07-XB

BST Application

Interval Tree

Your instinct, rather than precision stabbing, is more about just random bludgeoning.

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Stabbing Query

- Given a set of intervals in general position

  on the **x**-axis:

  $$S = \{s_i = [x_i, x_i^{'}] \mid 1 \le i \le n\}$$

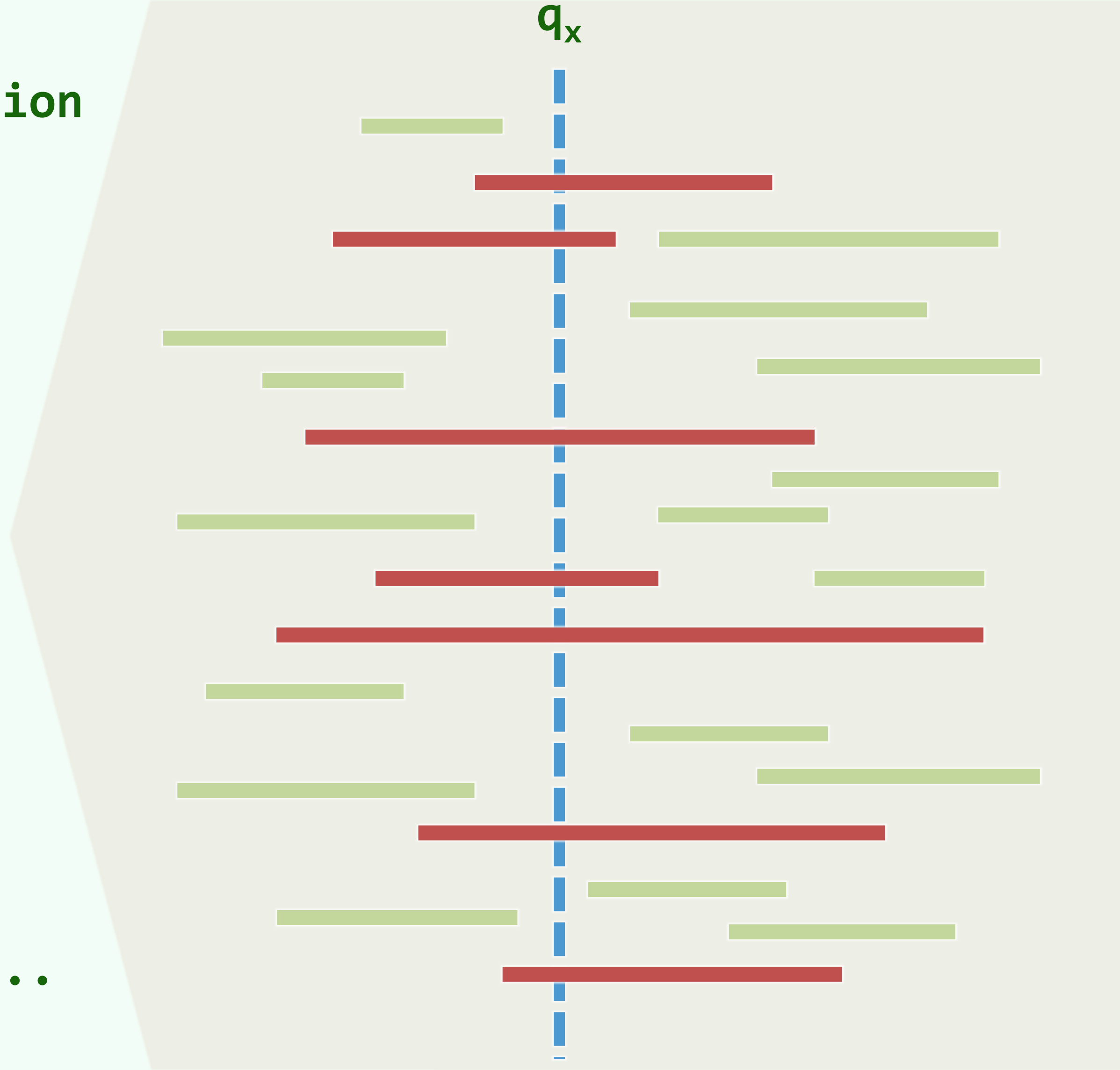
  and a query point q_x

- Find all intervals that contain q_x

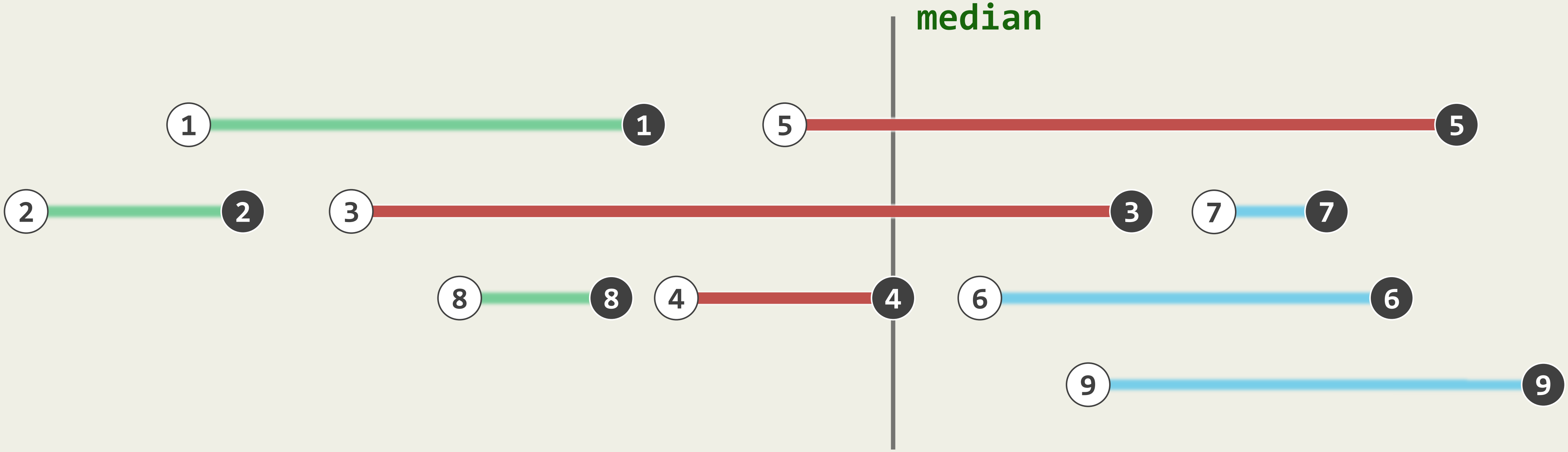
  $$\{s_i = [x_i, x_i^{'}] \mid x_i \le q_x \le {x_i}'\}$$

- To solve this query,

  we will use the so-called interval tree ...



Median

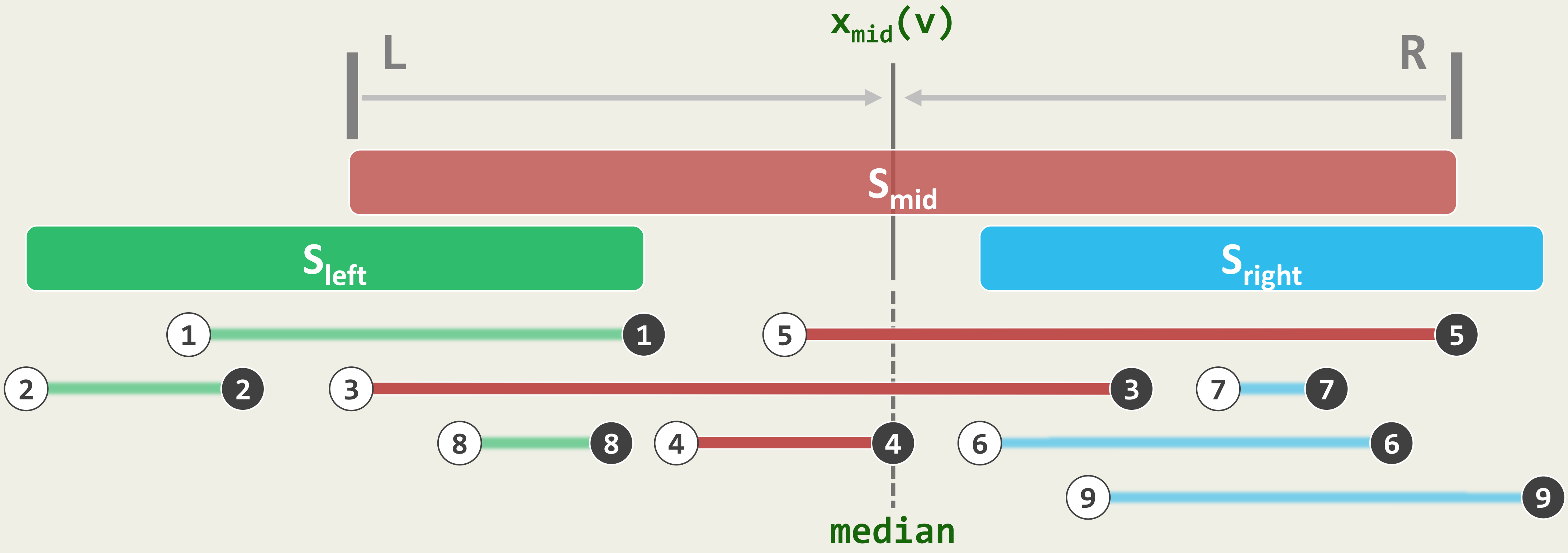


- Let $P = \partial S$ be the set of all endpoints

  (By general position assumption, $|P| = 2n$)

- Let $x_{mid} = median(P)$ be the median of P

Partitioning



- All intervals can be then categorized into 3 subsets:

$$S_{left} = \{\ S_i \mid x'_i < x_{mid}\ \} \quad S_{mid} = \{\ S_i \mid x_i \leq x_{mid} \leq x'_i\ \} \quad S_{right} = \{\ S_i \mid x_{mid} < x_i\ \}$$

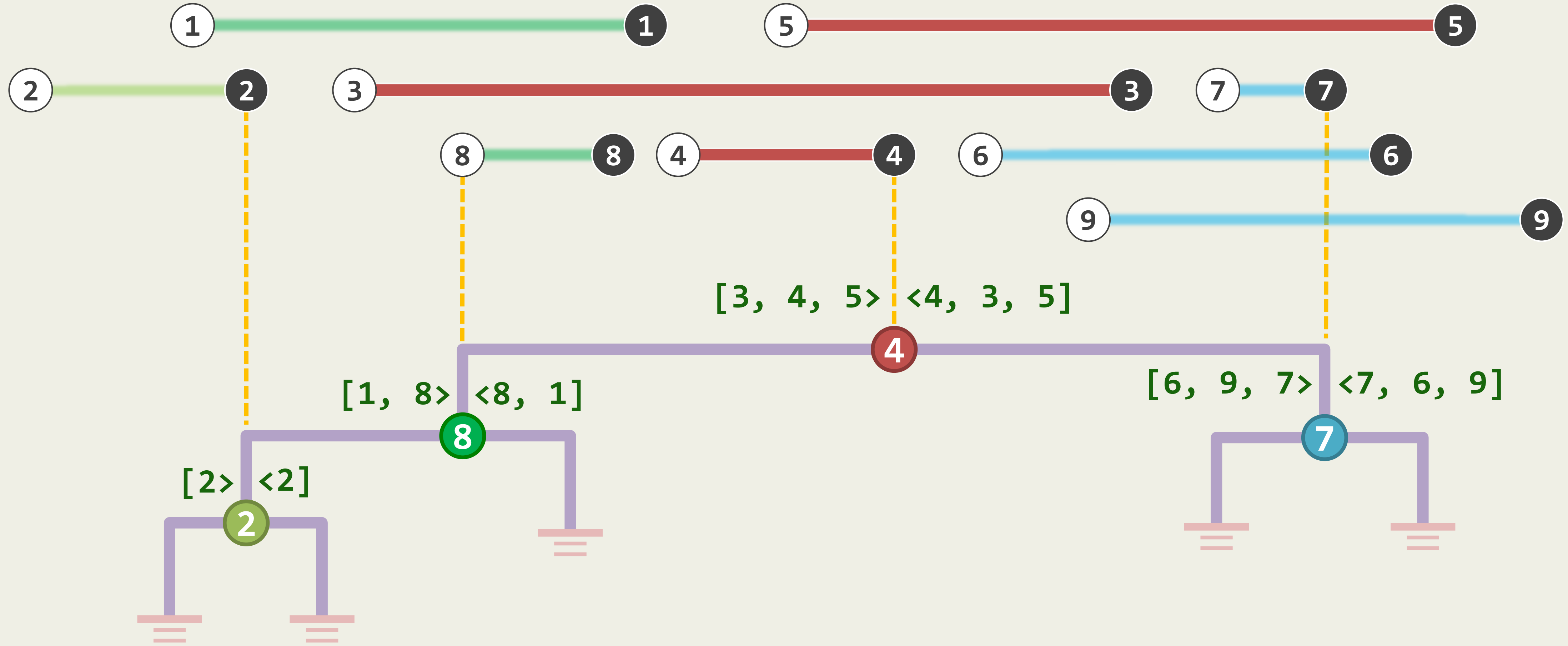
- $S_{left/right}$ will be **recursively** partitioned until they are empty (leaves)

Balance & $\mathcal{O}(\log n)$ Depth

$\max\{\ |S_{left}|, |S_{right}|\ \} \le n/2$

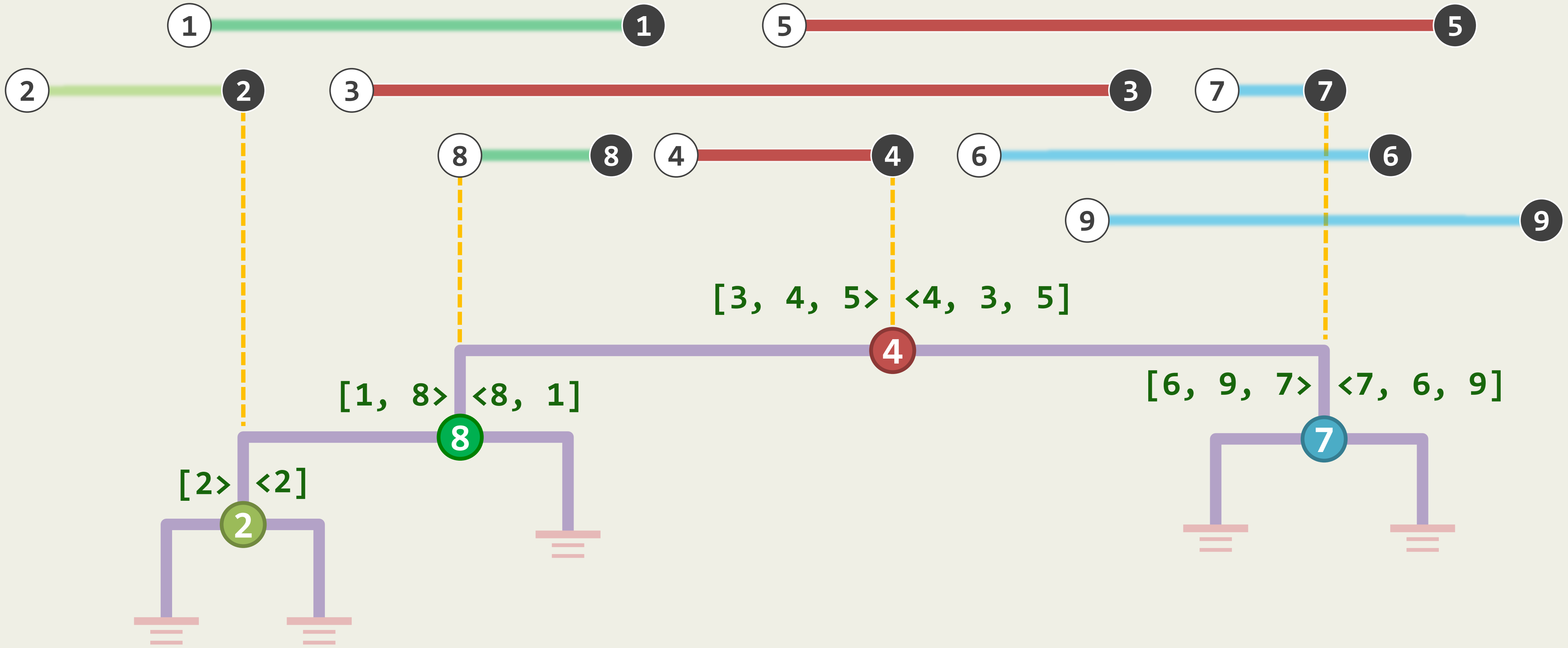
Best case: $|S_{mid}| = n$

Worst case: $|S_{mid}| = 1$



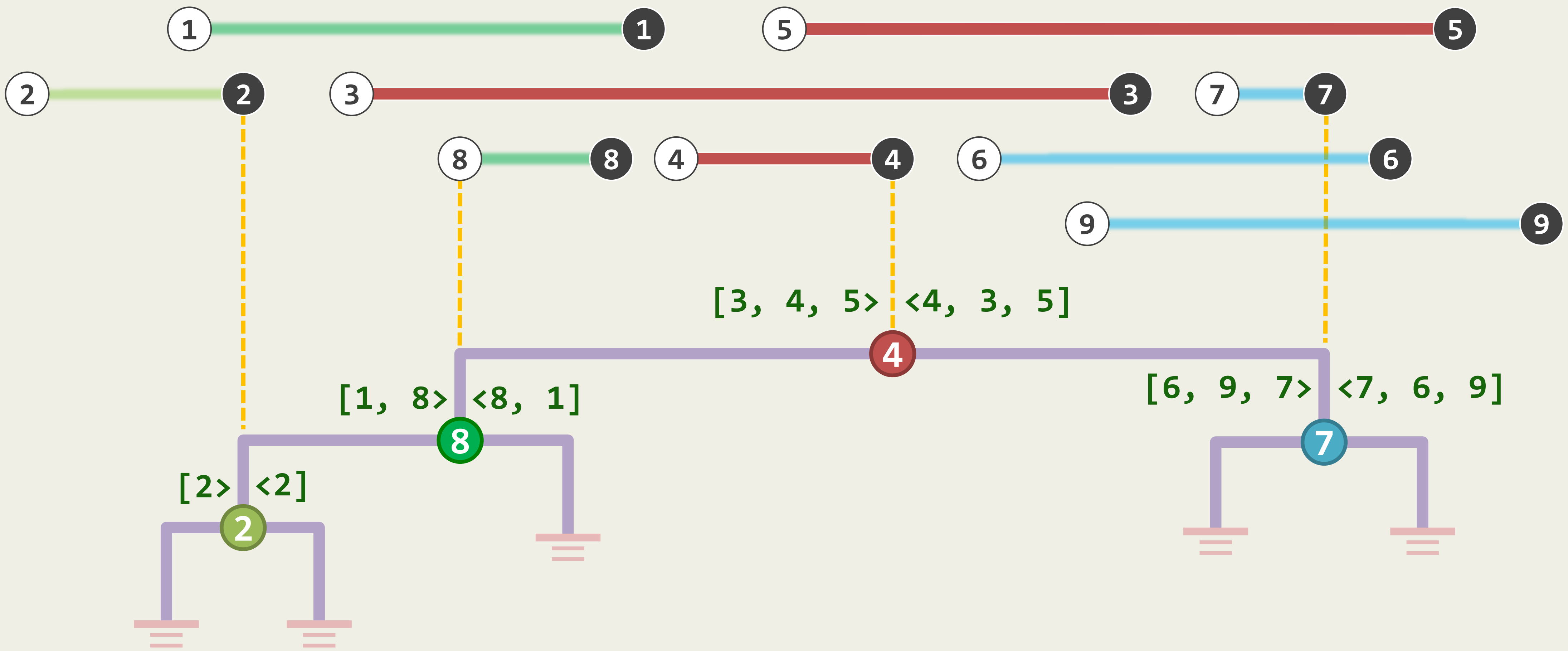
Associative Lists

❖ $L_{left/right}$ = all intervals of S_{mid} sorted by the left/right endpoints



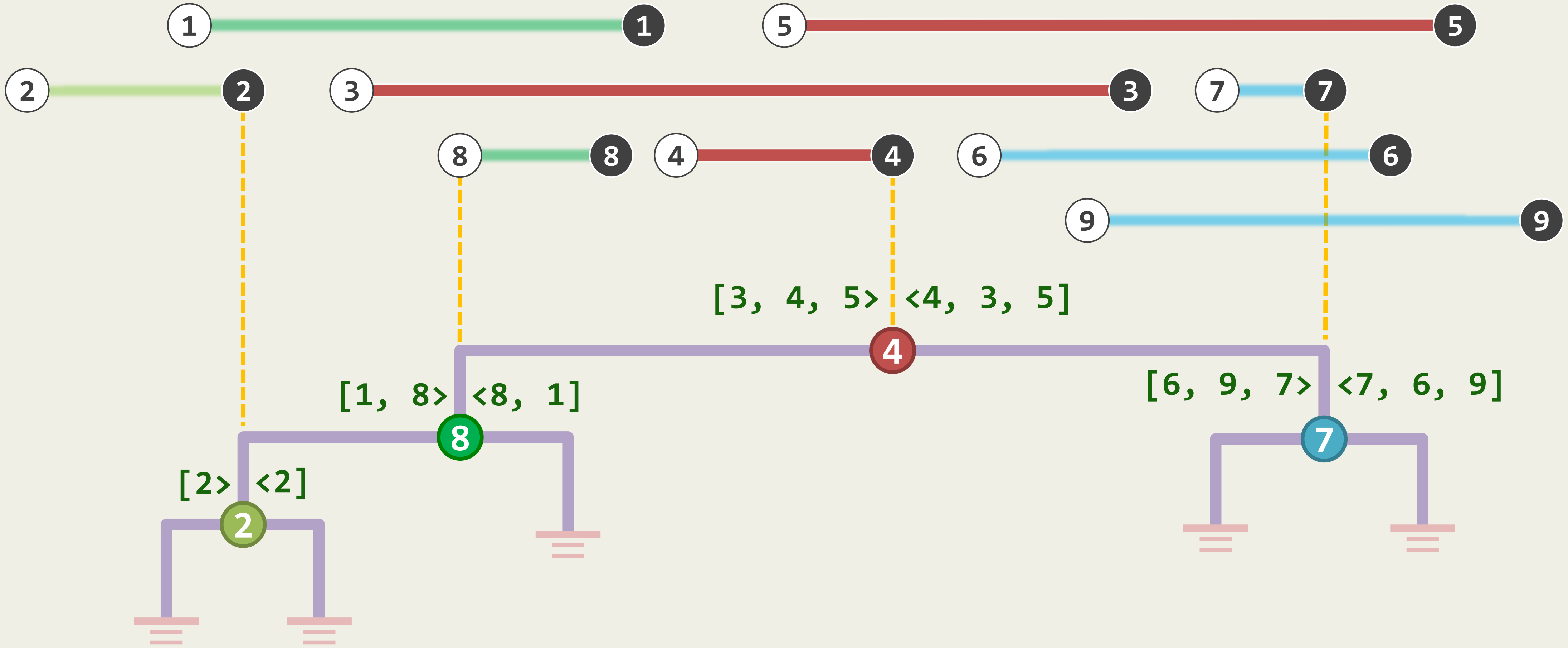
$\mathcal{O}(n)$ Size

- Each segment appears twice (one in each list)



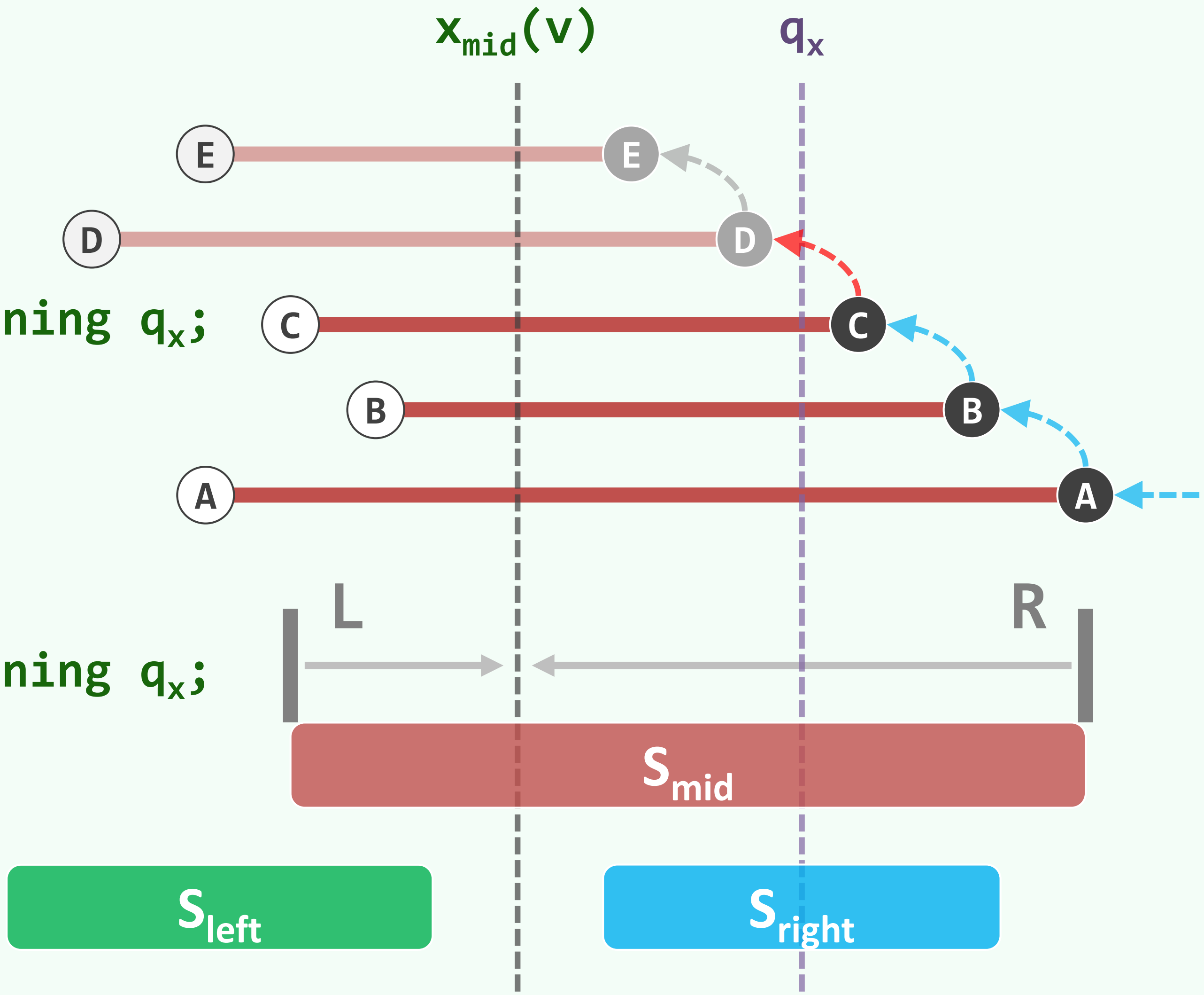
$\mathcal{O}$(nlogn) Construction Time

❖ Hint: avoid repeatedly sorting



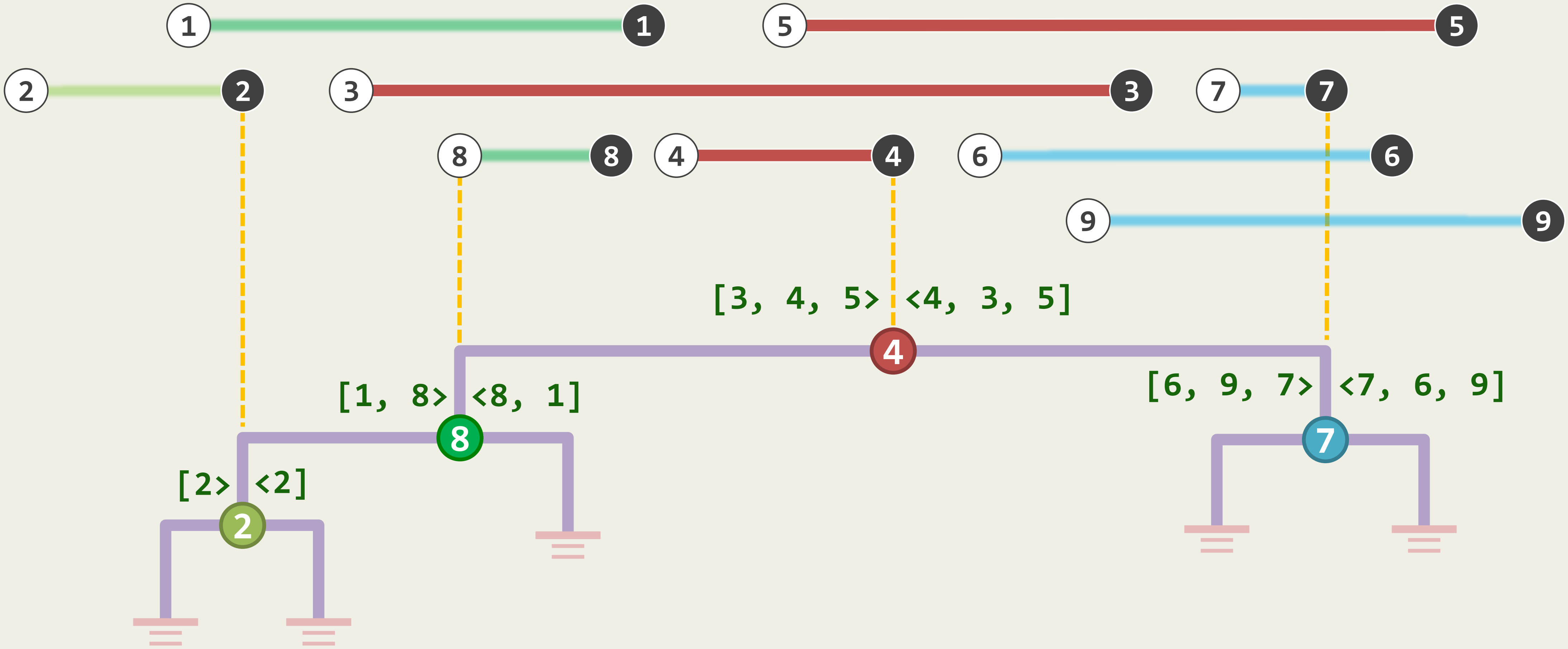
queryIntervalTree(v, q_x)

```
if ( ! v ) return;  //base
if ( $q_x$ < $x_{mid}(v)$ )
    report all segments of $S_{mid}(v)$ containing $q_x$;
    queryIntervalTree( lc(v), $q_x$ );
else if ( $x_{mid}(v)$ < $q_x$ )
    report all segments of $S_{mid}(v)$ containing $q_x$;
    queryIntervalTree( rc(v), $q_x$ );
else //with a probability ≈ 0
    report all segments of $S_{mid}$( v ); //both rc(v) & lc(v) can be ignored
```



$\mathcal{O}$(r + logn) Query Time

❖ Each query visits $\mathcal{O}$(logn) nodes //LINEAR recursion


1
1
5
5
2
2
3
3
7
7
8
8
4
4
6
6
9
9
[3, 4, 5> <4, 3, 5]
4
[1, 8> <8, 1]
8
[6, 9, 7> <7, 6, 9]
7
[2> <2]
2