Final Review

1. 1 Finite Automata

- Regular language is closed under
    - Union
    - Concatenation
    - Kleene star

1. 2 NFA

- Regular language can be recognized by
        - DFA / NFA / RegExp
- Union
    - Construct a DFA of $|M_1| \times |M_2|$ states

1. 3 Regular Expression

- Operations
    - Union
    - Concatenation
    - Kleene star
- Examples:
    - 1* ø = ø → Concatenate the empty set yields the empty set
    - ø* = {epsilon} → star operation puts together any number of strings from the language. Since the language has no elements, the star operation put 0 strings.

1. 4 Non-Regular Languages

- Pumping lemma
    - S = xyz, |y| > 0, |xy| ≤ p, xy^iz is in L

2. 1 CFGs

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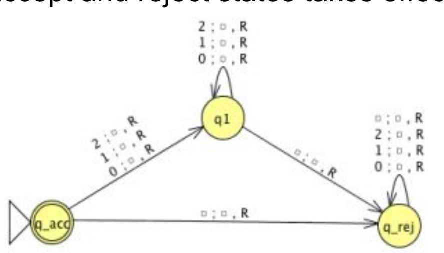
3. 1 Turing Machine

- Tape is infinite
- Special states: accept and reject states takes effect immediately.
    - Ex: M_1 

    - Once hit q_acc, input gets accepted.
- ∑ does NOT contain blank symbol
- T is the stack alphabet, blank symbol is in T and ∑ is a subset of T

- **Turing Recognizable**
    - If some ™ recognizes a language

- M fails to accept the input by entering q_{reject} or looping
- **Turing Decidable**
  - Always halts
  - <u>Every decidable language is Turing Recognizable</u>

<u>3. 2 Variants of TM</u>

- Multitape TM
  - Ordinary TM with multiple tapes
  - funct: $\delta(q_i, a_1, \ldots, a_k) = (q_j, b_1, \ldots, b_k, L, R, L, L, \ldots)$ if there are k states
  - Every multitape ™ has an equivalent single-tape ™

- Non-deterministic TM
  - At any point in a computation, machine may proceed according to several possibilities
  - funct: Q X T → P(Q X T X {L, R})
  - Every nondet TM has an equivalent deterministic TM

- Enumerator
  - If the enumerator does NOT halt, it may print an infinite list of strings (repetition possible)
  - A language is Turing recognizable **iff** some enumerator enumerates it.
  - **Proof:**
    - *M = "On the input w,*
      - *1. Run E. Everytime E outputs a string, compare with w*
      - *2. If w appears in the output of E, accept"*
    - *The other direction*
    - *E = "Ignore the input,*
      - *1. Repeat the following for i = 1, 2, 3, …*
      - *2. Run M for i steps on each input $s_1, s_2, s_3, \ldots, s_i$*
      - *3. If any computations accept, print out the corresponding s_j"*
    - *If M accepts s, eventually, it will appear on the list generated by E, and appears infinite times.*

<u>4. 1 Decidable Language</u>

- Decidable problems concerning Regular languages
  - A_{DFA} = {<B, w>| B is a DFA that accepts w}
    - A_{DFA} is a decidable language

<u>Indi-hw0</u>

- DOUBLE(L) is a subset of L • L
- The length of each string in STUTTLE(L) is even
  - Each string can be written as $w_1w_1w_2w_2\ldots w_nw_n$, and $|w_i| = 1$. Thus $|s| = 2n$

- **A set X is said to be closed under an operation OP if, for any element in X, applying OP to them gives an elements in X.**
- Concatenating two strings over the alphabet Σ gives a string over the alphabet Σ
- Power set is the set of set, not set of integers.

Grp-hw1

- (a U b)* → (a*b*)* (without union operation)
- Any set represented by some regular expression without kleene star is finite
- Reminder of binary number

Indi-hw2

- Every DFA is a NFA
- Conversion from NFA to DFA, if NFA has n states, then converted DFA has 2^n states

Grp-hw2

- Construction of kleene star
    - Add a new start state q_0, which is also a final state (because * accept epsilon)
    - Every transition stays the same
    - Add a spontaneous move from the original final state to the original start state

Indi-hw3

- Pumping lemma:
    - $S = xyz$, $|y| > 0$, $|xy| \leq p$, xy^iz is in L
- The stack alphabet of PDA is any finite superset of all symbols appeared.

Grp-hw3

- Pumping lemma:
    - If L has a pumping lemma p, then every string of length ≥ p, can be pumped.

Indi-hw4

- Every NFA can be converted to a PDA by not touching the stack

Grp-hw4

- To match the number of symbols, design a PDA only pushes the same symbol, and pops the same symbol.
- There exists a CFL over ∑ that is regular;
- There exists a CFL over ∑ that is non-regular
- Every regular language is CFL.

Indi-hw5

- Once TM hits special states, effect taken immediately.
- For a TM M that decides ø, M has to halt and reject every input.
- Halt → no loop

- Loop ex:
    - Self-loop, and write the same symbol when at the first symbol, move left.

Grp-hw5

- Configuration:
    - (input) + current state + the symbol of controller points to + (input)
- Decidable language closed under Kleene Star
    - If L is decidable, L* is also decidable
    - Proof:
        - Let D be a TM that decides L, then construct a TM D' =
        - "On input n:
            - If |n| = 0, accept
            - For all possible splits of w:
                - If D accepts all possible split, accepts
                - If D rejects, rejects;"

Indi-hw6

- If a language is recognized by some PDA, then this language is CFL
- Every CFL is decidable

Grp-hw6

- Decidable language is closed under complementation
- **According to indi-hw6, every CFL is decidable. By contrapositive, if a language is undecidable, then it is non-CFL.**
- There exist some non-CFL that are undecidable
    - Ex: A_{TM}
- There exist some decidable language whose subset is undecidable
    - Ex: $\sum^*$ and A_{TM}
- Ø is a subset of every set
- High level description
    - No states
    - No tape
- Prove decidability
    - Construct a TM M that always halts
    - Take advantage of A_{DFA}, E_{DFA}, EQ_{DFA}, …

Grp-hw7

- E_{TM} is unrecognizable and undecidable
- A_{TM} is recognizable and undecidable
- If A mapping reduces to B, and B mapping reduces to A, then A and B are on the same difficult level
- No languages can map reduces to ø except itself.
- No languages can map reduces to $\sum^*$ except itself.

- If A reduces to B, then it means that A is no harder than B.
- $HALT_{TM}$ only means TM M halts on input, does not necessarily accept or reject.
- **EQ_{CFG} is undecidable**