

Project #1  
Due: February 28, 2019

Consider the solution of a quadratic equation of the form

$$x^2 + bx + 1 = 0.$$

Naturally, the solutions are given by

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4}}{2} = -\frac{b}{2} \pm \frac{|b|}{2} \sqrt{1 - \frac{4}{b^2}}. \quad (1)$$

The numerical evaluation of the term

$$-\frac{b}{2} + \frac{|b|}{2} \sqrt{1 - \frac{4}{b^2}}$$

may be numerically inaccurate if  $b \gg 2$ . Here are two ways to avoid the problem:

1. Compute

$$\left( -\frac{b}{2} - \frac{|b|}{2} \sqrt{1 - \frac{4}{b^2}} \right)^{-1}. \quad (2)$$

2. Use a Taylor series approximation to evaluate

$$-\frac{b}{2} \left( 1 - \sqrt{1 - \frac{4}{b^2}} \right). \quad (3)$$

The objective of this project is to evaluate the accuracy of the original approach in (1) and the two alternatives in (2) and (3). You need to perform a numerical experiment to determine a threshold for the value of  $b$  such that when  $b$  is larger than this threshold, the accuracy of the roots is unsatisfactory.

To complete the project, you need to prepare a typed report that summarizes your findings and uses plots and tables to support your conclusions