串

13-F1

Karp-Rabin算法：串即是数

All things are numbers.

- Pythagoras (570 ~ 495 BC)

God made the integers; all else is the work of man.

- L. Kronecker (1823 ~ 1891)

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凡物皆数：Gödel Numbering

Kurt Gödel 1906-1978

- ❖ 逻辑系统的符号、表达式、公式、命题、定理、公理等，均可表示为**自然数**
- ❖ 每个有限维的自然数**向量**（包括字符串），都唯一对应于某个**自然数**
- ❖ 素数序列：p(k) = 第k个素数 = 2, 3, 5, 7, 11, 13, 17, 19, ...

$$\langle\, a_1\,,\ a_2\,,\ \ldots\,,\ a_n\,\rangle \ \rightleftharpoons\ p(1)^{1+a_1} \times p(2)^{1+a_2} \times \cdots \times p(n)^{1+a_n}$$

$$\langle \boxed{3}\quad \boxed{1}\quad \boxed{4}\quad \boxed{1}\quad \boxed{5}\quad \boxed{9}\quad \boxed{2}\quad \boxed{6} \rangle$$

$$2^{\boxed{4}} \times 3^{\boxed{2}} \times 5^{\boxed{5}} \times 7^{\boxed{2}} \times 11^{\boxed{6}} \times 13^{\boxed{10}} \times 17^{\boxed{3}} \times 19^{\boxed{7}}$$

- ❖ "godel" = $2^{1+7} \times 3^{1+15} \times 5^{1+4} \times 7^{1+5} \times 11^{1+12}$ = 139869560310664817087943919200000
- ❖ 若果真如RAM模型所假设的**字长无限**，则只需**一个**寄存器即可...

凡物皆数：Cantor Numbering

$$cantor_2(i,j) \ = \ [(i+j)^2 + 3\cdot i + j]/2$$

$$cantor_2(2,3) \ = \ [(2+3)^2 + 3\cdot 2 + 3]/2 \ = \ 17$$

$$cantor_2(3,2) \ = \ [(3+2)^2 + 3\cdot 3 + 2]/2 \ = \ 18$$

0	1	3	6	10	15
2	4	7	11	16	
5	8	12	17		
9	13	18			
14	19				
20					

Georg Cantor
(1845-1918)

$$cantor_{n+1}(\ a_1,\ \ldots; a_{n-1},\ a_n,\ a_{n+1}\ ) \ =$$

$$cantor_n(\ a_1,\ \ldots; a_{n-1},\ cantor_2(a_n,\ a_{n+1}\ )\ )$$

凡物皆数：Cantor Numbering

❖ 长度**有限**的字符串，都可视作d=1+|Σ|进制的自然数

"decade" = $453145_{(10)}$

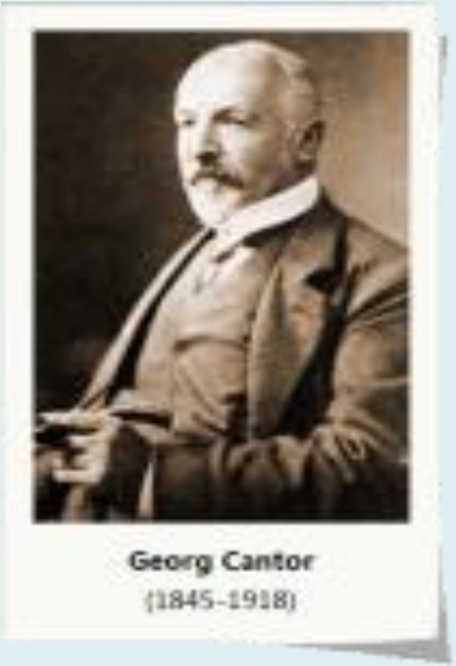
//d = 1 + ('i'-'a') = 10

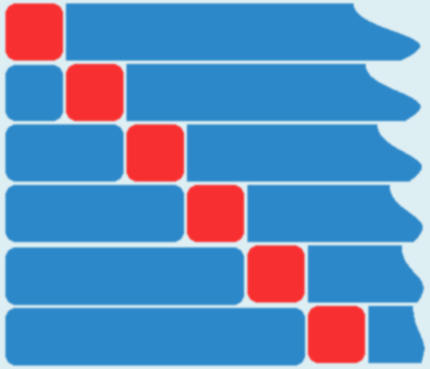
❖ 长度**无限**的字符串，都可视作[0,1)内的d进制小数

"bgahbhahbhdei..." = 0.2718281828459...

"fad<u>e</u>" = 0.614<u>5</u> = 0.614<u>4999...</u> = "fad<u>diii...</u>"

❖ Cantor's Diagonal: $\aleph_0 < \aleph_1$





串亦为数

- **十进制串，可直接视作自然数** //指纹（fingerprint），等效于多项式法

  P = "82818"　　　　T = 271$\boxed{82818}$28459045235360287471352

- **一般地，随意对字符编号{ 0, 1, 2, ..., d - 1 }** //设d = |Σ|

  于是，每个字符串都对应于一个 d 进制自然数 //尽管不是单射

  "CAT" = 2 0 19 $_{(26)}$ = $1371_{(10)}$ //Σ = { A, B, C, ..., Z }

  "ABBA" = 0 1 1 0 $_{(26)}$ = $702_{(10)}$

- **P在T中出现 仅当 T中某一子串与P相等** //为什么不是“当”？

- **这，不已经就是一个算法了吗？！** //具体如何实现?

- **问题似乎解决得很顺利，果真如此简单吗?** //复杂度?