and density function, and both of them have good mathematical properties, clear physical meaning and numerical characteristics such as mean value, variance which reflect the characteristics of function curve. Through setting these parameters can accurately fit the specific application of the utility function[10].

In view of the relationship between utility function and marginal utility function, the utility function of food allocation is constructed as follows.

$$U(x) = \begin{cases} 1 - e^{-\lambda(x-\mu)}, & x \geq \mu \\ 0, & x < \mu \end{cases} \tag{3.2}$$

The corresponding marginal utility function of food allocation is:

$$\frac{dU(x)}{dx} = \begin{cases} \lambda e^{-\lambda(x-\mu)}, & x \geq \mu \\ 0, & x < \mu \end{cases} \tag{3.3}$$

According to the function, as the food allocation increases, the total utility will gradually approach 1, but not equal to 1. When the allocation of food resources is $x_{i\max}$, the total utility is $1-\varepsilon$, where ε is the infinitesimal parameter. When the food allocation is allocated as $x_{i\min}$, the total utility is 0. Substitute $x_{i\max}$ and $x_{i\min}$ into formula (3.2), and the equation can be obtained as follows.

$$\begin{cases} 1 - e^{-\lambda(x_{max}-\mu)} = 1 - \varepsilon \\ 1 - e^{-\lambda(x_{min}-\mu)} = 0 \end{cases} \tag{3.4}$$

The calculated parameter value is as follows.

$$\mu = x_{min} \tag{3.5}$$

$$\lambda = \frac{-ln\varepsilon}{x_{max} - x_{min}} \tag{3.6}$$

2) Parameter definition

According to the criteria proposed by the United Nations Food and Agriculture Organization (FAO), families whose Engel coefficient higher than 59 percent are regard as poor families. At the same time, countries with a per capita GDP of $765 or less are classified as low-income and poor countries. We define the variables as follows.

Minimum food demand:

$$x_{imin} = w_i \cdot GDP_i \cdot En \tag{3.7}$$

where, w_i is the proportion of food consumption in GDP for country *i*. GDP_i is the product of population and GDP per capita. *En* is Engel's coefficient of critical poverty, and $En = 0.59$[11].

Maximum food allocation value:

$$x_{i\max} = m_i \cdot x_{i\min} \tag{3.8}$$

where, m_i is maximum regulation factor, related to environmental levels, population and food insecure. The maximum food allocation quantity is the product of the minimum demand and the maximum regulation coefficient.

$$s_i = \frac{\text{total pollutant discharge}}{\text{agricultural land area}} \tag{3.9}$$

where, s_i is the total pollutant emission per unit agricultural land area, which is normalized.

3.1.2 Establishment of Food Allocation Model

Based on the utility function and marginal utility function above, the food allocation model is constructed.

1) Objective function

When the total amount of food allocation is certain, the objective function is the maximum of the sum of utility of food allocation:

$$\max U = \sum_{i=1}^{n} U\left(x_i\right) \tag{3.10}$$

where, *n* is the number of countries in a food system.

2) Constraint condition

a. Total food constraint

The total amount of food allocation must be within the supply capacity of the food system, the constraint conditions is:

$$\sum_{i=1}^{n} x_i \le W \tag{3.11}$$

where, W is the total amount of food allocation, which can be regarded as unchanged in the short term.

b. Food demand constraint

Food allocation should not only meet the basic demand of residents, but also meet the maximum allocation value of the region in the food system.

$$x_{i\min} \le x_i \le x_{i\max} \tag{3.12}$$

where, $x_{i\min}$ and $x_{i\max}$ are the minimum demand and maximum allocation values in the food system of the *i-th* country respectively.

c. Constraint of sustainable development[12]

The impact of sustainable environmental development should be considered in food allocation, and the total pollution quantity should not exceed the total allowable pollution discharge. Therefore, the constraint conditions are as follows.

$$\sum_{i=1}^{n} s_i x_i \le W_s \tag{3.13}$$

where, s_i is the pollution coefficient in the food system of the *i-th* country, W_s is the total pollutant emission allowed for sustainable development.

3) The food allocation model construction

According to the constraint conditions and the objective function, the food configuration model is finally established:

$$\begin{cases} \max U = \sum_{i=1}^{n} U\left(x_i\right) \\ \sum_{i=1}^{n} x_i \le W \\ x_{i\min} \le x_i \le x_{i\max} \\ \sum_{i=1}^{n} s_i x_i \le W_s \end{cases} \tag{3.14}$$

3.2 Dynamic Evolution Model

3.2.1 Model Preparation

Lotka-Volterra Model[13] (set $\boldsymbol{P}$ & $\boldsymbol{Q}$) is an extension of logistic model, which can simulate the evolution trend of two populations under the condition of limited resources. It includes the following parameters.

n_1, n_2: maximum amount of $\boldsymbol{P}$ and $\boldsymbol{Q}$.

s_1: for the resources supporting $\boldsymbol{P}$, the consumption of unit quantity $\boldsymbol{Q}$ (relative to n_2) is s_1 times of unit quantity $\boldsymbol{P}$.

s_2: same as s_1.

r_1, r_2: the inherent growth rate of two species.

x_1, x_2: real time amount of two species.

And these parameters meet the following formulate.

$$\begin{cases} \dfrac{dx_1}{dt} = r_1 x_1 \left(1 - \dfrac{x_1}{n_1} - s_1 \dfrac{x_2}{n_2}\right) \\ \dfrac{dx_2}{dt} = r_2 x_2 \left(1 - \dfrac{x_2}{n_2} - s_2 \dfrac{x_1}{n_1}\right) \end{cases} \tag{3.15}$$

3.2.2 Establishment of Dynamic Evolution Model

We take set ***P*** and set ***Q*** as two populations, and solve their original food consumption, growth rate and other parameters. Referring to the Lotka-Volterra Model, we build a Dynamic Evolution Model to predict the time required to achieve the goal and the final stable value.

The Dynamic Evolution Model is established as follows.

$$\begin{cases} \dfrac{dx_1}{dt} = R_1 x_1 \left(1 - \dfrac{x_1}{N_1} - S_1 \dfrac{x_2}{N_2}\right) \\ \dfrac{dx_2}{dt} = R_2 x_2 \left(1 - \dfrac{x_2}{N_2} - S_2 \dfrac{x_1}{N_1}\right) \end{cases} \tag{3.16}$$

where x_1 is the food consumption of set ***P*** and x_2 is the food consumption of set ***Q***, R_1 and R_2 are the average growth rates of set ***P*** and set ***Q*** in recent ten years which meet the following formulate.

$$\begin{cases} R_1 = \dfrac{x_1(t) - x_1(t-10)}{10 x_1(t-10)} \\ R_2 = \dfrac{x_2(t) - x_2(t-10)}{10 x_2(t-10)} \end{cases} \tag{3.17}$$

N_1 is the maximum amount of set ***P***. Considering the distributive effect, the food consumption of set ***P*** should decrease monotonically, $N_1 = x_1(0)$.

N_2 is the maximum amount of set ***Q***, $N_2 = N_1$.

S_1 is the unit resource proportion of set ***P***, $S_1 = x_{1\min} / x_1(0)$.

S_2 is the unit resource proportion of set ***Q***, $S_2 = x_{2\min} / x_2(0)$.

IV. Solution and Verification of Our Model

4.1 Food Allocation Model

4.1.1 Solution of Food Allocation Model

To reduce the complexity of the food system, we simplify the global food system. According to the incidence of malnutrition in 2018, we have selected 110 countries with relatively complete data in the past decade. The data comes from the Food and Agriculture Organization of the United Nations (FAO).

Aiming to obtain the maximum equitable utility of food allocation, we input the data of food allocation x_i, minimum food demand $x_{i\min}$ and other data of 110 countries into the model, and obtain the following results.

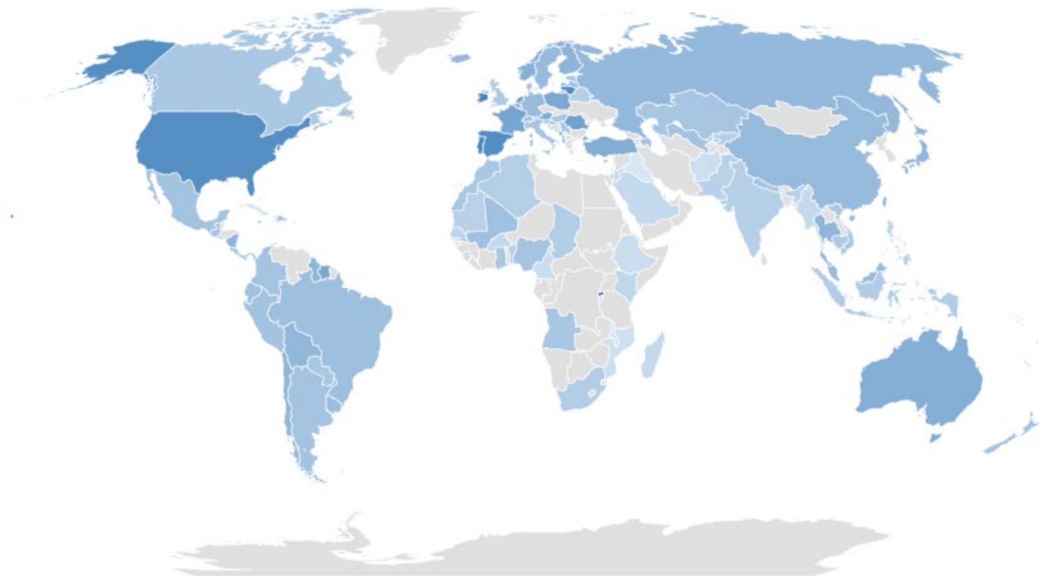
1) Food equity

Food allocation utility refers to the contribution of a certain amount of food consumption to the sustainable development and equity of regional food system. The initial value of the utility function $U(x)$ before food allocation is calculated as 64.52, which is increased to 75.76 under the optimization of the allocation model. This indicates that the equity of food allocation is improved and the food allocation utility is increased after optimization.

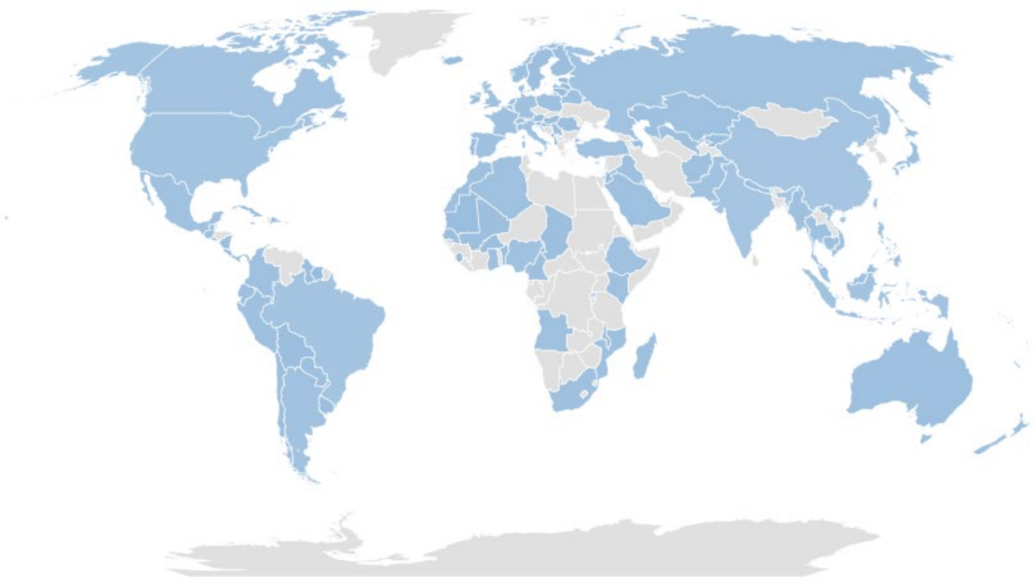
We define the hunger index as follows.

$$\text{hunger index}=\frac{x_i}{x_{i\min}} \tag{4.1}$$

In order to visually show the improvement of food equity, we display the hunger index x_i / $x_{i\min}$ before and after the optimization on the global map. Figure 1. (a) shows the hunger indicator distribution before optimization. Due to food insecurity and unequal food allocation, there is a big difference in hunger indicator for different regions. Figure 1. (b) shows that after optimization, the hunger indicator of all regions tend to be close, but there are still a few regions with unbalanced food allocation. We admit that change is difficult, but under the optimization of the model, the global food system is adjusted and homogenized. Rather than profitability, equity and sustainability become the new goals.



(a)



(b)

Figure 1. Hunger indicator allocation before and after the optimization:

(a) Before optimization. (b) After optimization.

2) Profitability

When a food system is optimized for equity and sustainability, countries no longer aim for profitability in food allocation. As a result, the allocation of food tends to be equitable and mutually beneficial. Define the profitability index:

$$\text{profitability index}=\frac{x_i - x_{i\,\min}}{x_{i\,\min}} \tag{4.2}$$

Radar map of global national profitability based on profitability index is showed in Figure 2.

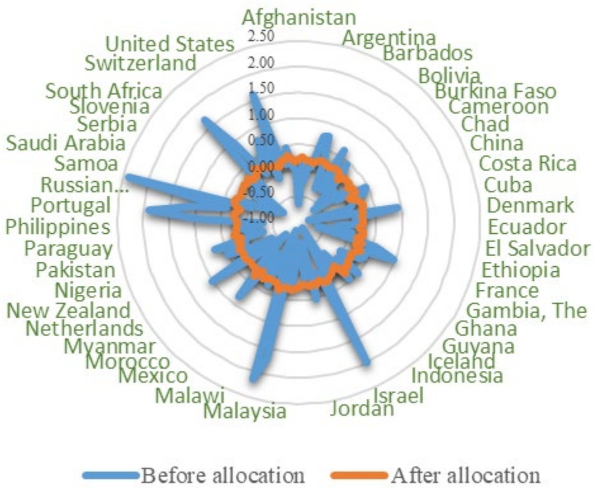


Figure 2. Global national profitability

4.1.2 Verification of Food Allocation Model

Referring to the incidence of malnutrition in 2018[14], 110 countries were divided into two groups: 45 food secure countries with an incidence of malnutrition < 3% (set ***P***), and 65 food secure countries with an incidence of malnutrition > 3% (set ***Q***). We substitute set ***P*** and set ***Q*** into the model to evaluate the flow direction and reasonable allocation of food resources, and test the rationality of the model with the index range of food insecurity rate and the food flow rate.

According to the Food Allocation Model, we get the food allocation values of 110 countries. In order to verify its rationality, this paper introduces the global food insecurity rate (*fir*) (68 countries) published by FAO as the standard value to test. At the same time, we define the food flow rate *(ffr)*[15] as:

$$ffr = \frac{x_i' - x_i}{x_i} \tag{4.3}$$

The bigger *ffr* is, the more relative inflow will be, with the higher *fir*. Vice versa. We standardize the data and draw the comparison image as shown in Figure 3.

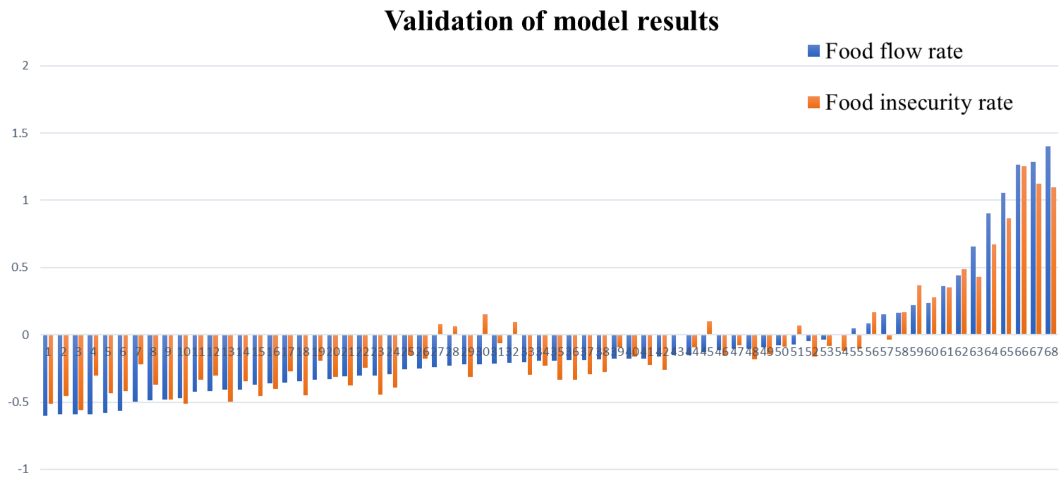


Figure 3. Validation of the model results

The correlation between the two groups is 93.6%, which verifies the rationality of the model.

4.1.3 Results Analysis

1) Optimized results

From the results of our model, the most intuitive results after optimization is food allocation around the world become uniform, all parts of the population are able to get enough food. According to the allocation of the model, the productivity of the population with severe food insecurity can be liberated to a certain extent, so that the total amount of food all over the world can be further increased. How to implement such a distribution is the focus of our discussion. We believe that the food consumption is determined by the local production, import and export. In order to achieve a more optimal result, there are two aspects to work, which can directly lead to the difference between the current system and the optimized system.

Firstly, countries can increase their food production, especially for countries and regions where food is in short supply. Food insecure problems are largely caused by food scarcity. If the production of each country is high enough, the problem of food insecurity can be alleviated partly even the allocation is not equitable enough. What needs noting is that that there has been sufficient food produced to feed all people all over the world, and rich countries hence do not have enough motivation to produce more food, while it is difficult for poor countries to increase food production in terms of funding and efficiency.

Secondly, food imports and exports of various countries need changing and regulating. For countries of $x_i > x_{i\max}$, which means food is secure enough, the difference between optimization and current is that some exports need to be increased, and imports can be appropriately reduced, so that more food can flow to countries that need them more. Conversely, countries of $x_i < x_{i\min}$ needs more imports and are appropriate for reducing exports to ensure that citizens can get the depending food.

2) Benefits and costs of changing the priorities

After we give high priority to equity and sustainability, which make $U(x)$ to its maximum, there are some benefits and also some costs.

Benefits are as follows.

- There are much fewer people who suffer from hunger. It is the biggest target of our model to overcome the problem of food scarcity.
- Environmental resources are reasonably allocated and are conducive to sustainable development. Each country can plant different food crops according to local conditions, and make sure that the farmland is not over-expanded, which can alleviate food scarcity while reasonably protecting the environment.
- Population resources are reasonably allocated. After optimization, the productivity of the countries that are in short supply of food will be liberated. They can engage in economic activities to improve the country's economic.

Costs are as follows.

- The main target of food production will no longer get profit, but to ensure people's normal life, which may lead to lack of enthusiasm for production, and thus will put a certain pressure on total production.
- It is not easy to change the import and export of various countries to the optimized results. Food-secure countries may not be willing to export food to food-insecure countries. In order to achieve the goal, it not only requires these countries to discard prejudices and work together to reach an agreement, but also requires international organizations such as the United Nations to play a role.
- For countries which rely on imports but are not economically strong, maintaining the food security of the population may require large external debts and expenditures, and even trigger economic and political instability, which make the economic situation deteriorate further.
- For countries dominated by agricultural economy, the efficiency and profitability may not be as good as before owing to the risk exporting food to the countries that are less able to afford it.

Meanwhile, we pay attention to the huge differences between developed countries and developing countries. This is because the level of food insecurity in developed countries is generally lower than in developing countries.

For developed countries, costs may be higher than benefits. This is because the profitability is more likely to decrease and they tend to absorb much more risk. But for some of the developing countries, benefits seem more than costs. The optimized result is absolutely better for food-insecure countries. The costs of developed countries and developing countries are also different. While profitability becomes to the costs of developed countries, it is the problem for some developing countries to protect their own economic system and have a higher debt burden.

4.2 Dynamic Evolution Model

According to the data of set ***P*** and set ***Q***, the calculation results of each parameter are as follows.

Table 2. The results of each parameter

$x_1(0)$/billion	$x_2(0)$/billion	$x_{1\min}$/billion	$x_{2\min}$/billion	x_1'/billion	x_2'/billion
2403.901	1410.915	1691.881	1543.980	1933.048	1761.609
$x_{1\max}$/billion	**$x_{2\max}$/billion**	**R_1**	**R_2**	**S_1**	**S_2**
2403.901	2403.901	0.018416703	0.022661384	0.369	0.637

4.2.1 Dynamic Evolution Curve

Substituting parameters into "Level 5 Runge-Kutta Formula of Order 4", we can get the dynamic evolution curve[16].

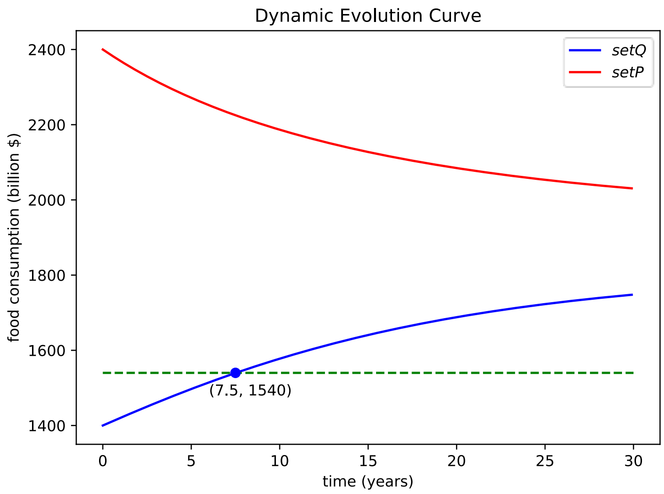


Figure 4. Dynamic evolution curve

4.2.2 Direction Field and Phase Trajectory Cluster

For ***P*** and ***Q***, when $\frac{dx_1}{dt}=0$ and $\frac{dx_2}{dt}=0$ are satisfied, we can get four equilibrium points.

$$\begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases}, \quad \begin{cases} x_1 = 0 \\ x_2 = N_2 \end{cases}, \quad \begin{cases} x_1 = N_1 \\ x_2 = 0 \end{cases}, \quad \begin{cases} x_1 = \dfrac{N_1\left(S_2 - 1\right)}{S_1 S_2 - 1} \\ x_2 = \dfrac{N_2\left(S_1 - 1\right)}{S_1 S_2 - 1} \end{cases} \tag{4.4}$$

Taking the last group, the coordinate of the corresponding phase trajectory[17] is $\left(x_{1stable}, x_{2stable}\right) = \left(1965, 1814\right)$.

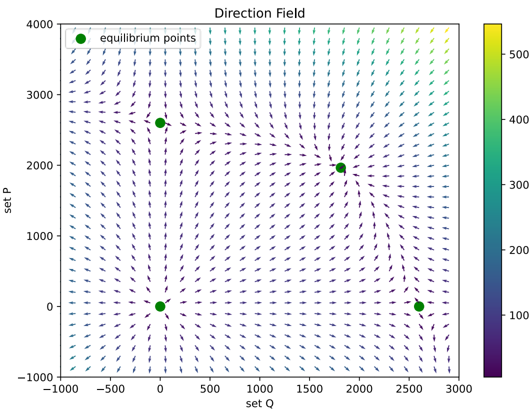


Figure 5. Direction field

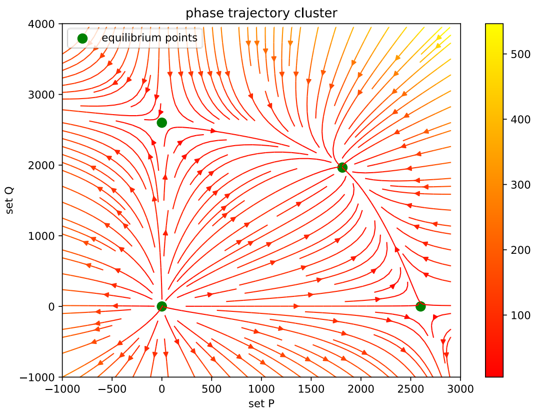


Figure 6. Phase trajectory cluster

4.2.3 Results Analysis

Analyzing the Figure 4 to 6, the food consumption of set $\boldsymbol{Q}$ countries can reach the critical level after 6.5 years ($x_{2\min} = 1540$), and it tends to be stable ($x_{2stable} = 1814$) after 30 years. The food consumption level of set $\boldsymbol{P}$ is always higher than that of set $\boldsymbol{Q}$ and tends to be stable ($x_{1stable} = 1965$) after 30 years.

The $x_{1stable} = 1965$ and $x_{2stable} = 1814$ predicted by the time model are similar to the final food allocation $x_1' = 1933$ and $x_2' = 1761$, which proves the rationality of the Dynamic Evolution Model and its adaptability.

V. Application of Our Model

5.1 A Developed Country Example - Spain

Spain is a typical developed country. Its industrial accounts for a relatively high proportion of its GDP, and its agricultural planting area is high in Europe. The results of indexes we calculate in the Food Allocation Model are shown in Table 3.

Table 3. Indexes of Spain

x_0	x_{min}	x_{max}	x'
565.69	211.22	313.16	253.72

If the production remains unchanged, Spain's food allocation will be reduced from 565.69 to 253.72 after allocation, which means that Spain will export more grain to food insecure countries, and it would sacrifice part of Spain's profitability to achieve this target. Therefore, accomplishing this goal requires both the support of the Spanish government, as well as the cooperation between countries, regions and international organizations, such as formulating corresponding policies to subsidize lost profitability.

5.2 A Developing Country Example - Iraq

Iraq is a developing country that has experienced wars in the 21st century, and it is also short of water. Agricultural land relies heavily on surface water and cannot be self-sufficient in food[18]. The results of indexes we calculate are shown in Table 4.

Table 4. Indexes of Iraq

x_0	x_{min}	x_{max}	x'
21.91	173.47	257.51	209.57

It is not difficult to find that the current level is much lower than x_{min}. To reach the allocated level, it is necessary to increase production and imports. To increase food production, more productivity, labor efficiency and higher planting technology are needed. To increase imports, the government may have to bear greater debt pressure, so it also requires cooperation between international organizations and countries, such as formulating corresponding tariff policies.

5.3 Adaptability and Scalability - BRICS

5.3.1 Application of Our Model in BRICS

It is important to apply the model to smaller food systems in order to analyze policy-based economic organizations and regional alliances.

With 42.58% of the world's population, BRICS accounts for about a quarter of the world's economy and contribute 50% to world economic growth[19]. The BRICS is made up of Brazil, Russia, India, China and South Africa. The five countries are closely linked in economy, which compose an independent system and can be regarded as a small food system.

We apply the model to the BRICS to analyze the equity and sustainability of the food system, and to verify the scalability and adaptability of the model.

5.3.2 Adaptability and Scalability of Our Model

1) Adaptability analysis

The adaptability of the model to food allocation in the BRICS is analyzed. We display the hunger indicator for BRICS before and after the optimization on the radar map. After optimization, the hunger index of BRICS countries tends to be equal, among which China and Russia will contribute to the food system to promote the equity of the food system.

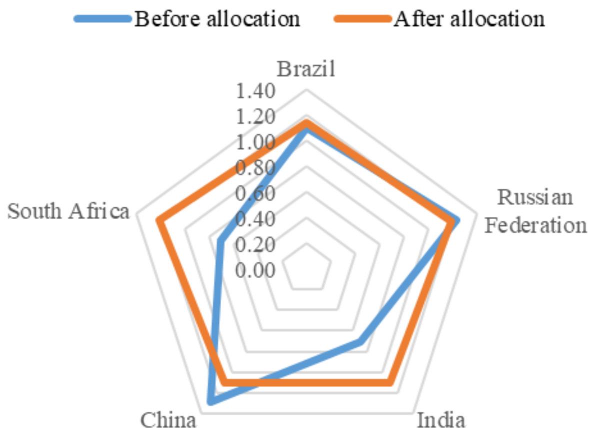


Figure 7. The hunger index for the BRICS

As shown in Figure 7, the allocation results of BRICS countries under the application of the global model and the BRICS model are close to each other, which verifies the adaptability of this model

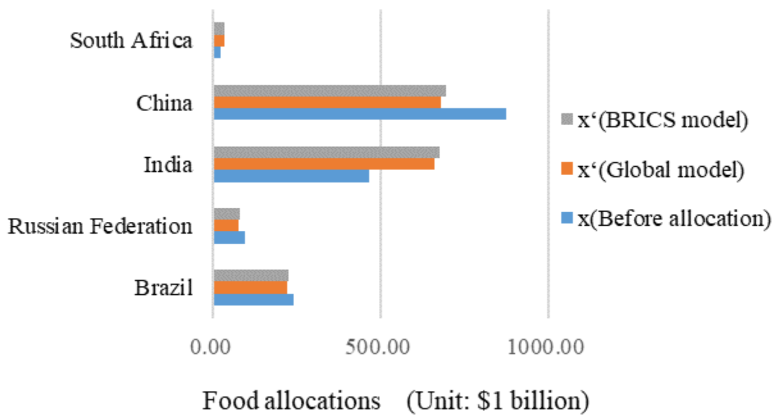


Figure 8. Comparison of food allocation in the BRICS

2) Scalability analysis

Smaller food systems often have characteristics that differ from those of the global system, for the different actual conditions. Indicators such as profitability and evolution time will be taken to discuss the scalability of the model.

a. Evolution time

According to the food allocation above, we divide Brazil, Russian Federation and China into set $\boldsymbol{P}$, and India and South Africa into set $\boldsymbol{Q}$. The results are as follows.

Table 5. The results of each parameter

$x_1(0)$/billion	$x_2(0)$/billion	x_{1min}/billion	x_{2min}/billion	x_1'/billion	x_2'/billion
1209.428	488.680	888.363	636.583	992.669	705.439
x_{1max}/billion	**x_{2max}/billion**	**R_1**	**R_2**	**S_1**	**S_2**
1317.095	943.807	0.022405	0.024545	0.246	0.619

Substituting parameters into "Level 5 Runge-Kutta Formula of Order 4", we can get the Dynamic Evolution Curve of BRICS.

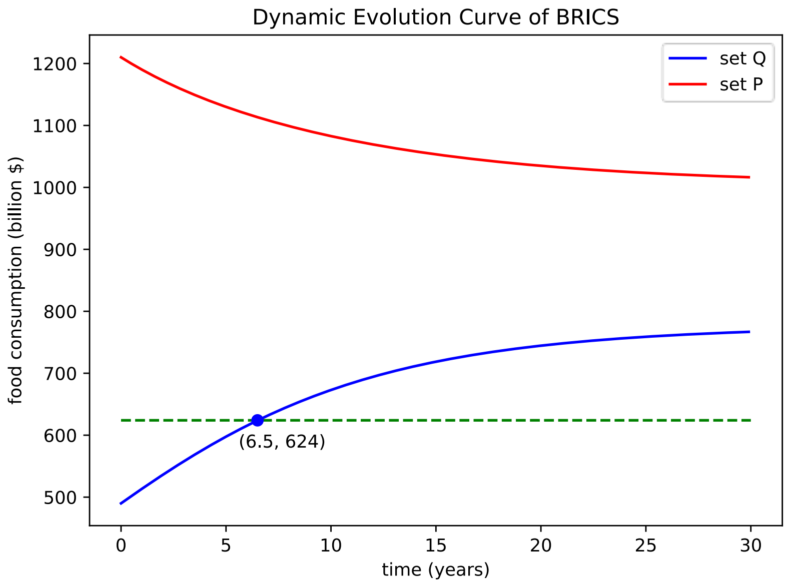


Figure 9. Dynamic Evolution Curve of BRICS

The curves of set $\boldsymbol{P}$ and set $\boldsymbol{Q}$ are finally stable at $x_{1stable}$ = 1006 and $x_{2stable}$ = 778.

Analyzing Figure 9, the food consumption of set $\boldsymbol{Q}$ can reach the pass level after 6.5 years ($x_{2\min} = 624$) and tends to be stable ($x_{2stable}$ = 778) after 30 years. The food consumption level of set $\boldsymbol{P}$ tends to be stable ($x_{1stable}$ = 1006) after 30 years. The $x_{1stable}$ = 1006 and $x_{2stable}$ = 778 predicted by the time model are similar to the food allocation $x_1' = 993$ and $x_2' = 705$.

As a smaller food system, the BRICS can reach the critical level of food equity more quickly due to the little strength gap of themselves. It only takes 6.5 years for the BRICS to reach the critical level, which is less than 7.5 years for the world.

b. Profitability

The BRICS is strong in economic strength, good food safety and environmental level. Therefore, the BRICS will play a more important role as food contributors in the global model, and the profitability of the global model is lower than that of the BRICS model, as shown in Figure 10.

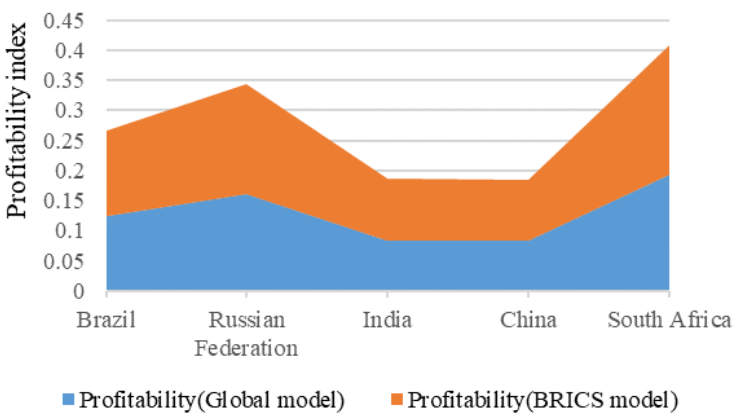


Figure 10. Comparison of profitability in the BRICS

VI. Sensitivity Analysis

In our Dynamic Evolution Model, the food growth rate R_1 and R_2 will directly lead to different result of the model. Considering that the grain growth rate will vary in actual situation and not be fixed, we verify the reliability of the model by changing the value of R_1 and R_2 in this section. The couple values of R_1 and R_2 are as Table 6 and we draw new dynamic evolution curves for these values, which are shown as Figure 11.

Table 6. New values of R_1 and R_2

R_1	R_2
0.16 (original)	0.12 (original)
0.2	0.16
0.12	0.08

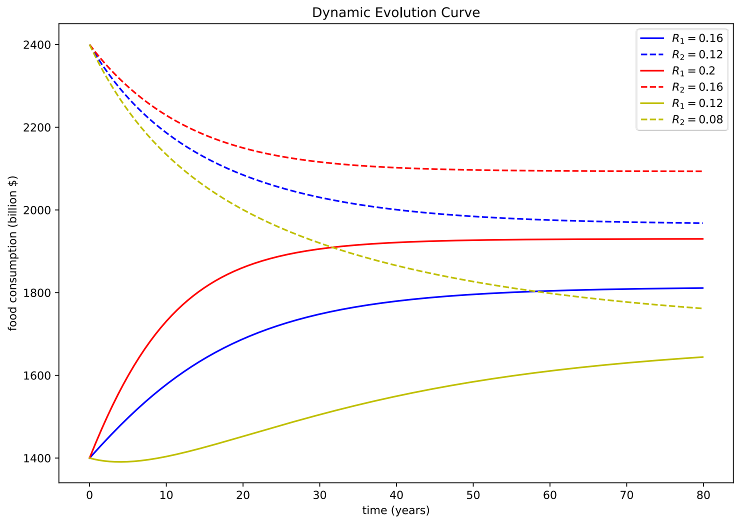


Figure 11. Sensitivity Analysis of R_1 and R_2

The red lines show the situation when indexes are above the original values, and time for $\boldsymbol{Q}$ will reach its $x_{\min}$ faster than before. Similarly, the yellow lines show the lower situation, which will take more time for $\boldsymbol{Q}$ to reach its $x_{\min}$. The common characteristic of these three cases is that they all eventually reach their own equilibrium points and the trends of these curves are the same. Therefore, it is reasonable to draw the conclusion that the model is not sensitive to the food growth rate.

VII. Model Improvement

We know that the current global food system is unstable and vulnerable to extreme events. Therefore, in order to build a complete model, the impact of extreme events should be considered. In this paper, we only consider the extreme events that have an effect on the food system, which can be divided into "Human activities", "Extreme weather" and "Pest invasion".

"Human activities" include events that affect food trade such as global epidemics and political conflicts. Since our Food Allocation Model has already considered import and export trade, such events will not be considered below.

"Extreme weather" includes fires, droughts, floods, etc. Such events are mainly manifested as abnormal temperature and precipitation[20].

"Pest invasion" is mainly manifested as locust plague invasion, and its outbreak is often related to population density[21].

7.1 Extreme Weather Index

The extreme weather disasters mainly embody in high temperature and rainstorm. Therefore, the probability of extreme weather ($P_{weather}$) can be calculated by filtering out the abnormal data of temperature and precipitation.

Assume that all temperature and precipitation data in a year follow normal distribution. Taking temperature as an example, we can get the probability density function of each temperature in a year $f(T)$.

$$f(T)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(T-\mu)^2}{2\sigma^2}} \tag{7.1}$$

where, μ is the annual average temperature and σ is the standard deviation of the temperature.

Then we define hot weather when the temperature $T>\mu+2\sigma$, and extreme hot weather when the temperature $T>\mu+3\sigma$.

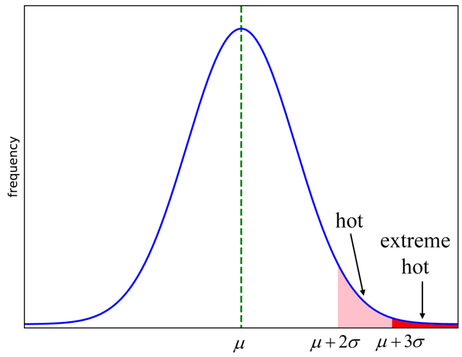


Figure 12. Temperature probability density distribution

Then the extreme hot weather index is $P(T)=\int_{\mu+3\sigma}^{\infty} f(T)dt$. Similarly, the extreme rainstorm weather index is $P(s)=\int_{\mu+3\sigma}^{\infty} f(s)ds$. Thus, the extreme weather index is $P_{weather}=\int_{\mu+3\sigma}^{\infty} f(T)dT+\int_{\mu+3\sigma}^{\infty} f(s)ds$.

7.2 Pest Invasion Index

The research of J. Buhl et al. showed that the occurrence of locust plague is related to population density. At medium density (25 to 62 locusts/m^2), locusts will move in groups. And when the density reaches ρ_0=72 locusts/m^2, the locusts begin to move in the same direction for several hours. J. Buhl et al. believe that at this defined density, locust swarms are likely to cross the "tipping point" and trigger a locust plague.

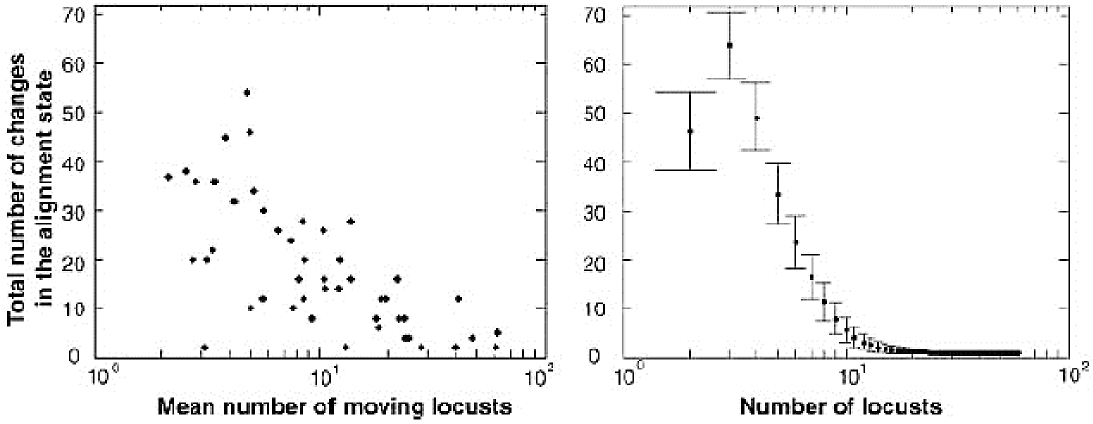


Figure 13. when $\rho > 72$, the locust swarms begin to move in an orderly manner[21].

Assuming that the locust density in a year follows a normal distribution, then the probability density function of locust density is $f(\rho)$.

$$f(\rho)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(\rho-\mu)^2}{2\sigma^2}\right) \tag{7.2}$$

Then the pest invasion index is $P_{pest}=\int_{\rho_0}^{\infty} f(p)\mathrm{d}\rho$.

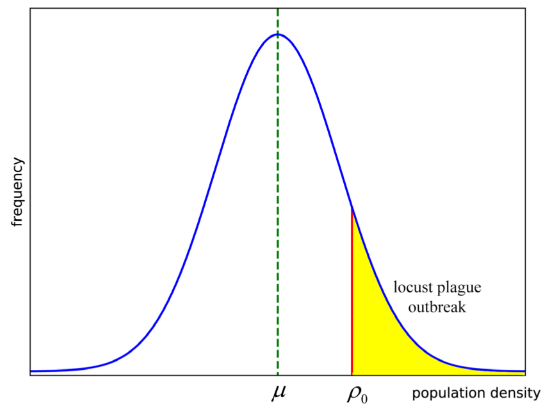


Figure 14. Population probability density distribution

7.3 Application of Extreme Event Indexes

When the extreme event indexes are considered, the parameters meet the following formulates.

$$\begin{cases} P_{event} = P_{pest} + P_{weather} \\ m_i' = P_{event} \cdot m_i \\ W' = P_{event} \cdot W \end{cases} \tag{7.3}$$

$$x_{i\max} = \begin{cases} m_i' x_{i\min}, & m_i' \geq 1 \\ x_{i\min}, & m_i' < 1 \end{cases} \tag{7.4}$$

The introduction of the extreme event indexes will limit the maximum food allocation value of each country, thus affecting the total amount of food allocation in the world and exacerbating the problem of food equity.

VIII. Conclusion

8.1 Our Contributions

The main contributions of our solution are as follows.

a. An equitable and sustainable solution for food allocation is proposed.

Guiding by the Marginal Utility Theory, we quantify food equity and establish the Food Allocation Model, which is able to optimize for profitability, equity and sustainability. The optimized result is visualized on the map for comparation.

b. A method to predict the food system evolution time is proposed.

Referring to the Lotka-Volterra Model, the Dynamic Evolution Model is established. It is predicted that the global food system is expected to reach the critical level in 7.5 years and reach stable level in 30 years. The evolution process is visualized in the direction field and phase trajectory cluster.

c. A feasible optimization scheme for a national food system and even global food system is provided.

The model is applied in food system of the world and the BRICS. We take Spain and Iraq as typical examples to support our research. And the benefits as well as the costs of the optimized system are also discussed. In addition, we provide several recommendations for global and national food system, hoping to offer guidance for the food system improvement.

d. The accuracy, scalability and adaptability of the model are verified.

The Food Allocation Model is applicable to food systems of different scale and regions. The accuracy of the model is verified by the correlation between the of insecurity rate and the food flow rate.

8.2 Strength

a. Strong theoretical basis

Both Food Allocation Model and Dynamic Evolution Model are constructed based on influential theories.

b. Good adaptability and scalability

The model base on a large amount of data which involve various indicators and countries with different food levels.

c. Stable and robust

By doing sensitivity analysis and introducing extreme event indexes to verify and improve the model, our model is able to cope with the current complex food system.

d. Relatively objective

Objective metrics and formulas are adopted in modeling. Subjective evaluation methods such as Analytic Hierarchy Process are strictly avoided.

8.3 Shortcoming

a. Lack of policy considerations

In international relations, it is difficult to take human factors such as national policies into account.

b. Static models

Dynamic factors such as population growth over time are not included in the model. The accuracy of the model can be improved through prediction and correction.

c. The complexity of food resource flows

In reality, the flow of food resources may be much more complex than the dynamic evolution model.

d. Data out of time and missing

Data for 2020 have not yet been updated. As a result, the latest data we use is for 2019 which is less representative than that for 2020. We only considered 110 countries due to the lack of data.

IX. References

[1] Unicef, and World Health Organization. "The state of food security and nutrition in the world 2017: Building resilience for peace and food security." (2017).

[2] FAO. "The State of Agricultural Commodity Markets 2020. Agricultural markets and sustainable development: Global value chains, smallholder farmers and digital innovations". Rome, FAO (2020).

[3] Xiande Li et al. "The impact of the new crown pneumonia epidemic on the global agricultural product market and trade and countermeasures." *Agricultural Economic Issues* 8(2020): 4-11.

[4] Victora, Cesar G., et al. "Maternal and child undernutrition: consequences for adult health and human capital." *The lancet* 371.9609 (2008): 340-357.

[5] Fei Lu. "Research on the Impact of Climate Change on China's Agricultural Export Trade". Diss. Jiangxi University of Finance and Economics, 2020.

[6] Chengxiong Xue. "my country's annual grain output is reduced by more than 50 billion kilograms due to disasters." *Liaoning Economic Statistics* 10 (2016): 34-34.

[7] "World Hunger, Poverty Facts, Statistics 2018. " World Hunger News, 29 Nov. 2018, https://www.worldhunger.org/world-hunger-and-poverty-facts-and-statistics/.

[8] Liu, Ya-qiu, et al. "Comprehensive evaluation of rural courtyard utilization efficiency: A case study in Shandong Province, Eastern China." *Journal of Mountain Science* 17.9 (2020): 2280-2295.
[9] Song, Ya-Nan, Qian Zhong, and Bin Liu. "Marginal utility function based networking resource scheduling." *Dianzi Xuebao(Acta Electronica Sinica)* 41.4 (2013): 632-638.
[10] Hu, Jian, and Sanjay Mehrotra. "Robust decision making over a set of random targets or risk-averse utilities with an application to portfolio optimization." *IIE Transactions* 47.4 (2015): 358-372.
[11] Wenting Qi. "Prediction of China's Grain Production and Demand and Net Import." Jiangxi University of Finance and Economics, MA Thesis (2020).
[12] Li Wang, et al." Study on green development evaluation system of grain production in Hunan Province." *Shanxi Agricultural Economy*.11(2020): 9-1162.
[13] Zhu, Chao, and G. Yin. "On competitive Lotka–Volterra model in random environments." *Journal of Mathematical Analysis and Applications* 357.1 (2009): 154-170.
[14] Food and Agriculture Organization. "FAO statistical yearbook". FAO (2018).
[15] Nakamura, Masafumi, et al. " The research direction of the United Nations Sustainable Development Goals is being adjusted." *scientific observation* 15.1 (2020): 38-47.
[16] Dormand, John R., and Peter J. Prince. "A family of embedded Runge-Kutta formulae." *Journal of computational and applied mathematics* 6.1 (1980): 19-26.
[17] Omurtag, Ahmet, Bruce W. Knight, and Lawrence Sirovich. "Dynamics of neuronal populations: The equilibrium solution." *SIAM Journal on Applied Mathematics* 60.6 (2000): 2009-2028.
[18] "Country Profile of Iraq." Ministry of Foreign Affairs of the People's Repu blic of China, 2020, https://www.fmprc.gov.cn/web/gjhdq_676201/gj_676203/yz_676205/1206_677148/1206x0_677150/.
[19] Jim O 'Neill, "20 years of BRIC countries: the world needs real global economic governance", China Business News 2021-02-01, A11, Global Economic Review.
[20] Liu Yang. "Study on the social and economic effects of global climate change on the estuary and coastal areas of the Yangtze River Delta." East China Normal University, 2014.
[21] Buhl, Jerome, et al. "From disorder to order in marching locusts." *Science* 312.5778 (2006): 1402-1406.

X. Appendix

10.1 Lingo Code for Food Allocation Model

```
Model1:
   Sets:
      XI/1..5/:x0,x,xmax,xmin,s;
   Endsets
```

```
    W = (@sum(XI: (x0)));
    WS = (@sum(XI: (x * s)));
    WS <= 15000;
    EE = 0.000001;
    DATA:
        ! data of 110 countries
xmin = 167.78, 42.94, 190.60, 139.06, 600.83, 112.76, 39.90, 44.86,
      129.38, 142.80, 51.58, 51.84, 51.24, 1945.44, 89.15, 73.34,
      113.81, 1167.26, 69.86, 84.53, 6286.09, 224.15, 52.56, 18.45,
      51.17, 50.37, 80.15, 47.97, 77.11, 444.23, 28.98, 20.97,
      492.99, 24.89, 302.25, 10.25, 100.29, 374.20, 134.35, 173.78,
      30.52, 100.12, 5.59, 6105.04, 1208.10, 369.21, 173.47, 32.97,
      40.09, 572.71, 53.25, 171.09, 44.94, 182.49, 231.96, 58.67,
      28.54, 8.70, 30.91, 12.64, 2.74, 118.54, 81.89, 142.30,
      86.11, 19.87, 569.56, 2.81, 162.62, 133.13, 242.41, 126.77,
      77.77, 100.28, 180.85, 29.18, 500.08, 53.98, 957.83, 18.85,
      131.40, 144.38, 481.37, 171.40, 46.42, 87.89, 652.10, 55.52,
      10.89, 18.95, 152.10, 71.56, 31.52, 34.53, 69.36, 20.95,
      260.79, 211.22, 4.60, 45.93, 38.43, 313.37, 371.55, 299.97,
      1474.50, 55.57, 148.75, 10.32, 50.31, 531.22
xmax = 248.75, 63.67, 282.59, 206.17, 890.79, 167.18, 59.16, 66.52,
      191.96, 211.72, 76.47, 76.87, 75.97, 2884.33, 132.18, 108.75,
      168.75, 1730.59, 103.57, 125.33, 9319.81, 332.32, 77.95,
      27.36, 75.88, 74.74, 118.84, 71.13, 114.33, 658.66, 42.98,
      31.09, 730.94, 36.91, 448.12, 15.26, 148.87, 554.79, 199.20,
      257.67, 45.26, 148.48, 8.29, 9051.42, 1791.16, 547.39, 257.21,
      48.88, 59.45, 849.13, 79.17, 253.67, 66.63, 270.56, 343.92,
      87.07, 42.31, 12.90, 45.84, 18.75, 4.09, 175.74, 121.42,
      210.99, 127.66, 29.47, 844.44, 4.17, 241.10, 197.38,
      359.42, 187.98, 115.32, 149.01, 268.13, 43.27, 741.43,
      80.04, 1420.11, 27.95, 194.81, 214.07, 713.71, 254.12,
      68.82, 130.31, 966.81, 82.34, 16.22, 28.24, 225.51,
      106.10, 46.73, 51.20, 102.86, 31.10, 386.65, 313.16,
      6.85, 68.09, 56.98, 464.61, 550.88, 444.74, 2186.11,
      82.39, 220.54, 15.36, 74.59, 787.62
x0 = 58.82, 48.42, 154.23, 145.95, 666.04, 193.83, 70.23, 53.78,
    94.47, 111.93, 84.26, 39.66, 74.61, 2410.00, 44.02, 67.11,
    43.17, 1168.01, 59.09, 124.40, 8751.03, 250.57, 56.70,
    18.45, 10.25, 97.01, 90.94, 52.24, 90.10, 337.50, 20.75,
    21.74, 180.66, 38.72, 596.14, 12.99, 143.57, 508.25,
    132.37, 175.31, 46.05, 100.85, 8.17, 4664.96, 1030.31,
    432.54, 21.91, 99.28, 51.18, 605.68, 55.17, 260.15,
    18.65, 195.02, 129.93, 44.00, 23.16, 10.20, 10.97, 34.20,
    8.60, 58.16, 13.03, 192.87, 105.06, 14.21, 642.54, 5.16,
```

```
    165.52, 43.60, 121.57, 138.64, 156.68, 60.80, 200.83, 44.34,
    551.09, 91.63, 679.19, 16.65, 133.37, 161.74, 335.41, 325.57,
    134.55, 184.26, 933.26, 188.43, 10.38, 21.91, 72.82, 44.18,
    31.93, 12.02, 78.31, 25.67, 221.83, 565.69, 8.76, 62.33,
    43.76, 467.13, 656.05, 277.72, 3863.47, 69.66, 168.77,
    14.97, 55.85, 528.49
s = 0.38, 0.39, 0.38, 0.38, 0.38, 0.38, 0.39, 0.38, 0.46, 0.38,
   0.39, 0.40, 0.38, 0.38, 0.39, 0.39, 0.39, 0.38, 0.38, 0.38,
   0.38, 0.38, 0.40, 0.39, 0.39, 0.47, 0.39, 0.40, 0.38, 0.39,
   0.41, 0.41, 0.39, 0.39, 0.38, 0.81, 0.39, 0.39, 0.38, 0.38,
   0.47, 0.38, 0.39, 0.40, 0.39, 0.92, 0.38, 0.39, 0.39, 0.38,
   0.38, 0.38, 0.61, 0.38, 0.38, 0.39, 0.40, 0.39, 0.61, 0.38,
   0.39, 0.38, 0.40, 0.38, 0.39, 0.38, 0.38, 0.38, 0.38, 0.39,
   0.38, 0.38, 0.40, 0.90, 0.90, 0.38, 0.39, 0.38, 0.40, 0.41,
   0.52, 0.38, 0.38, 0.90, 0.39, 0.39, 0.38, 0.38, 0.38, 0.38,
   0.38, 0.38, 0.78, 0.38, 0.38
   Enddata
End
max = (@sum(XI: 1 - @exp(@log(EE) / (xmax - xmin) * (x - xmin))));
(@sum(XI: (x))) <= W;
@for(XI: x <= xmax;);
@for(XI: x >= xmin;);
```

10.2 Index of Countries In Our Model

No	Country Name	No	Country Name	No	Country Name	No	Country Name	No	Country Name
1	Afghanistan	23	Costa Rica	45	Indonesia	67	Mexico	89	Samoa
2	Albania	24	Croatia	46	Iran (Islamic Republic of)	68	Montenegro	90	Sao Tome and Principe
3	Algeria	25	Cuba	47	Iraq	69	Morocco	91	Saudi Arabia
4	Angola	26	Cyprus	48	Ireland	70	Mozambique	92	Senegal
5	Argentina	27	Denmark	49	Israel	71	Myanmar	93	Serbia
6	Australia	28	Dominican Republic	50	Italy	72	Nepal	94	Sierra Leone
7	Austria	29	Ecuador	51	Jamaica	73	Netherlands	95	Slovenia
8	Azerbaijan	30	Egypt	52	Japan	74	New Caledonia	96	Solomon Islands
9	Barbados	31	El Salvador	53	Jordan	75	New Zealand	97	South Africa
10	Belarus	32	Estonia	54	Kazakhstan	76	Nicaragua	98	Spain
11	Belgium	33	Ethiopia	55	Kenya	77	Nigeria	99	Suriname
12	Benin	34	Finland	56	Kuwait	78	Norway	100	Sweden
13	Bolivia	35	France	57	Kyrgyzstan	79	Pakistan	101	Switzerland
14	Brazil	36	French Polynesia	58	Latvia	80	Panama	102	Thailand
15	Burkina Faso	37	Gambia	59	Lebanon	81	Paraguay	103	Turkey
16	Cambodia	38	Germany	60	Lithuania	82	Peru	104	United Kingdom
17	Cameroon	39	Ghana	61	Luxembourg	83	Philippines	105	United States of America
18	Canada	40	Guatemala	62	Madagascar	84	Poland	106	Uruguay
19	Chad	41	Guyana	63	Malawi	85	Portugal	107	Uzbekistan
20	Chile	42	Hungary	64	Malaysia	86	Romania	108	Vanuatu
21	China	43	Iceland	65	Mali	87	Russian Federation	109	Venezuela (Bolivarian Republic of)
22	Colombia	44	India	66	Mauritania	88	Rwanda	110	Viet Nam