ECSIM - a simple dynamic energy cost simulator[*]

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ECSIM is a simple dynamic energy cost simulator. It uses historical data about hourly electric energy costs together with average figures about the distribution of both energy consumption and solar energy production to compute the expected energy cost for a household. If a home battery is used, possibly attached to an electric vehicle, it can compute an optimal daily battery charging strategy that takes advantage of varying hourly tariffs to minimise the costs.

1   Model

Assume a house with a local grid to which electric appliances (consuming energy), solar panels (returning energy) and an electric vehicle with a battery (that can both be charge or return energy) are attached. Let

- e_u be the total energy consumed by the electric appliances in a year,
- e_v be the total energy consumed by the electric vehicle in a year, and
- e_s be the total solar energy returned by the solar panels in a year.

All energy is measured in kWh. The local grid of the house is connected to the electrical grid through a (smart) meter. This meter measures:

- $e[d,h]$ the hourly amount of electric energy consumed (positive) or returned (negative) to the grid (where day $d \in [0,\ldots,364]$ and hour $h \in [0,\ldots,23]$).

The hourly amount of energy consumed or produced depends on the amount of energy produced by the solar panels that hour ($e_s[d,h]$), the amount of energy consumed by appliances that hour ($e_c[d,h]$) and the amount of energy flowing into or out of the battery ($b[d,h]$). The latter is called the battery charging strategy, and is discussed later. We have

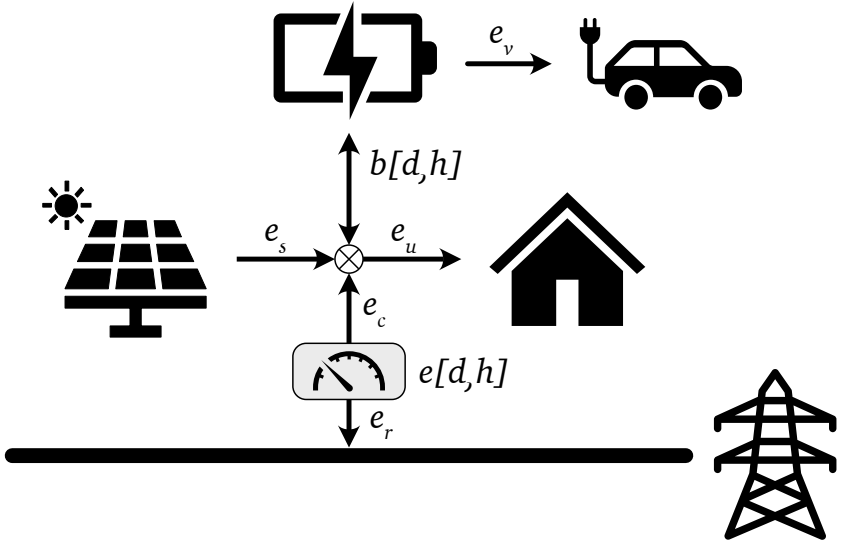$$e[d,h] = e_c[d,h] + b[d,h] - e_s[d,h].$$

Figure 1: Model

The amount of energy produced by the solar panels at a particular hour on a particular day is estimated using

  – information about the average fraction of yearly solar production produced each month, and
  – information about the average fraction of daily solar production produced each hour.

The same method is used to estimate the hourly energy consumption. Let

  – e_c be the total energy consumed from the gird in a year, and
  – e_r be the total energy returned to the grid in in a year.

We have

$$e_n = e_c - e_r = \sum_{d=0}^{354} \sum_{h=0}^{23} e[d,h].$$

Typically, some of the solar energy produced is immediately consumed by household appliances, and therefore some of the production and consumption is not measured by the meter. We have $e_c \leq e_u$ and $e_r \leq e_s$.

Assuming no losses when charging or discharging the battery, and provided the battery is empty at the start and the end of a year, we also have $e_n =$

$e_u + e_v - e_s$. This follows from the fact that whatever enters the battery needs to leave it at some point or is consumed by the electrical vehicle, and any excess solar energy not used to charge the battery needs to leave the local grid through the meter, and any additional energy necessary to charge the battery or run household appliances also needs to enter through the meter.

2   Typical electricity cost structure

The price you pay for electricity is determined by several factors. There are *fixed* costs (e.g. network operator connection costs, and energy supplier subscription fees) and *variable* costs (that depend on the amount of electricity, measured in kWh, consumed or returned). Fixed costs are ignored in the simulation.

The variable costs can be further divided into

- raw electricity price (EPEX) p (per kWh, changes every hour),
- energy supplier surcharge s (per kWh), and
- government taxes and levies t (per kWh).

Of these, only the electricity price itself is dynamic and changes every hour. The other factors are constant for a year. The tariffs for the next day are typically published at 15:00 the day before. These EPEX tariffs apparently differ for electricity consumption and energy returned (i.e. through solar panels or wind turbines). But most dynamic energy suppliers quote a *single* hourly tariff for both electrical energy consumed and returned. This is what the simulator also does (for technical reasons). We write $p[d, h]$ for the price for a kWh worth of energy on day $d \in [0, \ldots, 364]$ at hour $h \in [0, \ldots, 23]$.

Government taxes and levies *only* apply to the *net* energy consumption over the (annual) billing period. If you consumed e_c energy and returned e_r energy in a year, the total taxes and levies due are determined by $e_n = e_c - e_r$: if $e_n \leq 0$ *no* taxes or levies are due. In other words, if we define $Z(x)$ as

$$Z(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

then the total taxes due equal $Z(e_n) \times t$.

Energy supplier surcharges contribute to the energy cost in different ways. Some energy suppliers charge the surcharge *both* for energy consumed and

for energy returned. Others only charge the surcharge for the excess of energy consumed (i.e. like the government taxes).

The total cost for the electrical energy consumed for a full year is computed as follows

- For energy suppliers that charge the surcharge for both energy consumption and return, the total cost for electrical energy becomes:

$$Z(e_n) \times t + \sum_{d=0}^{354} \sum_{h=0}^{23} e[d,h] \times p[d,h] + |e[d,h]| \times s$$

  (where $|x|$ denotes the absolute value of x).
- For energy suppliers that charge the surcharge only for the excess energy consumed, the total cost for electrical energy becomes:

$$Z(e_n) \times (s+t) + \sum_{d=0}^{354} \sum_{h=0}^{23} e[d,h] \times p[d,h]$$

In the simulator, two separate surcharges s_n and s_x are defined, and following formula for the total cost is used:

$$Z(e_n) \times (s_x + t) + \sum_{d=0}^{354} \sum_{h=0}^{23} e[d,h] \times p[d,h] + |e[d,h]| \times s_n$$

Observe that the first case above corresponds to $s_n = s$ and $s_x = 0$ while the second case above corresponds to $s_n = 0$ and $s_x = s$.

Also observe that the total cost depends on the hourly energy consumption $e[d,h]$ which in turn depends on the hourly battery charge $b[d,h]$ (which does not influence the first term $Z(e_n) \times (s_x + t)$). The optimal cost is thus determined by the battery charge strategy.

3 Finding an optimal battery charge strategy

The battery in the electric vehicle is primarily used to power the vehicle of course. It can also be used to take advantage of the fluctuating dynamic prices by charging it when electricity is cheap and discharging it when electricity is more expensive. It can also be used to temporarily store solar energy to return it when local energy demand is high or electricity prices are high. To

accomplish this, a *battery charging strategy* $b[d,h]$ must be computed that instructs the battery when to charge or discharge.

The simulator computes this charging strategy, using the following (generic) method. It assumes it is given some advance knowledge of the hourly tariffs, future energy consumption and solar energy production. In practice, when energy tariffs for tomorrow are published at 15:00 today, the look-ahead interval is $33 = 24 + 9$ hours. So assume the following input (where H is the last hour in the look-ahead interval):

- $e_c[h]$, the hourly energy consumption, for $h \in [0, \ldots, H]$,
- $e_s[h]$, the hourly solar energy production, for $h \in [0, \ldots, H]$,
- $e_v[h]$, the hourly electric vehicle energy consumption, for $h \in [0, \ldots, H]$,
- $p[h]$ the raw hourly energy tariff, for $h \in [0, \ldots, H]$.

The output is a battery charging schedule $b[h]$ for these hours.

Let $e[h] = e_c[h] + b[h] - e_s[h]$. The goal is to minimise

$$G(b[]) = \sum_{h=0}^{H} e[h] \times p[h] + |e[h]| \times s_n$$

under certain constraints. (Observe that any energy consumed by the electric vehicle is hidden in the battery charge.) Observe that this is equivalent to minimising

$$G'(b[h]) = \sum_{h=0}^{H} b[h] \times p[h] + |b[h]| \times s_n.$$

Define

$$B[h] = b_0 + \sum_{i=0}^{h-1} b[i] - e_v[h]$$

as the charge of the battery at hour h (where b_0 is the initial battery charge when the simulation is started). Recall that the electrical vehicle consumes its energy through the battery. The constraints are the following.

- $-C \leq b[h] \leq C$, for all $h \in [0, \ldots, H]$: the battery cannot be charged or discharged more than allowed by the electrical installation.

- $0 \leq B[h] \leq B$, for all $h \in [0, \ldots, H+1]$: the battery cannot create power out of nothing and cannot be charged beyond its capacity. Note this also guarantees that the battery is always sufficiently charged to drive the vehicle as specified by $e_v[h]$.[1]
- $b[h] = 0$ if $e_v[h] > 0$, $h \in [0, \ldots, H]$: when using the car, the battery cannot be charged.

where

- C is the maximal hourly charge/discharge for the battery (11 kWh for typical electrical installations in the Netherlands).
- B is the maximum battery capacity.

Because the simulator needs to optimise for a formula containing the absolute value of the variable $b[h]$ we use the following trick: introduce additional ghost variables $b'[h]$ that serve as this absolute value in the (modified) optimisation formula

$$G''(b[], b'[]) = \sum_{h=0}^{H} b[h] \times p[h] + b'[h] \times s_n$$

with the following additional constraints:

- $b[h] \leq b'[h]$, for all $h \in [0, \ldots, H]$, and
- $-b[h] \leq b'[h]$, for all $h \in [0, \ldots, H]$.

Then if $0 \leq b[h]$, then $0 \leq b[h] \leq b'[h]$ by the first rule, while if $b[h] < 0$ then $0 < -b[h] \leq b'[h]$ by the second rule. As G'' decreases as $b'[h]$ decreases, the optimal choice for $b'[h]$ is either $b[h]$ in the first case or $-b[h]$ in the second case. In other words: $b'[h] = |b[h]|$.

4 Simulating the annual cost

For each month m in the year, and each day d in the month, the simulator computes the amount of solar energy produced and the amount of household energy consumed by

- first spreading the annual production and consumption over the months according to the (average) fraction of energy produced and

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[1] Specifying unreasonable vehcile usage constraints (eg demanding the car can immediately be used when the simulation starts) may prevent the simulator from finding a solution.

consumed per month (stored in the tables `SOLAR_PER_MONTH` and `CONSUMPTION_PER_MONTH`, and

- then divide this by the number of days in this month.

The vehicle energy consumption is evenly divided over all days of the year. This gives e_u, e_v, and e_s for the day to be simulated.

The simulator is given some time to look ahead in the future: at hour η it is given access to all tariffs p until midnight next day. The look ahead (measured in hours) then equals $48 - \eta$ hours (24 when $\eta = 0$).

The simulator then uses e_u, e_v, and e_s to compute the vectors $e_c[h]$, $e_s[h]$, $e_v[h]$, and $p[h]$, for $h \in [0, \dots, H]$ (where H is the look ahead). Here h is the hour of the date relative to η (and for simplicity the simulator assumes that today's and tomorrow's daily energy consumption and production values are the same). It uses information about the fraction of energy produced and consumed per hour (stored in the tables `SOLAR_PER_HOUR` and `CONSUMPTION_PER_HOUR`) for this.

It then calls the generic method to compute an optimal battery charge strategy for the *full* look ahead period, but then uses the returned battery charge strategy $b[]$ to compute the actual costs only for the next 24 hours (starting at η) as the cost for today. It also computes the remaining battery charge after these 24 hours (and uses this as the initial charge in the simulation for the next day),