07-B6

BST Application

kd-Tree: Complexity

肉眼看不清细节，但他们都知道那是木星所在的位置，这颗太阳系最大的行星已经坠落到二维平面上了。

有人嘲笑这种体系说：为了能发现这个比例中项并组成政府共同体，按照我的办法，只消求出人口数字的平方根就行了。

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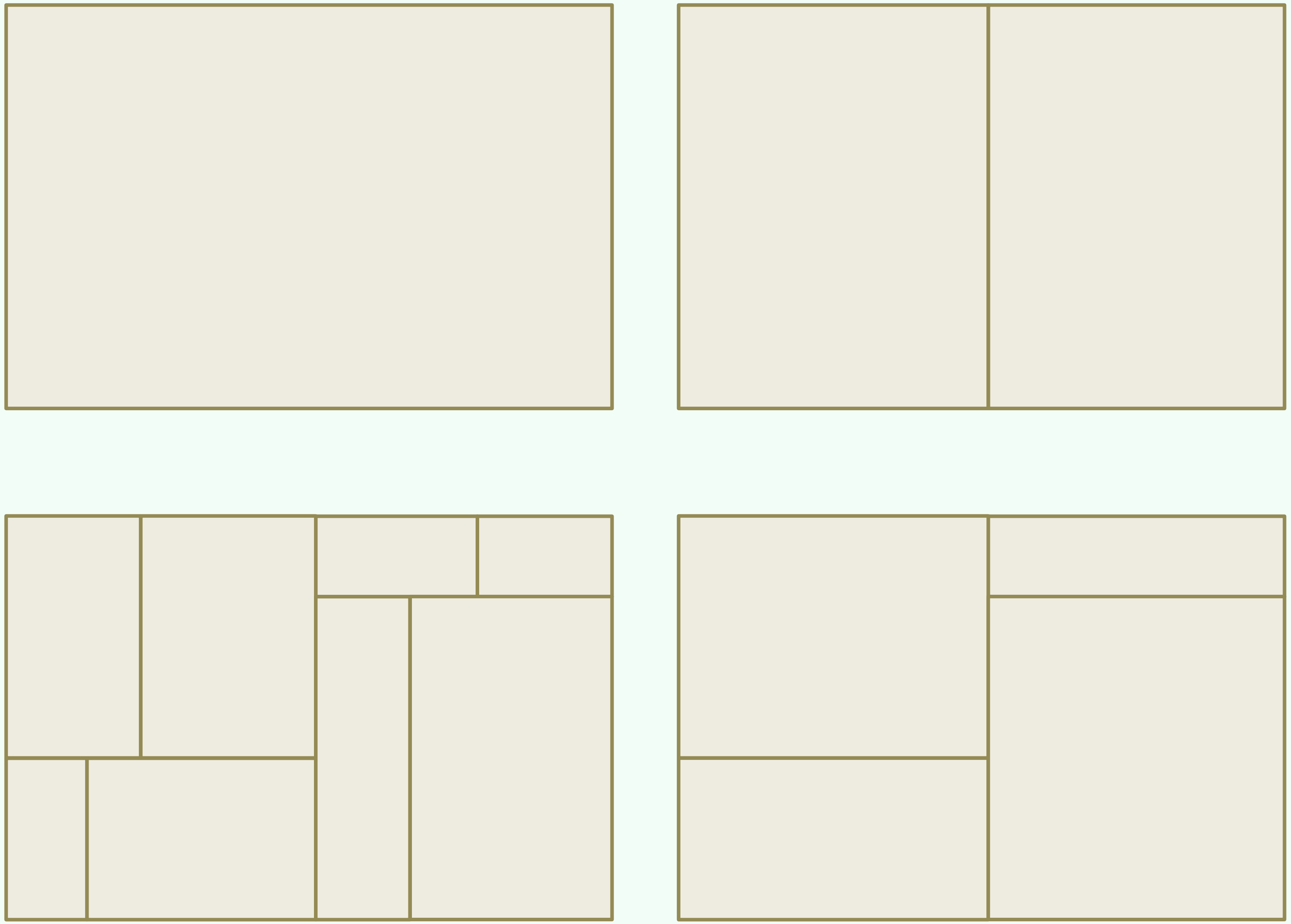
Preprocessing

- $T(n)$

  $= 2*T(n/2) + \mathcal{O}(n)$

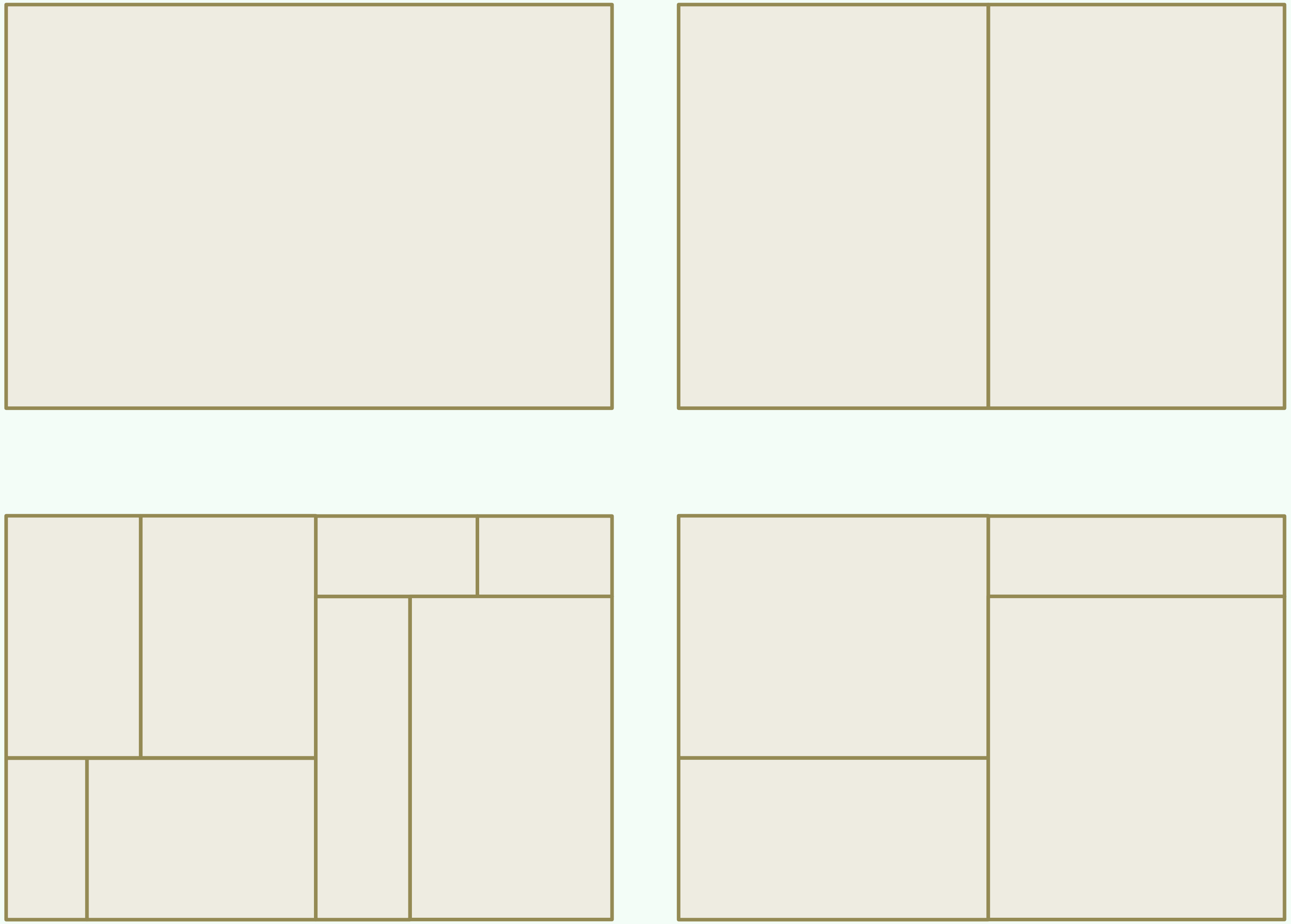
  $= \mathcal{O}(n\log n)$



Storage

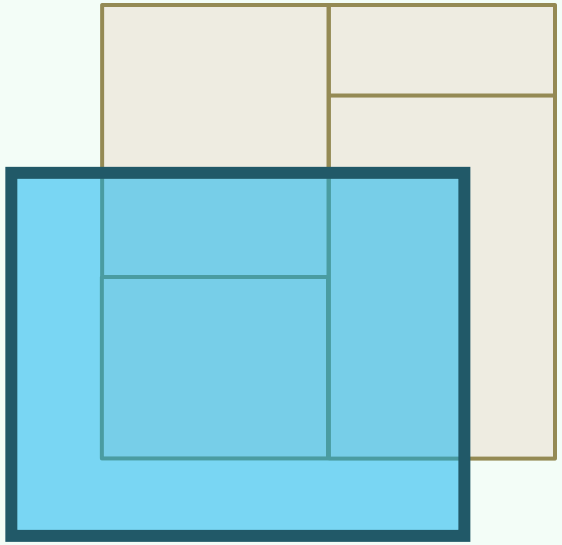
- The tree has a height of $\mathcal{O}(\log n)$

- $$\begin{aligned} & 1 \\ + & 2 \\ + & 4 \\ + & \dots \\ + & \mathcal{O}(2^{\log n}) \\ = & \mathcal{O}(n) \end{aligned}$$

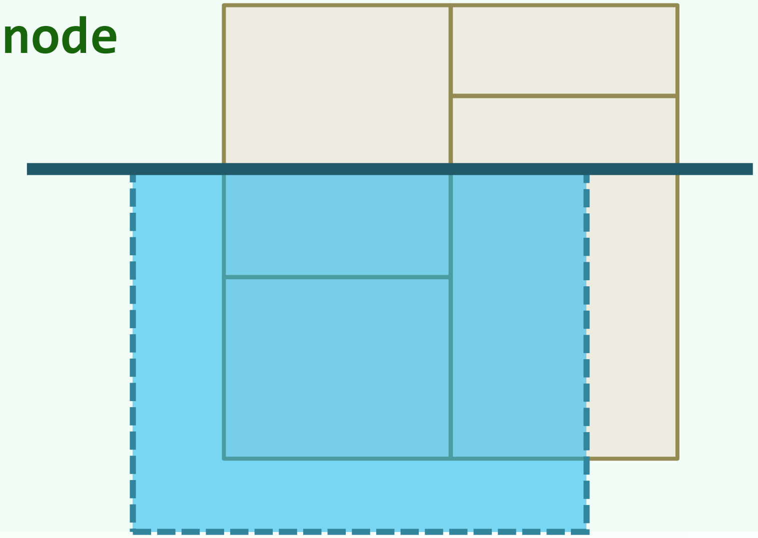


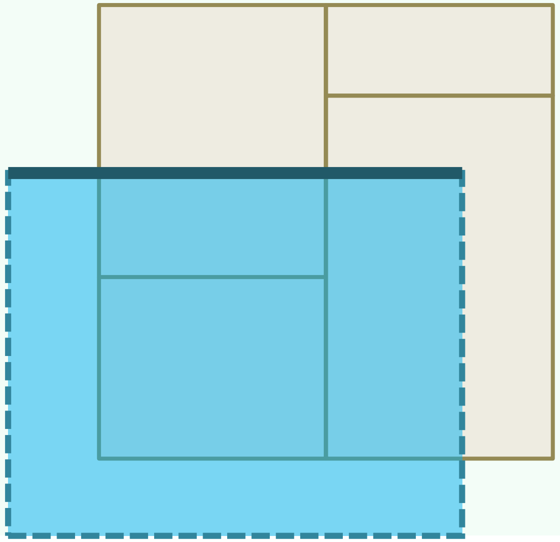
Query Time

- Claim: Report + Search = $\mathcal{O}(r + \sqrt{n})$
- The searching time depends on Q(n), the number of
  - recursive calls, or
  - sub-regions **intersecting with** R (at all levels)



- No more than **2** of the **4** grandchildren of each node will recurse
  - $Q(1) = \mathcal{O}(1)$
  - $Q(n) = 2 \cdot Q(n/4) + \mathcal{O}(1)$





- Solve to $Q(n) = \mathcal{O}(\sqrt{n})$

Beyond 2D

- Can 2d-tree be extended to kd-tree and help **HIGHER** dimensional GRS?

  If yes, how efficiently can it help?

- A kd-tree in k-dimensional space is constructed by

  recursively divide $\mathcal{E}^d$ along the $\boxed{1^{st}, 2^{nd}, ..., k^{th}}$ dimensions

- An orthogonal range query on a set of n points in $\mathcal{E}^d$

  - can be answered in $\mathcal{O}(r + n^{1-1/d})$ time,

  - using a kd-tree of size $\mathcal{O}(n)$, which

  - can be constructed in $\mathcal{O}(n \log n)$ time