Collaborative Study Guide CSE 20

Add topics you think that are worth including on an index card or questions you have about certain topics and any answers you find or problems you come up with. Do whatever you want that is productive and helpful for both you and everyone else here. Good Luck on the midterm! - Kevin- $\sqrt{\frac{1}{6}}$

NOTEWORTHY TOPICS

- **Quantifiers ^ Predicates** (Do 1.4 problems 17, 29, and 39 for review)
  - Quantifiers
    - Existential (∃) - There is
    - Universal (∀) - For all
    - Don't forget nested quantifiers, (∃ x ∃ y, etc. 1.5 problem 9, if you can do this without confusion, you're good.)
      - 9. Let L(x, y) be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements. a) Everybody loves Jerry ∀xL(x, Jerry)
    - De Morgan-ish
      ¬∀x P(x) = ∃x ¬P(x)
      ¬∃x P(x) = ∀x ¬P(x)
    -
- **Proof**

Theorem: $\forall x P(x)$ over a given domain.
Strategy (1): Let x be arbitrary element of the domain. WTS P(x) is true.
Strategy (2) if domain finite: Enumerate all x in domain. WTS P(x) is true.
Strategy (3) Proof by contradiction: Assume there is an x with P(x) false. WTS badness!

Theorem: $\exists x P(x)$ over a given domain.
Strategy (1): Define x = …. (some specific element in domain) WTS P(x) is true.
Strategy (2) Proof by contradiction: Assume that for all x, P(x) is false. WTS badness!

Theorem: $P \to Q$ over a given domain.
Strategy (1): Toward direct proof, assume P and WTS Q.
Strategy (2): Toward proof by contrapositive, assume Q is false and WTS P is also false.
Strategy (3) Proof by contradiction: Assume both P is true and Q is false. WTS badness!

(Source: Lecture 11, slide 4)

Credit: Karapet Topchyan

- Direct Proof
    - **Assume the hypothesis, prove the conclusion**
    - **A direct proof shows that a conditional statement p → q is true by showing that if p is true, then q must also be true, so that the combination p true and q false never occurs.**
    - **EX.** Prove that if 5 divides 2a for a being an element of an integer, then 5 divides a
        - Assume 5 divides 2a (2a % 5 == 0) is true, then WTS is that 5 divides by a. Since 2a is divisible by 5 this means that 2a = 5j, j being an arbitrary positive integer. We can also state that j is even because 2(a) is even (lemma) and 5 is odd, so for 5j to be even, j must be even, since odd * even is even. By definition of even, we can rewrite j to be 2k, k being an integer. 5(2k) = 2a. Expand this out and you get 10k = 2a, which means a is equal to 5k. 5 time any integer is a number divisible by 5, therefore, a is also a number divisible by 5.
- Contrapositive Proof
    - **make use of the fact that the conditional statement p → q is equivalent to its contrapositive, ¬q → ¬p.**
- Contradiction Proof
    - **we first assume the negation of the conclusion. We then use the premises of the theorem and the negation of the conclusion to arrive at a contradiction.**
        - **In a proof of p → q by contradiction, we assume that ¬q is true**

- **Lemmas**
    - Rational Numbers
        - To be rational a and b cannot have any common factors,
    - Even numbers
        - If some integer k is even, it must be divisible by 2, such that we can write $k = 2c$ where c is an arbitrary integer. $k \bmod 2 = 0$
    - Perfect Squares:
        - An integer n is a perfect square iff $n = c^2$ (where c is an arbitrary integer)
    - Irrational Numbers:
        - $\sqrt{2}$ is not rational
            - Corollary: There are irrational numbers x,y such that x^y is rational

Credit: Karapet Topchyan

- **Strong Induction**
    - P(i) true for all $1 \le i \le k$ then P(k+1) is true
    - Assume that base case through k is true to prove k+1. Useful for multi-proof problems
    - To show that some statement P(n) is true about all positive integers n,
        - Verify that P(1) is true.
        - Let k be an arbitrary positive integer. Show that if [P(1)^P(2)^....^P(k)] then P(k+1) is true. This is the strong induction hypothesis. Difference being that we can use all *k* statements to prove P(k+1), rather than just the statement P(k)
            - Ex. Every positive integer can be written as the product of primes.
            - Show that if *n* is an integer greater than 1, then *n* can be written as the product of primes.
            - Solution: Let P(*n)* be the proposition that *n* can be written as the product of primes.
            - Basis Step: First case we need to establish is P(2) as it is the first prime number case. It can be written as 2 * 1, which is a product of primes
            - Inductive Step:The inductive hypothesis is the assumption that P(j) is true for all integers j with $2 \le j \le k$, that is, the assumption that j can be written as the product of primes whenever j is a positive integer at least 2 and not exceeding k.
            - Every amount of postage of 12 cents or more can be made with just 4 and 5 cent stamps
            - Base Step: Prove that 12, 13, 14, 15 are true
- **Structural Induction**
    - To prove that P(n) is true for all positive integers n, where P(n) is a propositional function, we complete two st
- **Recursive Definition**
    - **Recursion -** defining an object in terms of itself.
    - Recursively Defined Functions
        - BASIS STEP: Specify the value of the function at zero.
        - RECURSIVE STEP: Give a rule for finding its value at an integer from its /values at smaller integers. (Inductive definition)
            - Recursively defined functions are well defined. Unambiguous
- **Induction Proof**
    - BASIS STEP
        - Establish that the statement is true about 0
    - INDUCTION STEP
        - Assume induction hypothesis () P(k), WTS: P(k+1)

Credit: Karapet Topchyan

- Ex. Prove that $2^n<n!$ For n being an element of all positive integers (**Z+**) and n is greater than 3
- Base case would be n = 4
    - LHS: $2^4=16$ < RHS: $4! = 24$
    - BASE CASE holds true
- Inductive Hypothesis: Assume n = k is true, WTS: k+1 is true
    - $2^k < k!$ Is true
    - $2^{(k+1)} < (k+1)!$ Expand this out $2^k * 2 < k! * (k+1)$. Since we know that $2^k < k!$ Is true and k + 1 is a number greater than 4 we can conclude that 2* a number (2^k) is less than k+1 * a number (k!).

- **Sets**
    - Definition: unordered collection of elements
    - Empty Set: There is not x where x is an element of A and x is an Element of B
        - Cardinal value (i.e. size) of the empty set is 0
        - $\varnothing = \{\} = \{x : x \neq x\}$
    - Power Set:
        - Size of a power set is $2^{|a|}$, where a is a set
    - Cartesian Product: $|A \times B| = |A| \times |B|$
    - Rational Numbers:
        - **Z** integers
        - **Q =** {p/q | p $\in$ **Z**, q $\in$ **Z**, and q $\neq$ 0}
            - and p, q are in lowest terms (no common factors)
        - Lemma: $\sqrt{2}$ is not rational
            - Corollary: There are irrational numbers x,y such that x^y is rational
    - Subset:
        - $A \subseteq B$ means $\forall x(x \in A \rightarrow x \in B)$
    - Operations on Sets:
        - $A \cap B = \{ x \mid x \in A \wedge x \in B\}$ (Intersection of sets-results in a set that has elements that are both in set A and set B)
        - $A \cup B\backslash = \{ x \mid x \in A \vee x \in B\}$ (Union of sets- results in a set that has elements that are in either set A or set B or in both)
        - $A - B = \{ x \mid x \in A \wedge x \notin B\}$ (Difference of A and B- results in a set that has elements that are in set A and at the same time not in the set B)
        - $A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$ (Cartesian Product- results in a set with ordered pairs in which the x coordinate comes from the set A and the y coordinate comes from the set B)
- **Binary Strings**
    - Lambda is the empty string
    - Kleene Star
    - $2^k < k!$ is true

<u>**Credit: Karapet Topchyan**</u>

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- **Definitions**
    - Divisibility: a|b ("a divides b") , aka $\frac{b}{a} \varepsilon$ **Z** (b is an integer multiple of a)
    - Evens and odds
    - Disjoint if intersection of two sets is empty

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