07-C

BST Application

Multi-Level Search Tree

几株不知名的树，已脱下了黄叶

只有那两三片，多么可怜在枝上抖怯

它们感到秋来到，要与世间离别

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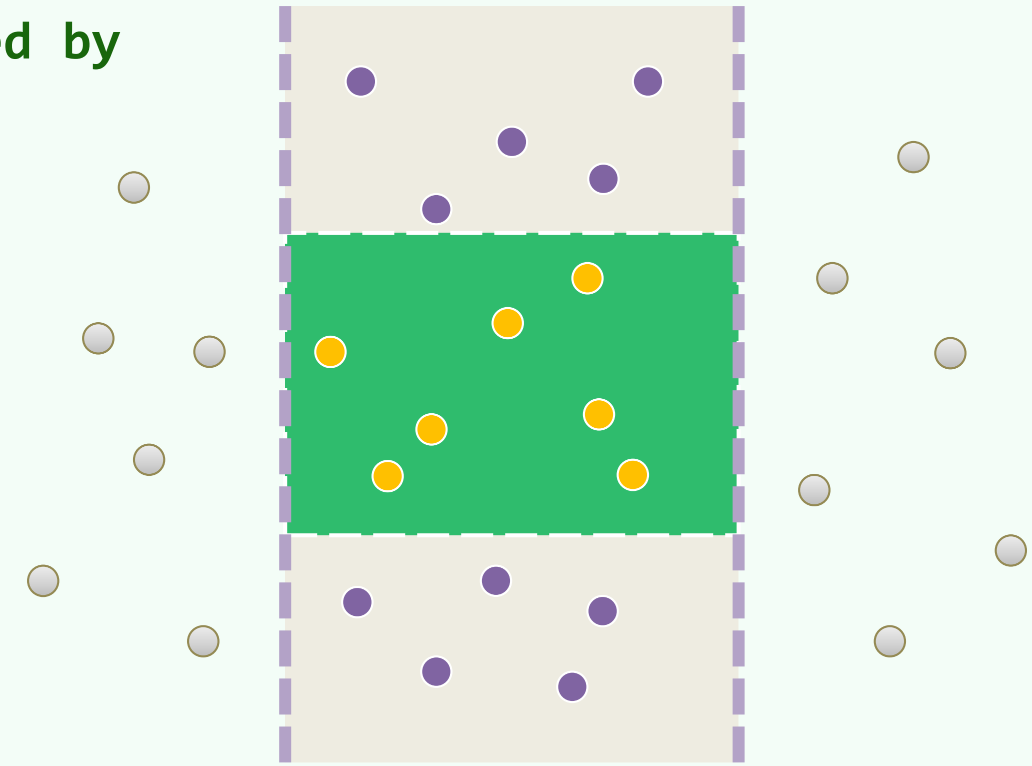
2D Range Query = x-Query + y-Query

❖ Is there any structure which answers range query **FASTER** than kd-trees?

❖ An m-D orthogonal range query can be answered by
the **INTERSECTION** of m 1D queries

❖ For example, a 2D range query
can be divided into two 1D range queries:

- find all points in $[x_1, x_2]$; and then
- find in these candidates those lying in $[y_1, y_2]$



Worst Cases

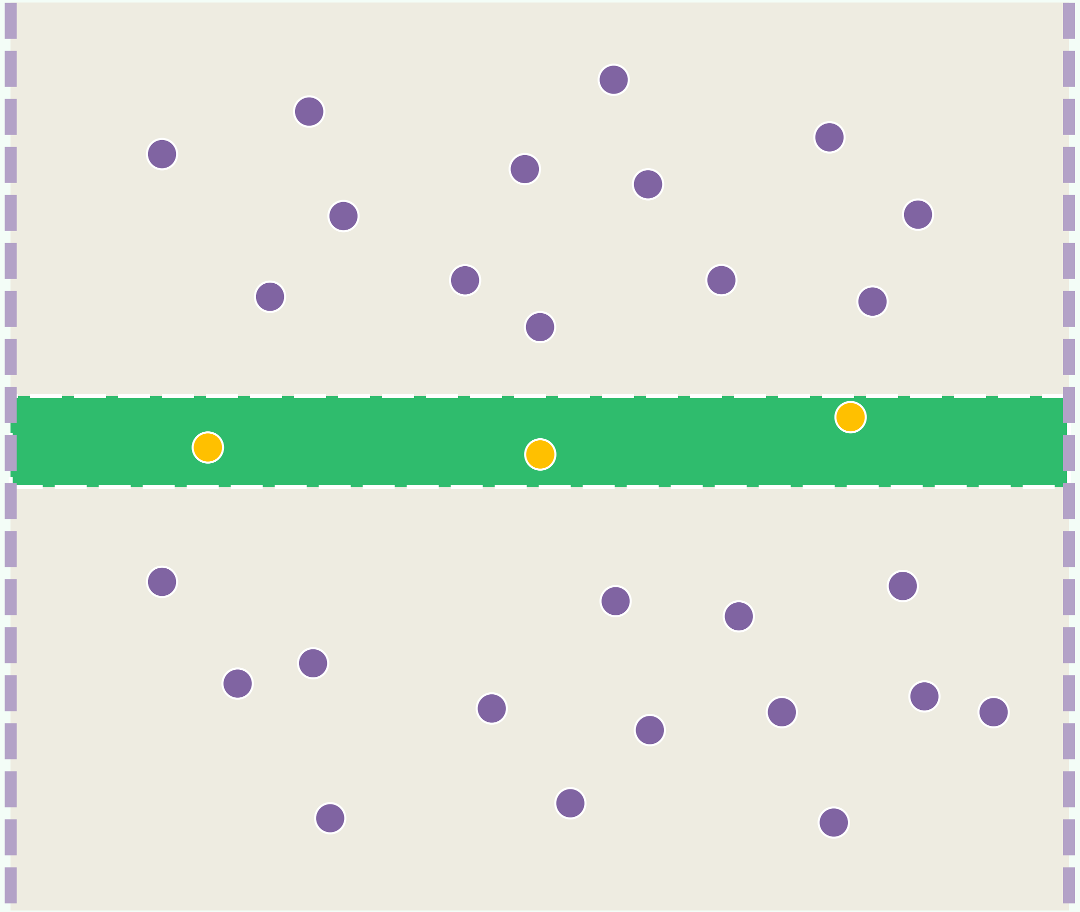
- Using kd-trees needs $\mathcal{O}(1+\sqrt{n})$ time. But here ...
- The x-query returns

  (almost) all points whereas

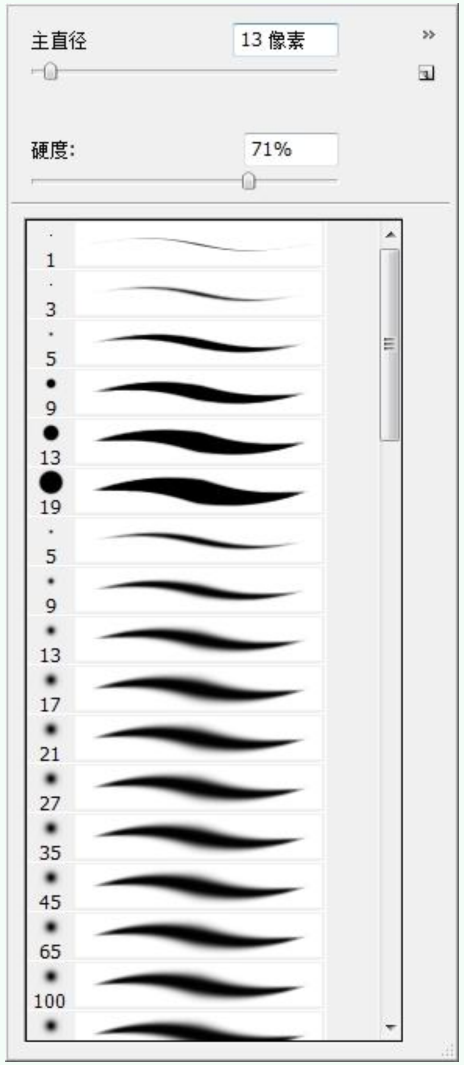
  the y-query rejects

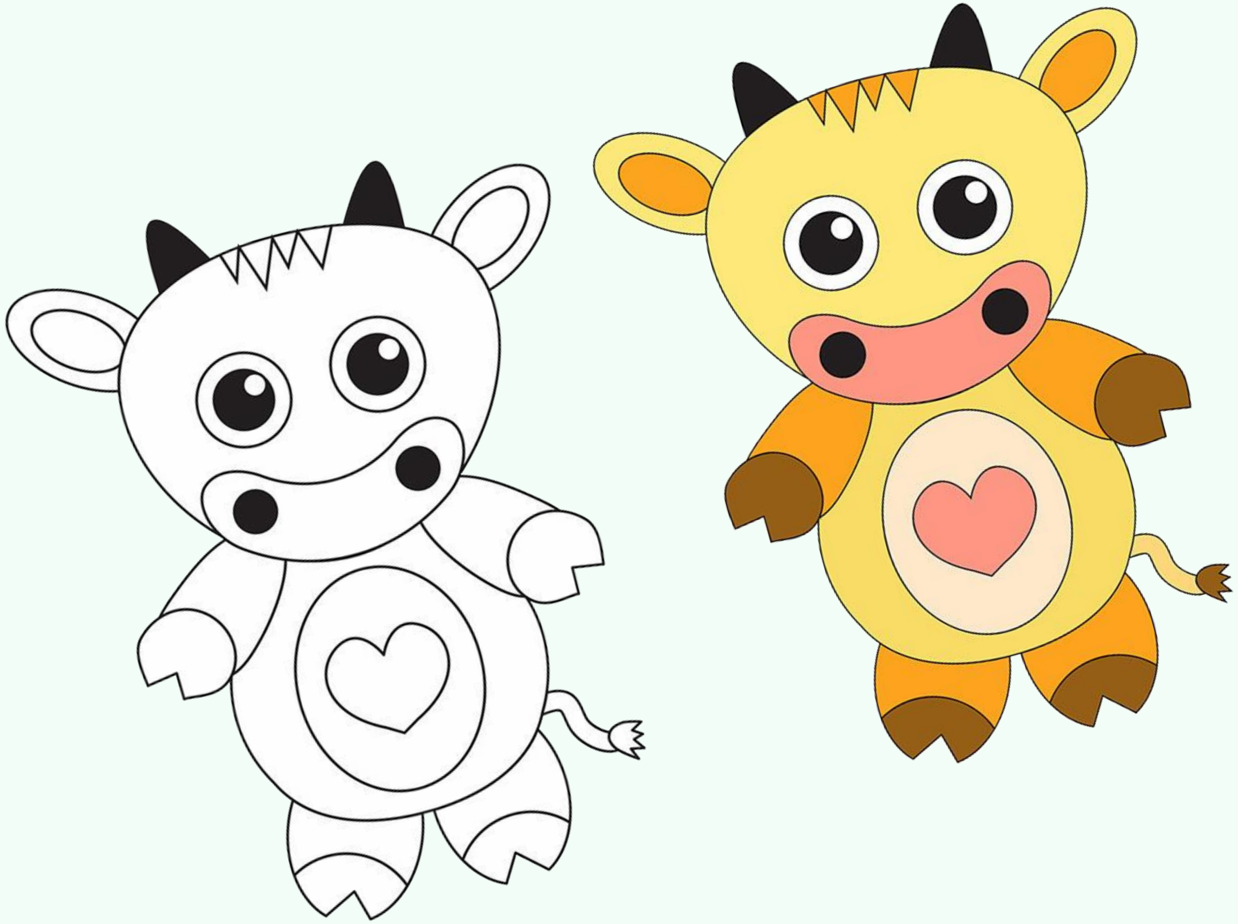
  (almost) all
- We spent Ω(n) time

  before getting r = 0 points



Painters' Strategy





2D Range Query = x-Query * y-Query

❖ Tree of trees

- build a 1D BBST (called x-tree)

  for the first range query (x-query);

- for each node v in the x-range tree,

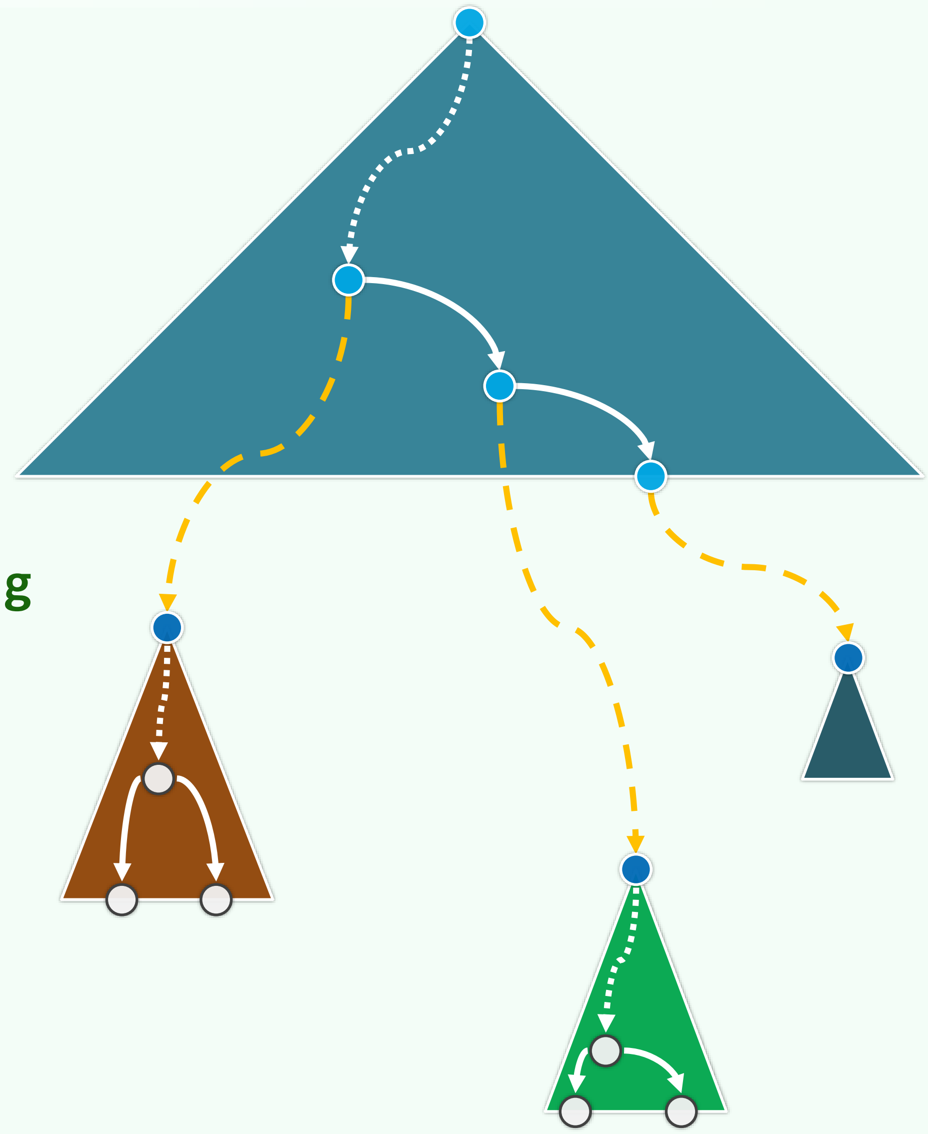
  build a y-coordinate BBST (y-tree), containing

  the canonical subset associate with v

❖ It's an x-tree of (a number of) y-trees,

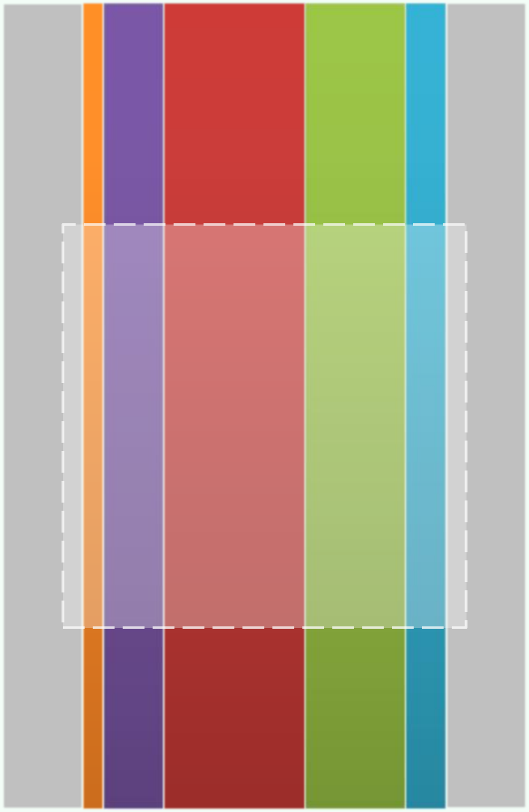
  called a Multi-Level Search Tree

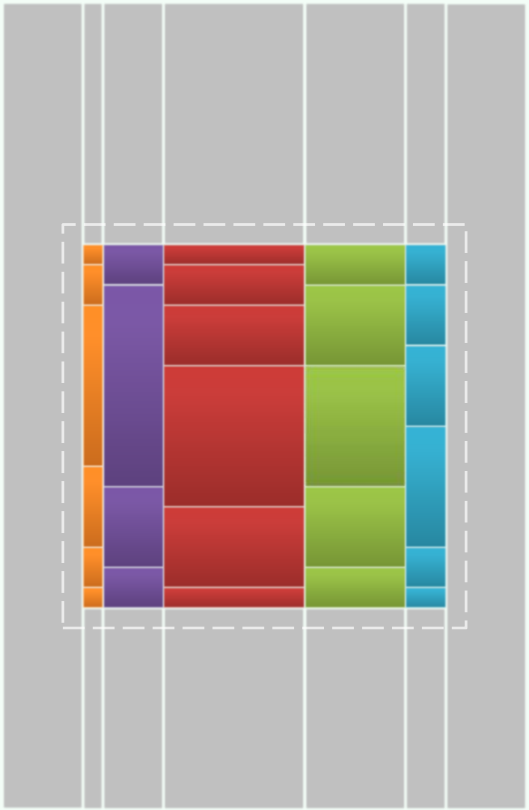
❖ How to answer range queries with such an MLST?

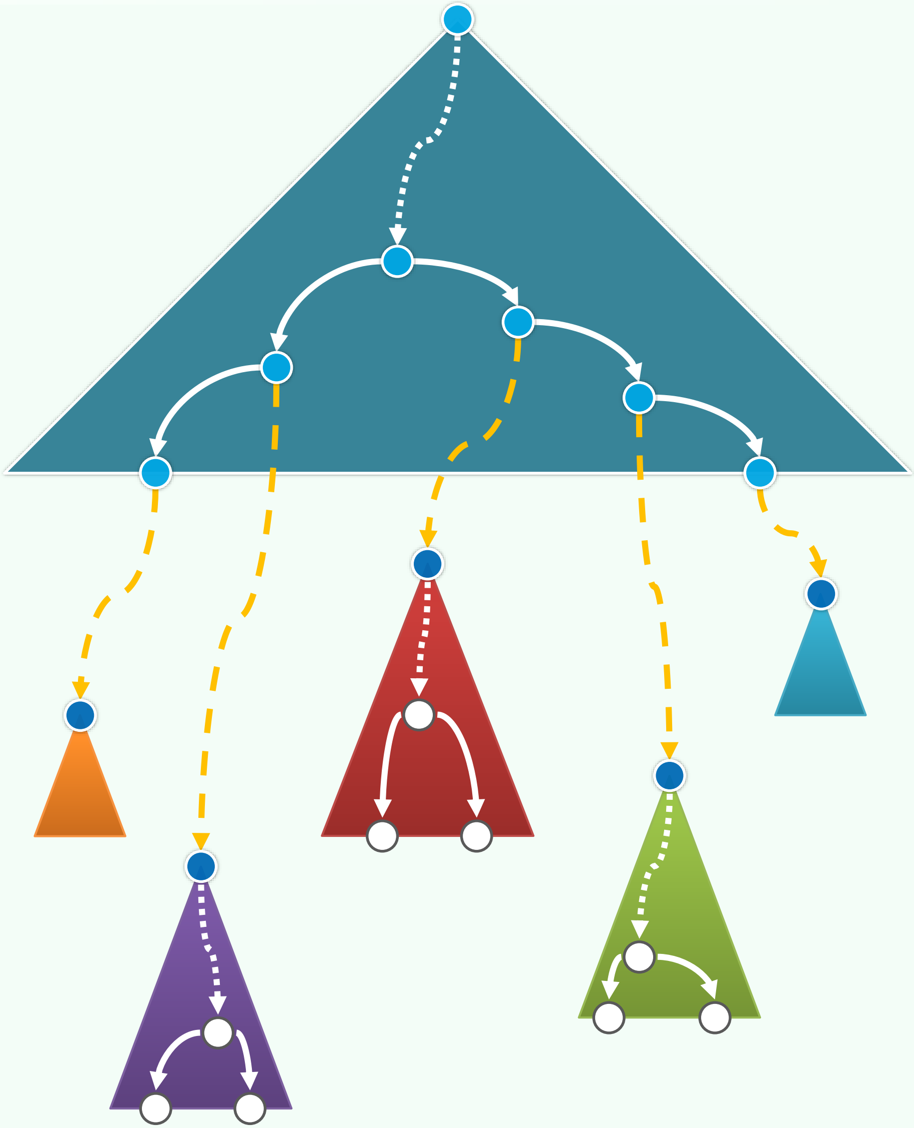


2D Range Query = x-Query * y-Queries

❖ **Query Time** = $\mathcal{O}(r + \log^2 n)$ ~ $\mathcal{O}(r + \log n)$







Query Algorithm

1. **Determine the canonical subsets of points that satisfy the first query**

   // there will be $\mathcal{O}(\log n)$ such canonical sets,

   //   each of which is just represented as

   //   a node in the x-tree

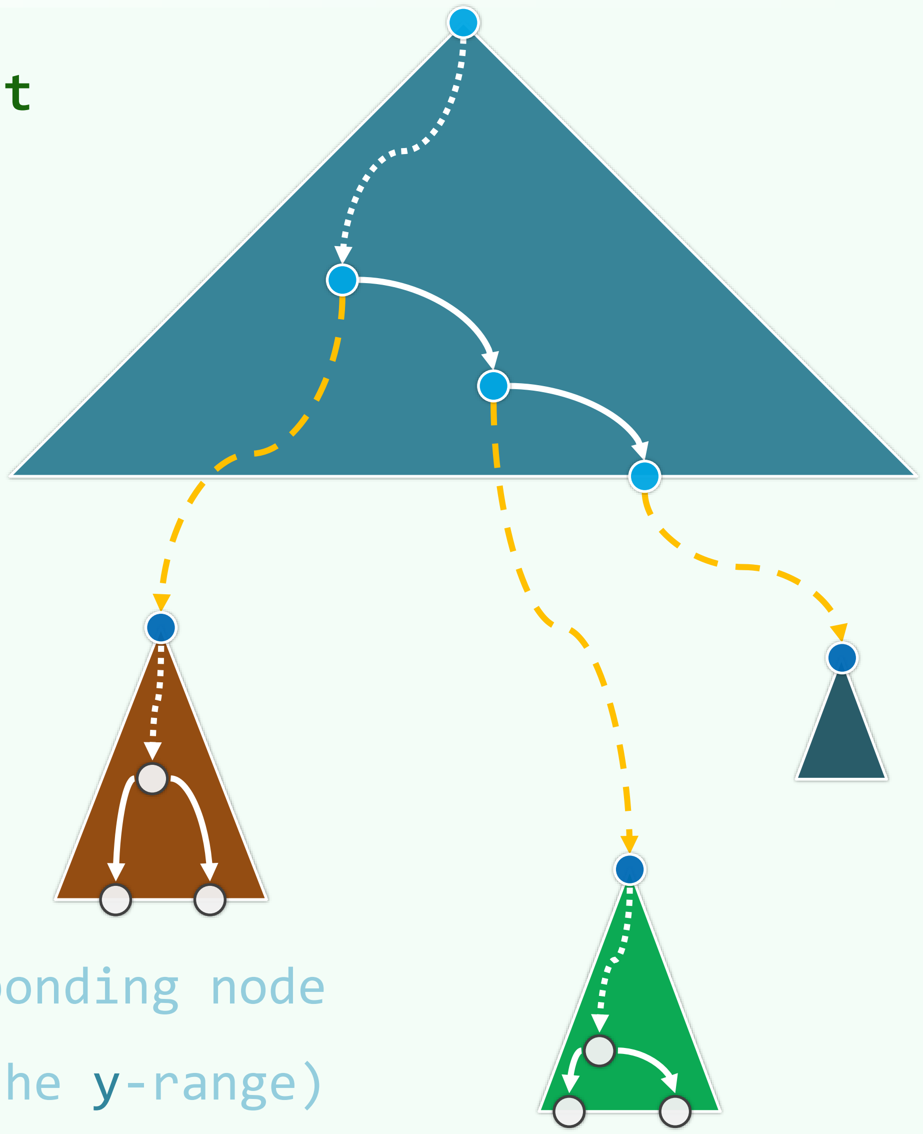
2. **Find out from each canonical subset which points lie within the y-range**

   // To do this,

   //   for each canonical subset,

   //   we access the y-tree for the corresponding node

   // this will be again a 1D range search (on the y-range)



Complexity: Preprocessing Time + Storage

❖ A 2-level search tree

for n points in the plane

can be built

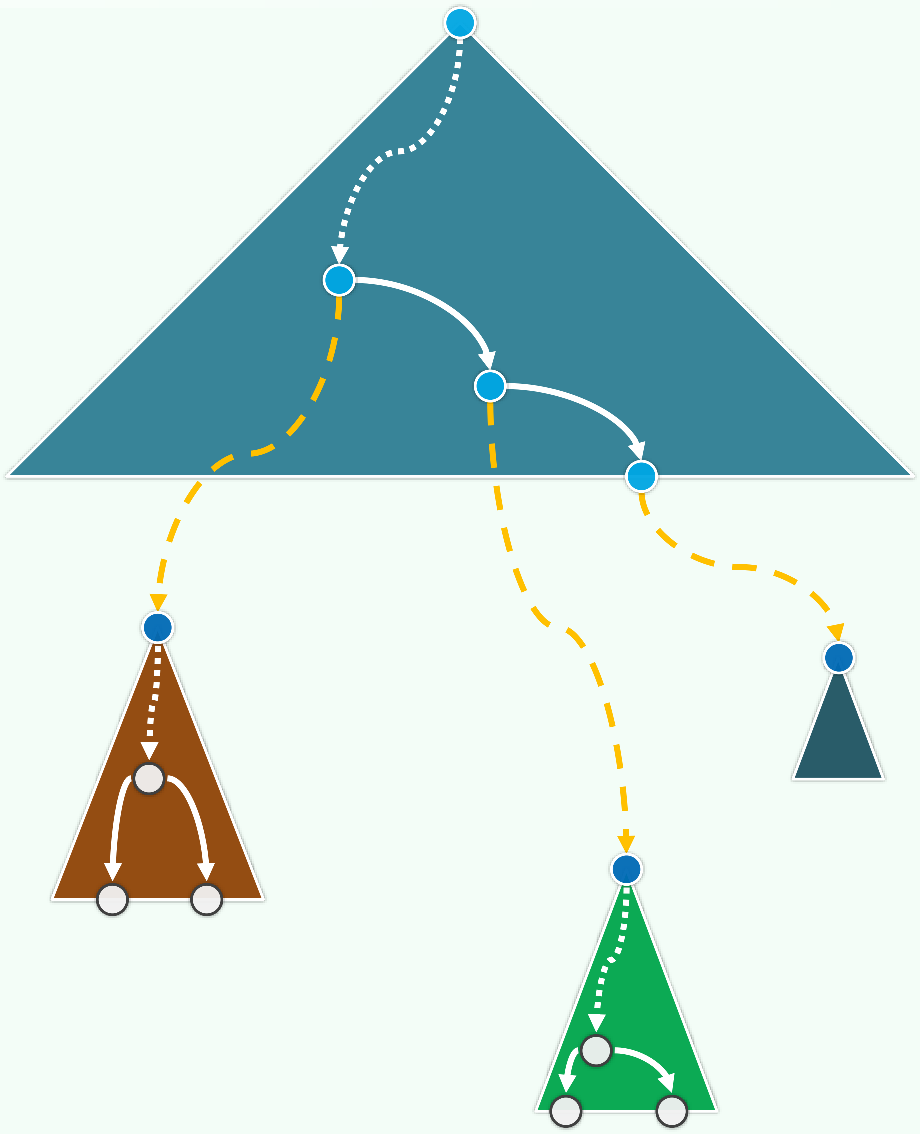
in $\mathcal{O}(n \log n)$ time

❖ Each input point is stored in $\mathcal{O}(\log n)$ y-trees

❖ A 2-level search tree

for n points in the plane

needs $\mathcal{O}(n \log n)$ space



Complexity: Query Time

❖ Claim: A 2-level search tree for n points in the plane

answers each planar range query

in $\mathcal{O}(r + \log^2 n)$ time

❖ The x-range query needs $\mathcal{O}(\log n)$ time

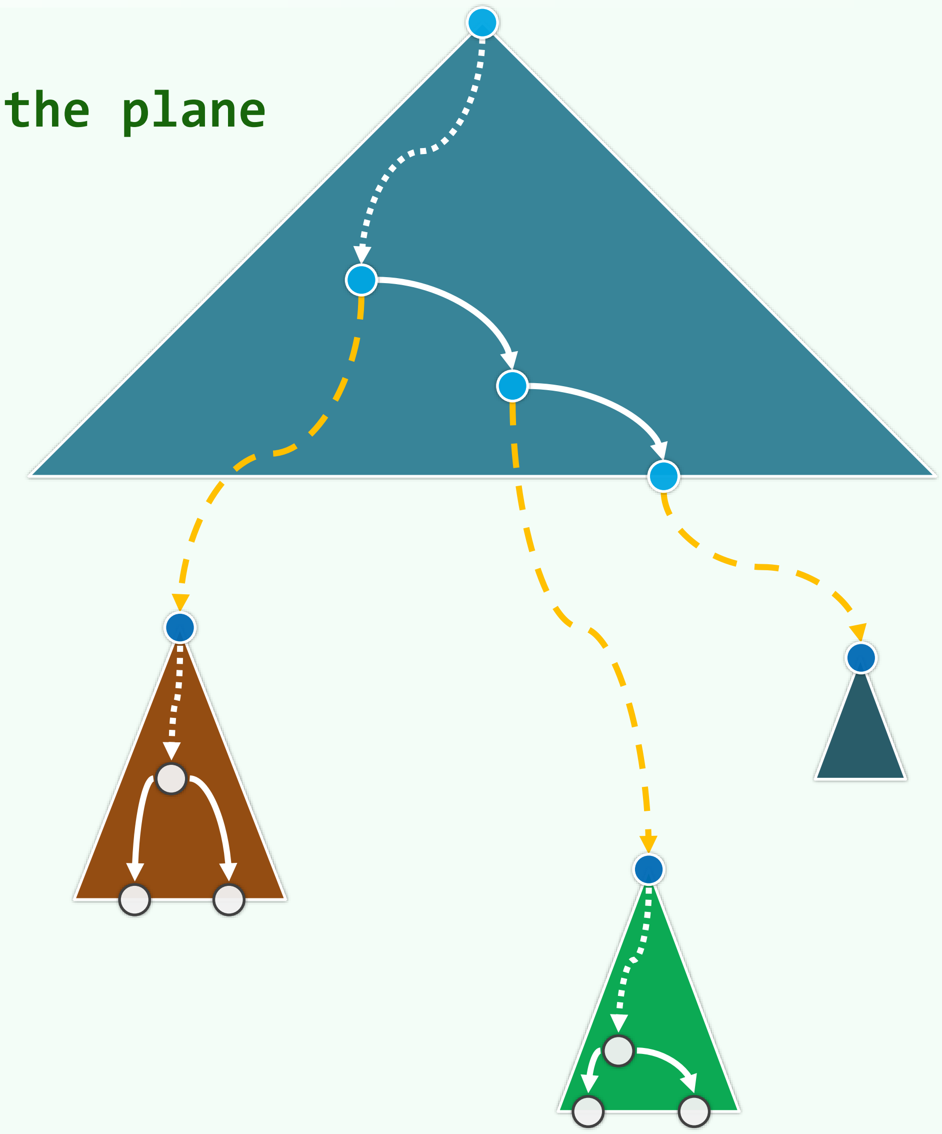
to locate the $\mathcal{O}(\log n)$ nodes

representing the canonical subsets

❖ Then for each of these nodes,

a y-range search is invoked,

which needs $\mathcal{O}(\log n)$ time



Beyond 2D

- Let S be a set of n points in $\mathcal{E}^d$, d ≥ 2
  - A d-level tree for S uses $\mathcal{O}(n \cdot \log^{d-1} n)$ storage
  - Such a tree can be constructed in $\mathcal{O}(n \cdot \log^{d-1} n)$ time
  - Each orthogonal range query of S can be answered in $\mathcal{O}(r + \log^{d} n)$ time
- For planar case, can the query time be improved to, say, $\mathcal{O}(\log n)$?