07-XA

BST Application

Range Tree

邓俊辉
deng@tsinghua.edu.cn

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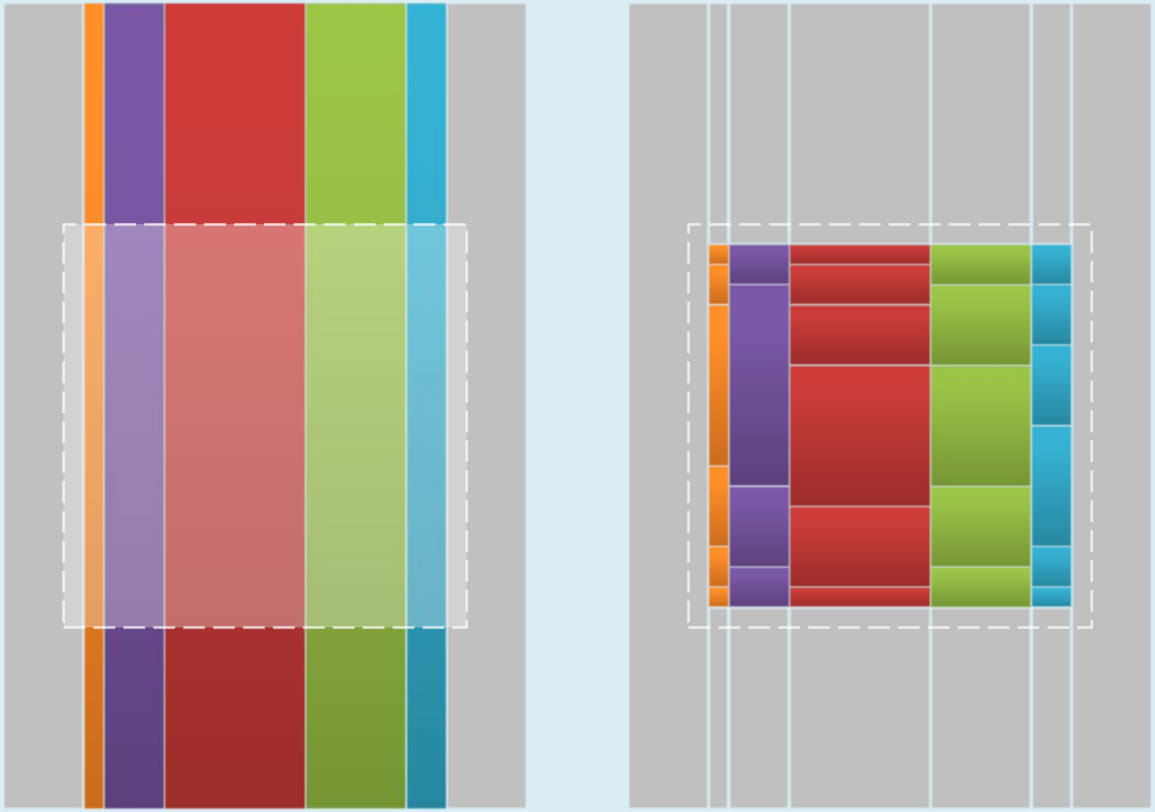
Coherence

❖ For each query, we

- need to repeatedly search DIFFERENT y-lists,

- but always with the SAME key



BBST<BBST<T>> --> BBST<List<T>>

- Each y-search is just

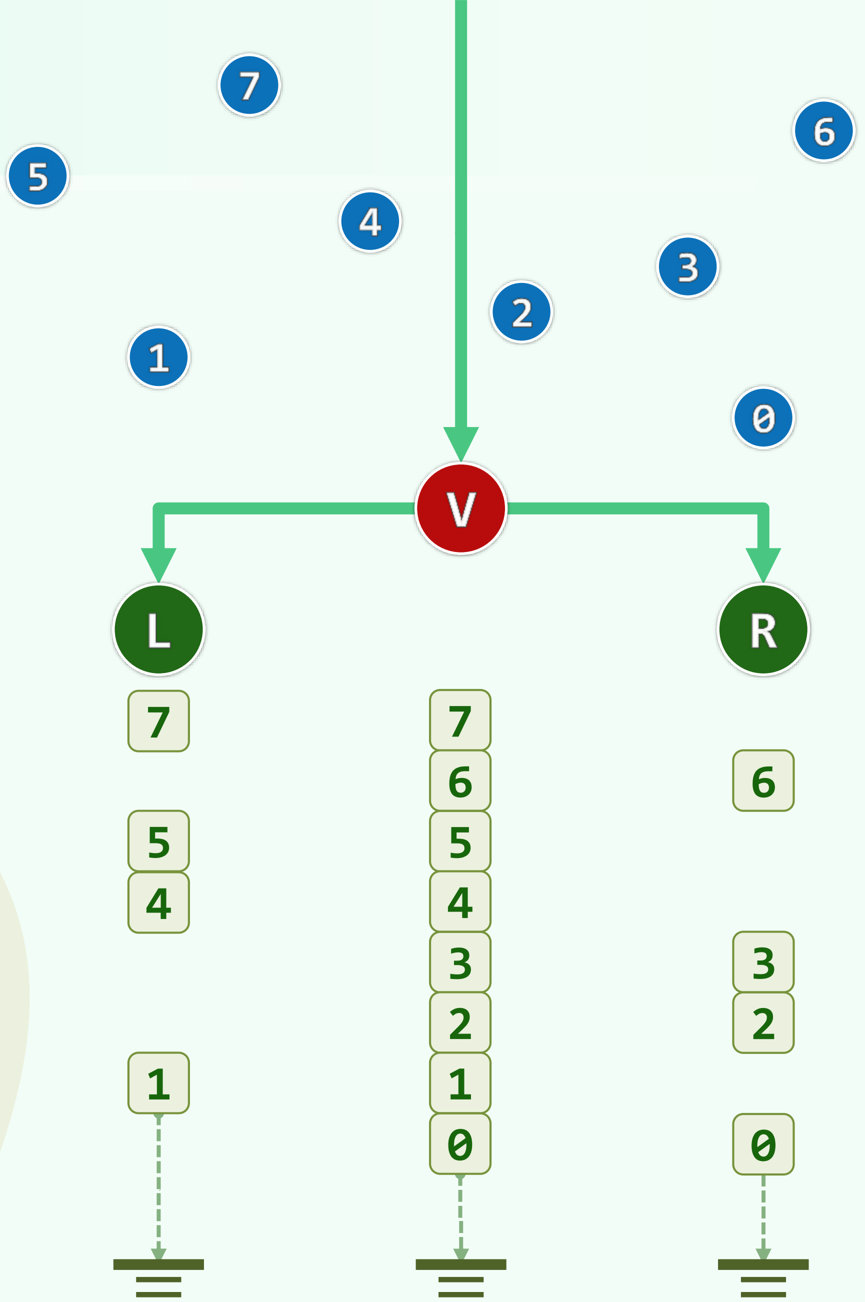
  a 1D query without further recursions

- So it not necessary

  to store each canonical subset

  as a BBST

- Instead, a sorted y-list simply works



Links Between Lists

❖ We can CONNECT all the different lists

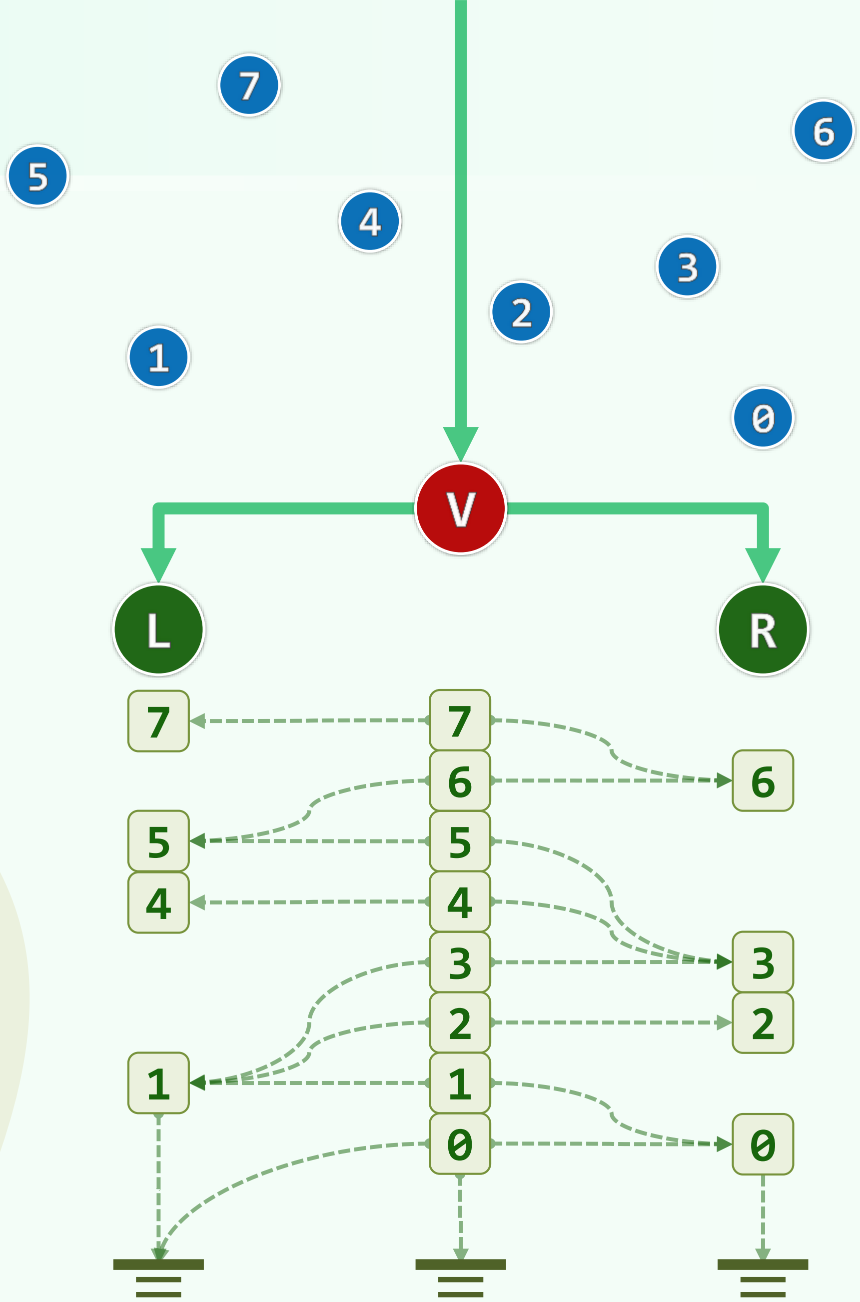
into a SINGLE massive list

❖ Thus, once a parent y-list is searched,

we can get, in $\mathcal{O}(1)$ time,

the entry for child y-list by

following the link between them



Using Coherence

❖ To answer a 2D range query, we will

do an $\mathcal{O}(\log n)$ search

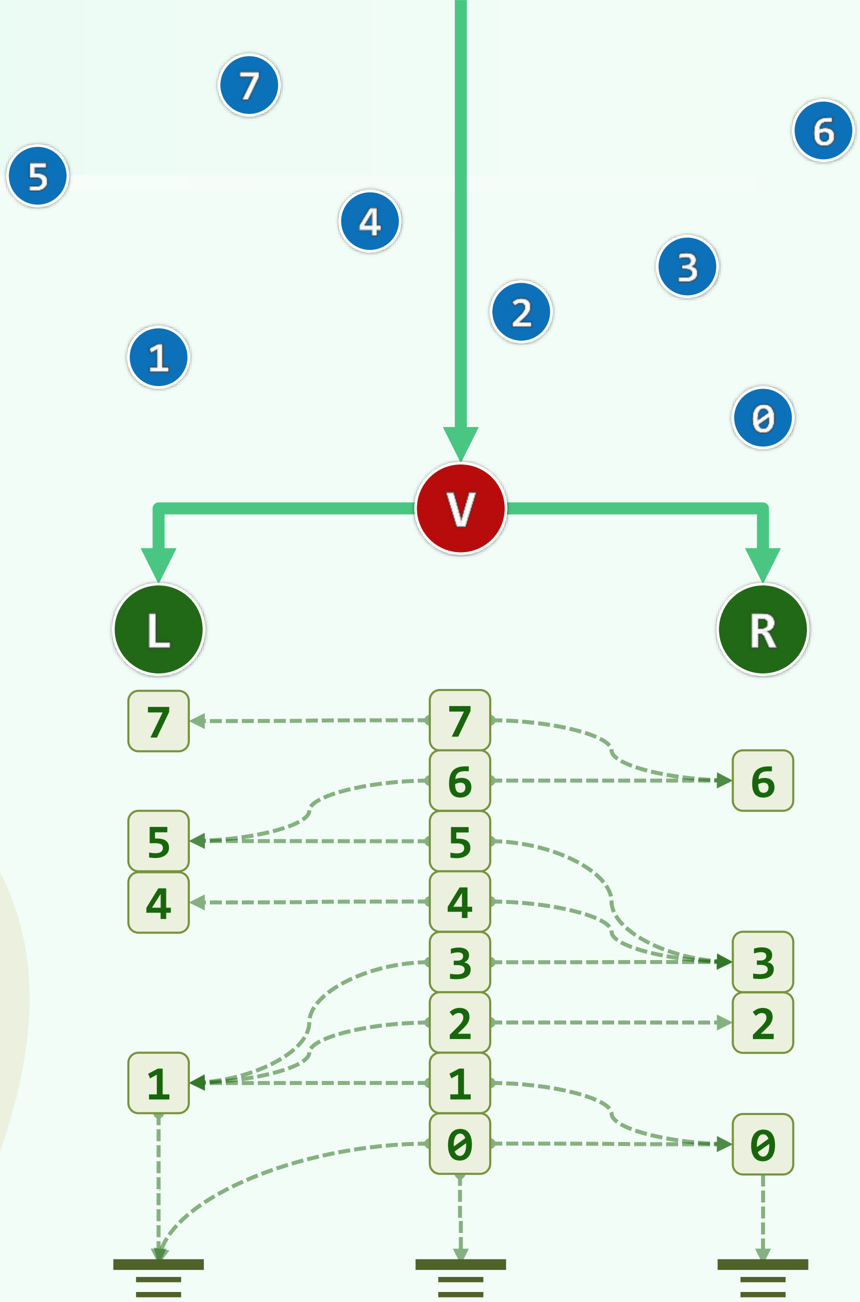
on the y-list for the LCA

❖ Thereafter, while descending the x-tree, we can

keep track of the position of y-range

in each successive list in $\mathcal{O}(1)$ time

❖ This technique is called ......



Fractional Cascading

❖ For each item in A_v,

we store two pointers to

the item of NLT value

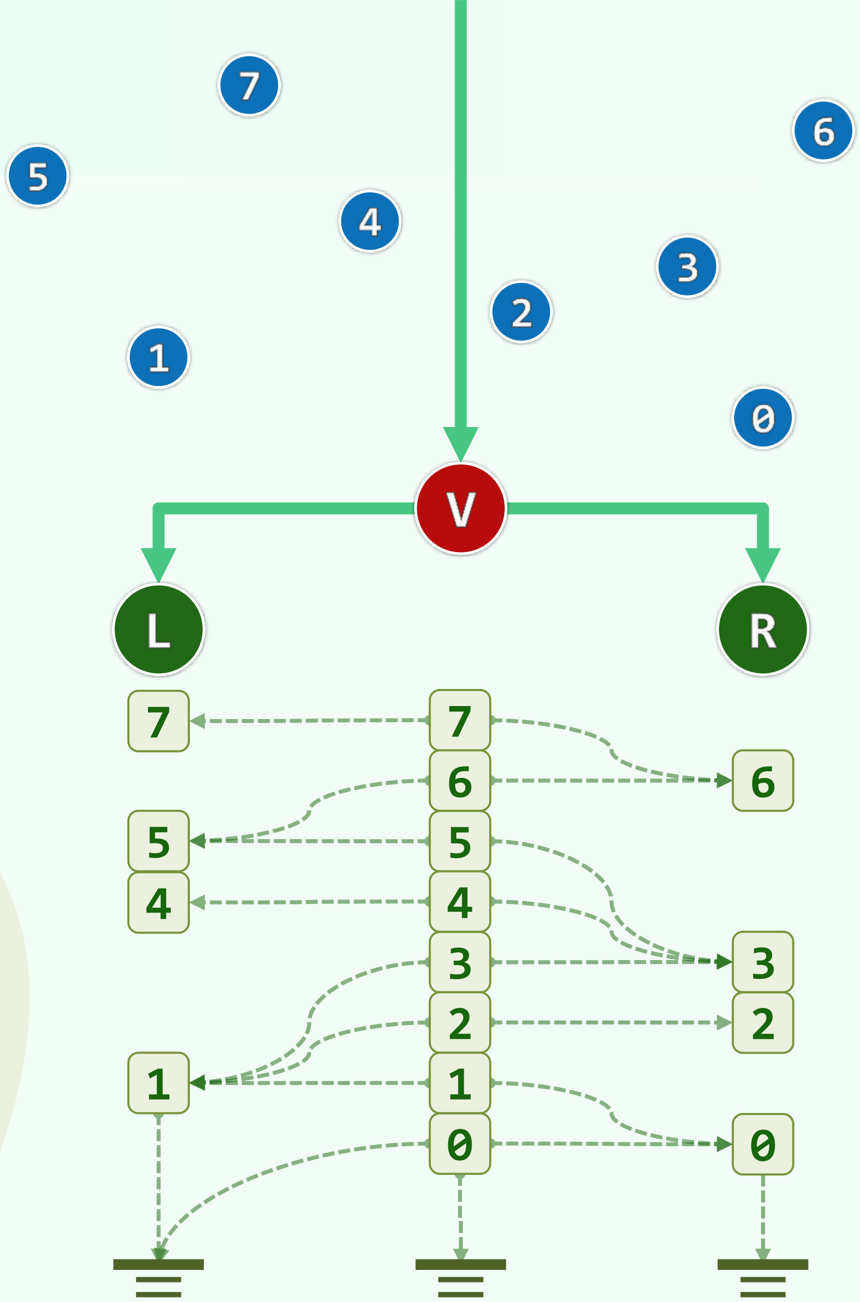
in A_L and A_R resp.

❖ Hence for any y-query with q_y,

once we know its entry in A_v, we can

determine its entry in either A_L or A_R

in $\mathcal{O}(1)$ additional time



Construction By 2-Way Merging

- Let V be an internal node in the x-tree

  with L/R its left/right child resp.

- Let A_v be the y-list for v and

  A_L/A_R be the y-lists for its children

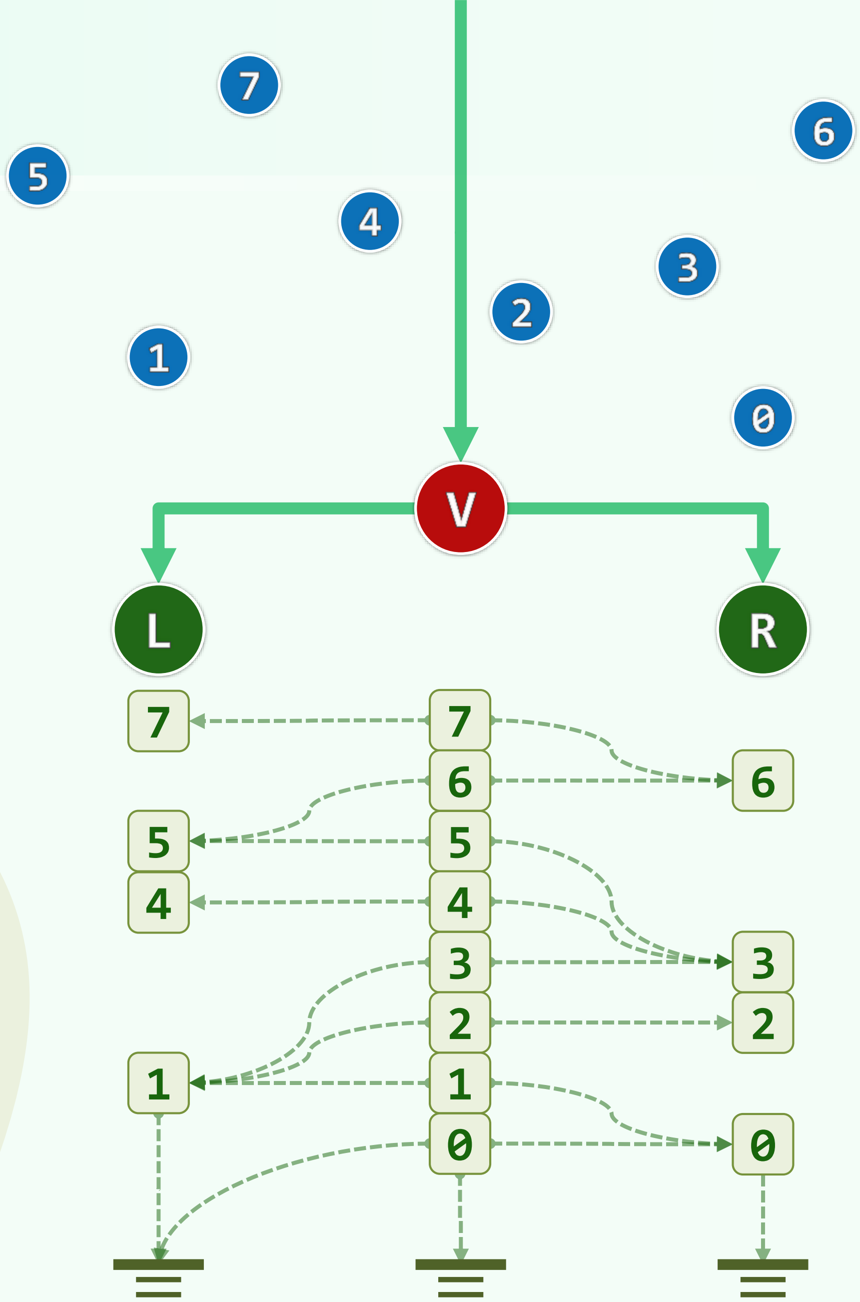
- Assuming no duplicate y-coordinates, we have

  - A_v is the disjoint union

    of A_L and A_R, and hence

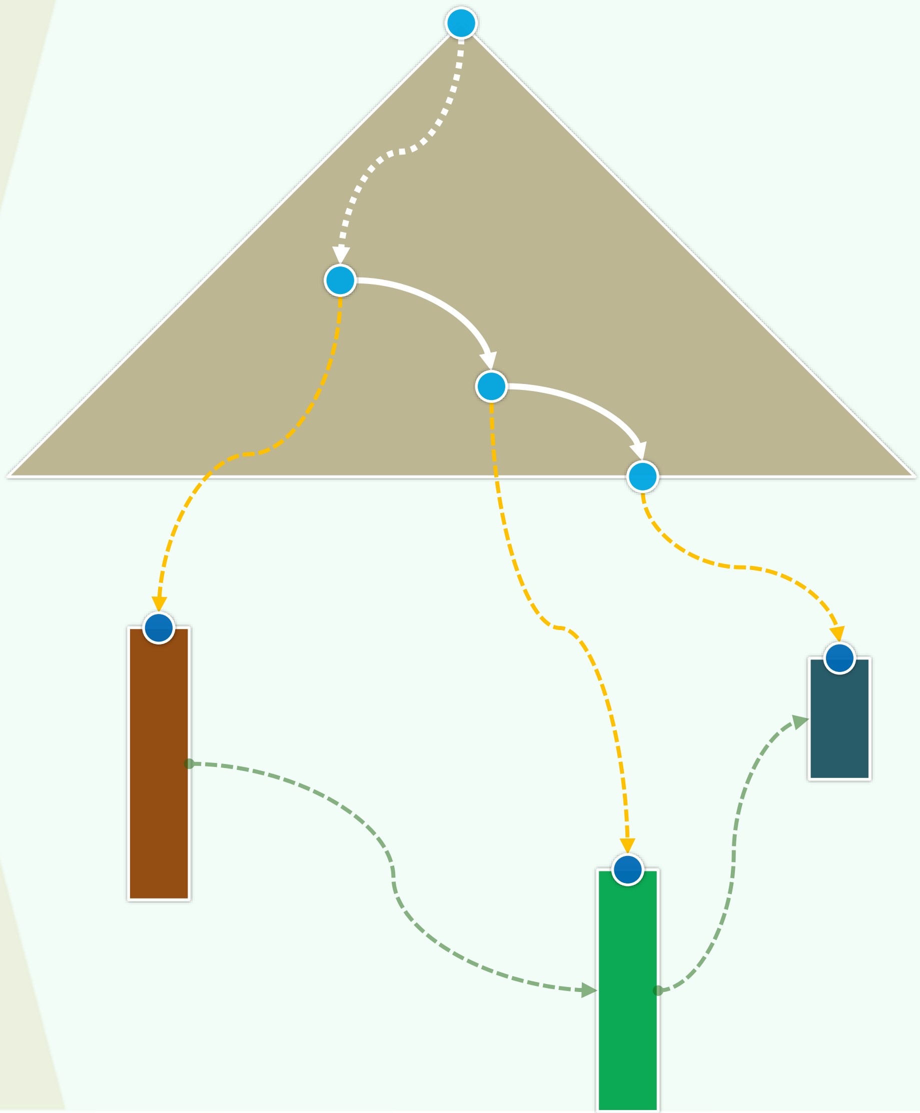
  - A_v can be obtained by

    merging A_L and A_R (in **linear** time)



Complexity

- An MLST with fractional cascading
  is called a range tree
- A y-search for root is done in $\mathcal{O}(\log n)$ time
- To drop down to each next level, we can
  get, in $\mathcal{O}(1)$ time, the current y-interval
  from that of the prior level
- Hence, each 2D orthogonal range query
  - can be answered in $\mathcal{O}(r + \log n)$ time
  - from a data structure of size $\mathcal{O}(n \cdot \log n)$,
  - which can be constructed in $\mathcal{O}(n \cdot \log n)$ time



Beyond 2D

❖ Unfortunately, the trick of fractional cascading

can **ONLY** be applied to

the **LAST** level the search structure

❖ Given a set of n points in $\mathcal{E}^d$,

an orthogonal range query

- can be answered in $\mathcal{O}(r + \log^{d-1} n)$ time
- from a data structure of size $\mathcal{O}(n \cdot \log^{d-1} n)$,
- which can be constructed in $\mathcal{O}(n \cdot \log^{d-1} n)$ time