Sophomore Housing Assignments Using TTC-based Partitioned Housing Markets

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1 Aims

Within the current housing assignment system at Harvard College, the matching between blocking groups and houses is a quasi-random process. There are 12 undergraduate houses (and one additional house that is inaccessible via lottery). These houses are partitioned into 4 neighborhoods, each with 3 houses. Each student chooses other students with which to form a *blocking group* of size $1 \leq s \leq 8$, and each blocking group enters a lottery, in which they are assigned a house, more or less at random [1]. There are also *linking groups*, in which two blocking groups can choose to link together, guaranteeing that the two blocking groups will be placed in houses P and Q respectively, with $P \neq Q$, where P and Q are in the same neighborhood.

Blocking/linking groups do not get to submit preferences over which houses they would like to enter. Unfortunately, one of the neighborhoods (the Quad) is quite far away from much of the rest of campus; some students mind this a lot, and others do not care as much about this problem. However, these students are assigned to a Quad house or non-Quad house more or less at random, which is not student-optimal in terms of satisfying preferences. We would like to investigate alternative mechanisms for reassigning housing after the initial random assignment that might be more student-optimal, and understand some of the advantages and tradeoffs that must be made to adopt such approaches.

One of the mechanisms that has been well-explored in the area of assignment and exchange is the top-trading-cycles mechanism. This paper will have three main contributions.

- **Exploring existing TTC mechanisms:** Since the formalization of the TTC mechanism by Scarf and Shapley (1974) [2], the TTC has been applied in multiple contexts. One of the most salient is the school-choice context, in which students and schools with a limited number of positions submit preference orderings and are matched according to a modified TTC mechanism. By understanding the limitations of TTC mechanisms within this context, we will develop a robust top-trading-cycles variation that serves the housing allocation issue at Harvard.

- **Proposal and analysis of modified TTC mechanisms for Harvard's housing allocation:** Within our theoretical analysis, we will analyze multiple axes of our own modified TTC mechanisms. We will propose two mechanisms that Pareto dominate both Harvard's current black box assignment (which may be an RSD assignment) in all of our empirical results (unless the Harvard assignment or RSD assignment is already Pareto optimal). We will also address fairness, computational feasibility, and how our mechanism considers diversity to improve student-interaction. Our analysis highlights the interplay between achieving theoretical benchmarks and practical implementation concerns.

- **Empirical analysis of modified TTC mechanisms for Harvard's housing allocation:** We provide a thorough breakdown of our TTC mechanisms. Through our code, we also provide results that show the number of trades that occur. We measure the total utility to show that our mechanism optimizes trades relative to a baseline method, thus supplementing the theoretical analysis of limitations to follow.

These three aims provide us with a fundamental and relatively comprehensive understanding of the existing TTC mechanisms, and allow us to better contextualize our modified TTC mechanism within existing literature and show its usage for specific use cases.

2 TTC in Literature

The Top Trading Cycles (TTC) mechanism is widely considered for conducting swaps on items assigned to agents when money is not involved and all agents have already been assigned an item. The classic example of the application of TTC is that of a housing market, formed after agents have been assigned houses (perhaps by RSD or some other mechanism). The top-trading cycles mechanism proceeds as follows: all agents make simultaneous claims about their preference order on items. In each round, we form a directed edge from each agent to their most preferred item, and remove all agents that form a cycle and trade. This process is repeated until no agents are left. For the housing markets problem, this algorithm is core-selecting, strategy-proof, Pareto-optimal and individual-rational (Parkes). As such, this combination of properties provides a foundation for a trading mechanisms that initially appears suitable for the trades we focus on in the Harvard housing assignment problem.

To better understand how to integrate our restrictions regarding the size of blocking groups, we look at different modifications of TTC. For instance, an extensive corpus has been written about the school assignment problem, which is a framework with which to analyze variations of the TTC mechanism. The TTC mechanism for school assignment proceeds as follows. Each student points at their most preferred school and each school points at the student that it most prefers. Because of the finite number of students and schools, there must exist a cycle, and the algorithm assigns students to the schools the students point to, reducing the capacity of each school as this mechanism matches students and schools. Schools with full capacities drop out over time, changing the graph. This mechanism is Pareto optimal, strategy-proof, and individually rational. We ended up proposing a few mechanisms where houses do not have preferences over students, making this different from two-sided matching.

Previous papers have approached similar issues with assigning desirable goods to agents within the school choice debate. In 2005, Abdulkadiroglu et al. published a paper detailing TTC as a strategy proof mechanism for matching between students and schools that is more robust than the Boston School Mechanism (a deferred acceptance algorithm). [3]. Pycia and Unver (2011) studied modifications of TTC to deal with school choice [4]. However, in their model, different agents cannot own the same house/commodity. That is, they study mechanisms that match students with schools that are so-called submatchings: no two agents will receive the same house. They were able to obtain a mechanism called the Trading Cycles (TC) mechanism that is individually strategy proof and Pareto efficient for this problem. According to the background knowledge we have accumulated for this task, we therefore find a gap in the current literature we focused on addressing through alternative TTC mechanisms, as evaluated against a basic RSD mechanism.

3 Our Mechanism Model

Our case study differs from the above papers because in this setting, the number of blocks (agents) is far larger than the number of houses, and blocks have joint ownership over the same house as opposed to individual ownership. In our market, each house represents a cluster of blocking groups that may want to trade out of the house. These groups may be different sizes, and will point to a house instead of a specific group to trade into within that house. However, we saw some great potential in adapting TTC-like mechanisms for housing allocation, and from these papers we decided that we would propose similar schemes where all blocking groups submit preferences, we proceed with a black box (perhaps random) assignment of houses to blocking groups, and then we run a TTC variant in order to facilitate exchanges that are strictly preferable for all blocking groups involved. This would allow us to generate housing allocations that Pareto dominate a black box assignment or any random assignment of houses that is not already Pareto optimal.

4 Assumptions and Fundamental Design Decisions

We make the assumption that individuals in a blocking group form a united front and agree on one set of submitted preference orderings. We believe this is a fair assumption to make because if an individual cares more about their own choice of house than the people with whom they are blocking, they would have the opportunity to simply join another blocking group that plans to submit an alternative set of preference orderings over houses, or they could form their own blocking group of size 1. Additionally, it is reasonable to assume that a blocking group would only even enter a market if the group decided to do so as a whole; requiring a form signed by all members of the block would be an easy way to assure consent of all members of the block. Furthermore, we assume that the preferences are strict over all of the houses, and that any improvement over the original house is desirable.

When deciding how to evaluate our model, we limited the total blocking group range from $[100, 400]$, as indicative as what we believe to be the minimum and maximum number of groups that would have preferred to participate in the trading cycle. Our code would continue to work for any number of groups up to 10000 groups (according to what we tested), and reaches reasonable results, but it is important to take note that if we have an extremely limited number of groups, we observe that the groups will not be able to trade because the market is not liquid enough. Our model can also be expanded to different ranges of blocking groups, without a major change in runtime, to account for related situations within other contexts.

To understand how our TTC mechanism compares to other existing mechanisms, we use a quasi-RSD mechanism that trades with the same preferences. Mechanism 3, which serves as a non-TTC baseline, gives us an understanding of how a simpler RSD-based mechanism might also function within a similar context (and which we conjecture to possibly be strategy-proof).

5 Mechanism 1: Random Rankings Over Blocking Groups

First, each blocking group submits a single strict preference ordering over all houses.

We partition the set of all blocking groups into submarkets, where each submarket contains only blocking groups of a single size. Thus there will be 8 submarkets for the blocking groups of size 1-8. The motivation for this scheme is as follows. We want to be able to facilitate trades between blocking groups in house H_1 with blocking groups in house H_2, where the total size of the groups in H_1 is equal to the total size of the groups in H_2. The issue is that if we allow for all possible combinations of sizes, then the problem becomes combinatorially difficult. However, if we just allow for trades between pairs of blocking groups that add up to the same size, then the number of possible trades is more feasible to deal with. We start with blocking groups whose total sizes are large, and only trade among them, because as trades occur, large blocking groups have fewer and fewer opportunities to undergo trades, while small blocking groups will have many opportunities to be combined with other small groups as part of trades.

Furthermore, this particular design is suitable because it allows for combinations of smaller groups that could trade with larger groups. In our model assumptions and in the code implementations, we choose to ignore this aforementioned combinatorial trading for two main reasons. One reason is that if we allow for combinations of groups to trade with each other (e.g. groups of sizes 3,5 to trade with groups of sizes 2,3,3), finding optimal trades becomes much more computationally expensive due to the combinatorial nature of finding suitable trades. Another reason is that from a practical standpoint, it is not guaranteed that rooming assignments are tradable within houses; two groups of sizes 3,5 may find it hard to trade with three groups of sizes 2,3,3 because there aren't suites and rooming configurations within the houses that work to make this transfer possible.

Consider one such submarket. Every house will assign a random ordering over blocking groups within the submarket. Each house will keep a ledger that contains the ordering of each blocking group. This ledger is sorted by a random dictatorial ranking mechanism within the house. The top group (by ordering) in each house points to their most desired house. As a result, there should be 12 groups (one from each house), each pointing to its most desired group with the same size in the most desired house. If this house is their current house, then they leave the market. If a cycle is detected, the blocking groups along that cycle will trade on that cycle, and exit the market. Each house that was involved in the cycle then moves

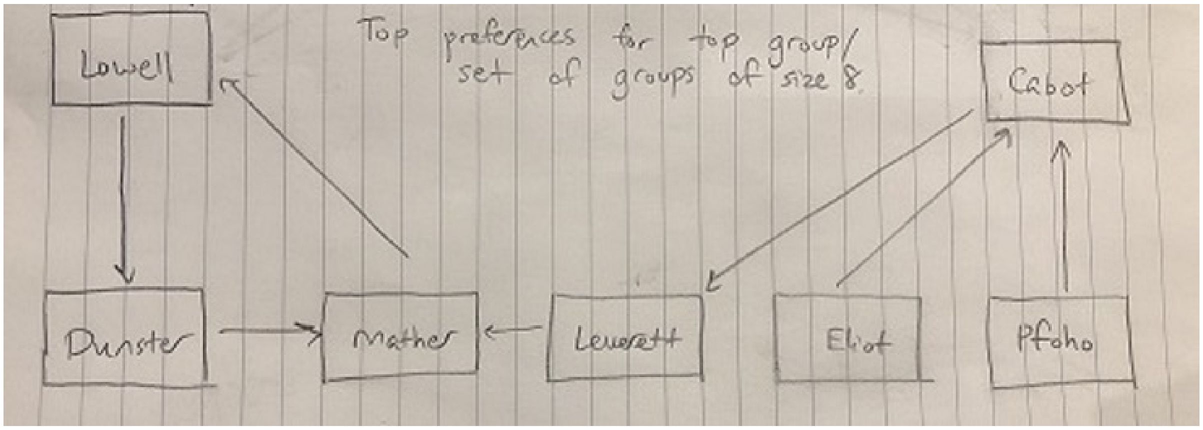

Figure 1: Round k in TTC mechanism, where the top group in each house currently trading in the size 8 trading points to their most preferred house. Notice, only the top 8-size groups will be traded amongst Lowell, Dunster, and Mather. Here, each house name represents a blocking group of size 8 from each of those houses that is currently pointing to their most preferred mechanism.

to the next block in the ledger within each house. We will keep looking for cycles and repeat until there are no more cycles.

When there are no more cycles and the mechanism has stalled, we pick a house uniformly at random from the houses that are not currently on the bottom group in their ledger. We move to the next highest group in the random dictatorial ranking by ranking in this house, and have it point to its most desired house. We then look for cycles and enact trades on these cycles until none remain. We repeat the above process until we reach the bottom group on the ledger in each house.

After running this mechanism on the most-preferred house for each blocking group within each house, we have each blocking group that is currently untraded point to their second most preferred house. We run the exact same process as above. We repeat, traversing down the preference orderings for remaining groups until either they prefer their own house more than any other swap among remaining houses that still have actively-looking groups. In this way, we do not consider trades between groups with different positions in their preferences - for instance group A from Eliot which prefers Dunster the most (rank 1) would not trade with group B from Dunster that prefers Eliot with preference 2. Resolving this limitation would require much more computation and complexity, but could be improved with future models.

6 Mechanism 2: Weighted Directed Graph on Houses

We explore an alternative TTC-based model.

All blocking groups first submit a strict preference ordering over all houses. They are then assigned houses through a black box mechanism according to the way that Harvard currently assigns housing. After the initial housing allocation, trades occur according to this modified mechanism. Each house is modeled as a node in a graph. We then run a modified version of the TTC mechanism, as follows. Each blocking group *points* to their most desired house remaining on the market. We have a separate submarket for groups of each size. For example, blocking groups of size 8 will only trade with other blocking groups of size 8. Thus, for each size s of blocking groups, there is a corresponding submarket.

Consider one submarket. For each pair of houses (A, B), we count the number of requests from house A to house B, and form a directed edge (A, B) with weight corresponding to the number of such requests. We then find cycles in our graph where each edge in the cycle has positive weight. Suppose we find a cycle with minimum weight w. Whenever we detect a cycle, we trade w blocking groups along each edge in the cycle (where we can choose any of the available blocks for trade to do this with). These blocking groups leave the market.

Thus we modify our graph, subtracting out the weights from edges where necessary to represent this change. If an edge has zero weight after subtracting, it is removed from the graph. We continue finding cycles, doing the above until we cannot find any more cycles. Then, we move on to the next round of TTC, and each blocking group points to their most desired house still on the market (i.e. there are still blocking groups in the market in that house). We run the above once again, and finish when everyone has left the market or there are no more changes that could be made in order to trade among the remaining groups.

See Figure 2 for an example of this mechanism in action.

7 Mechanism 3: RSD-based design

We explore an RSD-based mechanism design that serves as an alternative to the previous TTC-based models that we have designed. All blocking groups report a strict preference ordering over houses. We again break down the mechanism into the submarkets of differently-sized blocking groups. For each blocking group size, the mechanism runs as such:

- For each individual submarket (e.g. the submarket considering only blocking groups of size 8), we choose a random blocking group, group A from one house, house D. Each blocking group then points to their most preferred house, house E. Note that this mechanism chooses one house at random first, and then chooses a blocking group within that house at random, which means that a skew in the distribution would not be accounted for when choosing a house uniformly at distribution).
- Within this group's most-preferred house, we find whether there exists blocking groups that currently prefer house D to house E, and we find the group that most-prefers this change (ie. group D is ranked highest among the ones that prefer D to E. Let us denote this blocking group as group B.
- We conduct a swap between these two houses between group A and group B. If there are not suitable blocking groups, we restart the loop and wait to see whether there are groups that would trade with this house's lower preferences in later rounds.
- We restart this loop, and this loop continues until all individuals have been considered within the loop.

This mechanism is "RSD-based" because in every round, the master of the lottery runs which house to choose their top preference. It then chooses a person to swap with in the most-preferred house based on the other individuals' rankings. Its outcomes are generally deterministic when all preferences are finalized and the rules of the mechanism completely govern the shifting of the blocking groups between houses. This quasi-dictatorial mechanism therefore allows for more control by the people running the mechanism in choosing which groups will leave the group, perhaps with the pursuit of achieving some strong diversity baseline. We conjecture and wish to show in future work that this mechanism has properties of strategy-proofness, by comparison with current mechanisms that are strategy-proof.

8 Empirical Results

Access to code: `https://github.com/dylanli073/housingTTC`

We calculated this metric by dividing the total number of students swapped over the number of students that were considered. If we consider the expected number of students per blocking group to be 4.5 students, given a uniformly distributed sizing of blocking groups, we get that the expected total number of students per 100 blocking groups is 450 students. Therefore, we consider the number of students traded in every mechanism over 10 averaged trials, and use that to calculate the average percentage of students that have been traded for every number of blocking groups.

We have empirical results for Mechanism 1, Mechanism 2, and Mechanism 3 (Baseline), for blocking groups (without considering linking groups). We generated blocking groups with preference orderings and

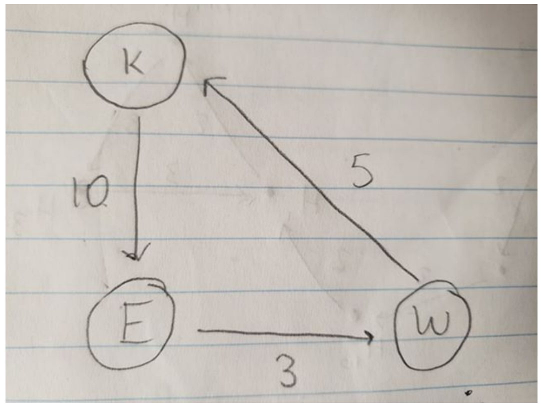

Figure 2: An example of Mechanism 2 in action. Suppose in some round, 10 Kirkland blocks point at Eliot, 3 Eliot blocks point at Winthrop, and 5 Winthrop blocks point at Kirkland. If this cycle is found, then 3 blocks each are traded from Kirkland to Eliot, from Eliot to Winthrop, and from Winthrop to Kirkland. 3 is the minimum capacity of edges along this cycle.

block sizes uniformly at random, and varied the number of blocking groups from 100 to 400 by increments of 50. We generated 10 samples for each number of blocking groups and assigned them houses randomly. Then we ran the mechanism and took the average percentage of students (not blocks, but students) that were part of some swap in the mechanism. This indicates the proportion of students who ended up in a better house than their originally-assigned house. Those who do not trade remain with their current housing. All graphs here show that our mechanisms Pareto Dominate the existing Harvard mechanism, as we build on those results with these mechanisms.

See Figure 3 for a graph of our results for Mechanism 1. We found that for the first mechanism, assuming the existence of 400 blocking groups, on average, 46.7% of students were swapped into a better house, indicating a good amount of liquidity.

See Figure 4 for a graph of our results for Mechanism 2. We found that for the first mechanism, assuming the existence of 400 blocking groups, on average, 75.2% of students were swapped into a better house, indicating extremely high liquidity. This mechanism produced our best results empirically, with greater liquidity (proportion of people involved in trades) than either Mechanisms 1 or 3. We believe this is because in this model, when we find cycles, we trade the largest possible number of groups along that cycle as we can, while in the other models, we only trade along cycles where one block moves along an edge at a time.

See Figure 5 for a graph of our results for Mechanism 3. We found that for the third mechanism, assuming the existence of 400 blocking groups, on average, 55.8% of students were swapped into a better house, indicating better liquidity than Mechanism 1. In all three cases, as we increase the number of blocking groups, there are more groups to possibly do trades with, creating more liquidity, and this explains why the proportion of people who are included in trades increases alongside the number of blocking groups. We should expect that for uniformly randomly generated initial assignments to houses for blocking groups, that the proportion of people traded should not exceed 11/12, since 1/12 of people on average will be given their first preference house, and this is what we observe in our best mechanism (Mechanism 2), where the proportion of people involved in trades continues to increase with the number of blocking groups on the market, but where the rate of increase slows down as the proportion approaches 80%. Additionally, the rate of increase towards 11/12 was closely fit by a linear model for the first mechanism and a log model for the second. It is important to note that neither of these would hold as the number of blocking groups grows since they trend above 1, but they give a good approximation for how the proportion would grow given realistic market participation.

Observe here that the housing TTC mechanism performs differently between the two TTC mechanisms and the RSD-based mechanism. Specifically, we see that trading with weighted cycles performs the best.

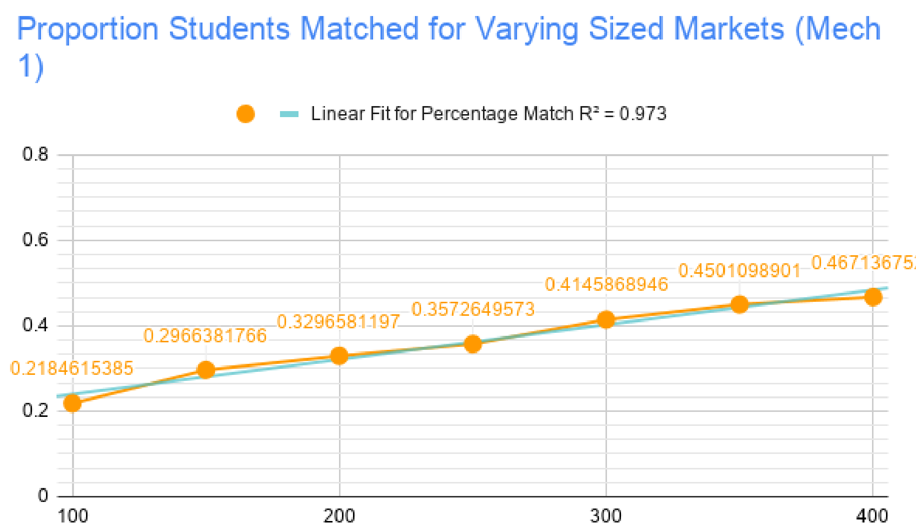

Figure 3: Mechanism 1. The y-axis is the proportion of students who undergo swaps, and the x-axis is the number of blocking groups.

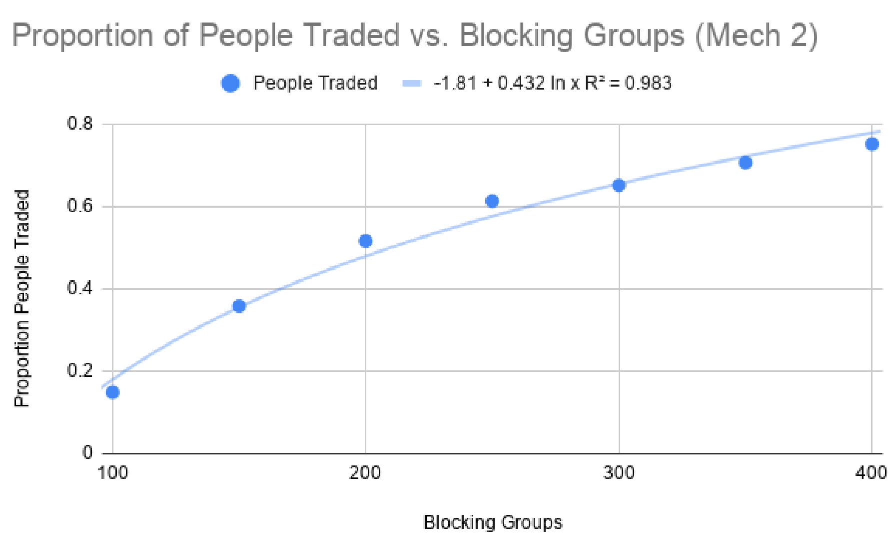

Figure 4: Mechanism 2. The y-axis is the proportion of students who undergo swaps, and the x-axis is the number of blocking groups.

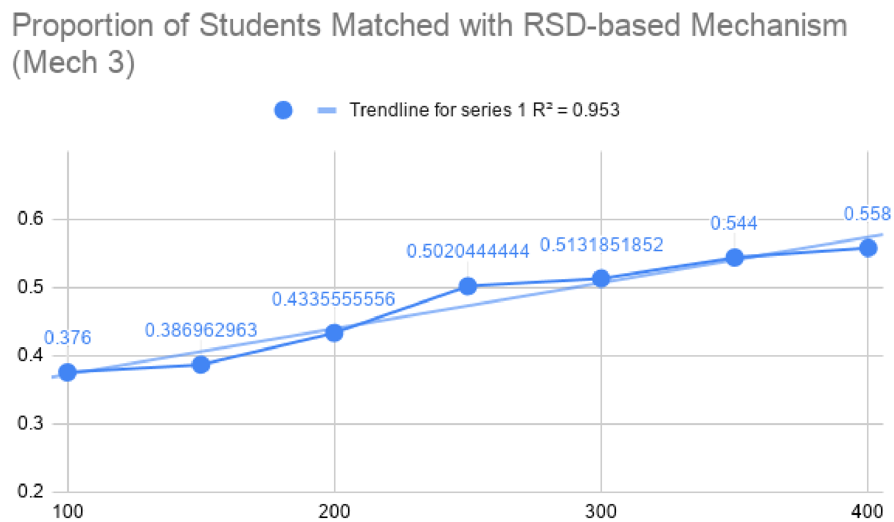

Figure 5: Mechanism 3 (Baseline). The y-axis is the proportion of students who undergo swaps, and the x-axis is the number of blocking groups.

Because it considers everybody's preferences at once, it allows for increased liquidity compared to Mech 1, which trades one group per house at a time. As a result, it is not surprising that the proportion of people who are trade starts to approach a limit as the number of blocking groups increases. This percentage of people traded is best-represented as a log-model as a first-approximation for these cases in the range of $[100, 400]$.

We also observe that mechanism 3 performs significantly better than mechanism 1, which was originally surprising to us. However, upon further inspection, this makes sense because it matches groups that have each other ranked differently. For instance, group A in Dunster can trade with group B in Eliot, even if group A prefers Eliot the most and group B prefers group A third in their preference ranking. As a result, this market has greater liquidity that can is reflected in the empirical results. We hypothesize that in future tests, the TTC-based Mech 1 that relaxes this constraint (which could be computationally difficult to achieve) would perform at least as well. Notice that mechanism 2 (a TTC-based mechanism) performs extremely well relative to the RSD-based one, perhaps because it considers everybody's preferences at the same time without constraint.

It is difficult to choose a specific mechanism to use within the final housing market system. However, based on these empirical results, it appears fundamental to the efficiency of the market that the restrictions in trading among groups be lifted, perhaps even more so than the specific type of mechanism use. Indeed, it appears that mechanism 2, which has some of the fewest restrictions in trading scores the highest, and mechanism 1 with the most constraints performs the poorest in terms of matching individuals. We hope to explore this work further by relaxing constraints on mechanism 1 and tuning the total liquidity at any point within each of the markets created.

9 Discussion and Limitations

One problem with both Mechanisms 1 and 2 is that it does not maximize the total number of trades that can occur, at each step of the modified TTC (i.e. it is not Pareto optimal). In particular, we do not find cycles to trade on in any particular order; we just trade whenever we find cycles. Once we select a cycle to do trades on, we may not be able to reuse an edge because all possible blocking groups along that edge may have been exhausted.

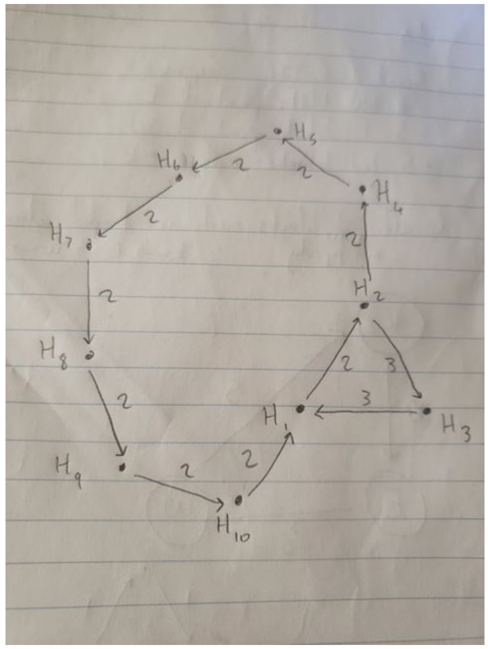

Figure 6: An example of what could go wrong with arbitrary order of cycle detection.

See Figure 6. For a relatively extreme example for Mechanism 2, suppose in the blocking group market, edge (H_1, H_2) has weight 2, edge (H_2, H_3) has weight 3, and edge (H_3, H_1) has weight 3. Suppose also that we have a cycle $H_1 \to H_2 \to H_4 \to \cdots \to H_{10} \to H_1$. Then if we discover the short cycle $H_1 \to H_2 \to H_3 \to H_1$ first, we will trade along that cycle first, trading 2 blocks along each edge, for a total of 6 blocks traded to more preferred houses. However, we will no longer be able to trade along the cycle $H_1 \to H_2 \to H_4 \to \cdots \to H_{10} \to H_1$ because now (H_1, H_2) has no blocks with which to trade. This does not maximize the number of possible trades; it would have been far better to trade 2 blocks along each edge in the long cycle, for a total of 18 trades, and so in this case, such a mechanism may be arbitrarily suboptimal (if in theory we had more houses, and put those houses on the long cycle).

This is not resolved by a greedy approach either. One greedy approach could be to find all simple cycles in the graph, and then rank them in decreasing order of how many trades they could elicit by choosing them.

However, as we can see in the example with Figure 7, this would not lead to optimal trades either, in either Mechanism 1 or 2.

We suspect the problem of finding the edges that maximize the total number of trades is intractable. Our model simply guarantees that we will find a cycle if it exists, and maximize the number of trades along the found cycles.

An obvious limitation with Mechanism 3 is that, even though it is relatively simple to implement and generates even more trades than Mechanism 1, it misses cycles of any length greater than 2, and so is clearly not Pareto optimal either.

9.1 Diversity

Harvard once allowed students to submit preferences over houses when doing sophomore housing. The primary reason that this system was abandoned in lieu of alternative methods that were more or less random seems to be that they felt that certain houses developed certain reputations (e.g. Eliot House developed the reputation of being elitist and the top preference of rich students).

One way we might be able to mitigate this is to create further trading markets based on various factors and axes that Harvard wishes its houses to be diverse in, like gender, race, ethnicity, income, religion, etc.

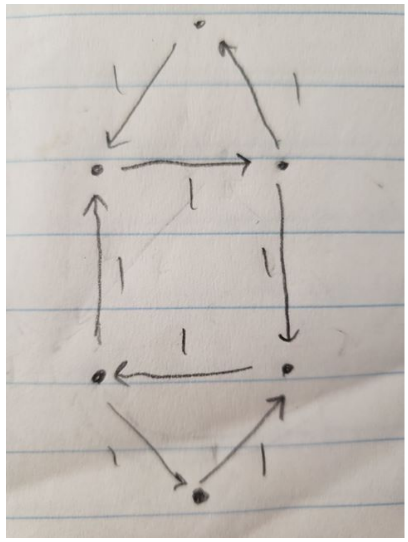

Figure 7: In this example, a greedy process would select the middle square cycle, for a total of 4 trades, leaving the two 3 cycles above and below infeasible, even though if we traded along these cycles, we would obtain a total of 6 trades.

The tradeoff is, of course, that with smaller and smaller markets, the number of possible trades might decrease, leading to an outcome that looks more and more like the original randomized black box house allocation. The design decision on part of the administration would then be to decide where to settle on the spectrum between random assignments and student-preferring optimal assignments.

In this particular case, we actually claim that using these submarkets allows housing to be even *more* mindful of diversity and the intentions of what houses want their demographic to look like. Random assignments may still accidentally lead to unbalanced demographics; by trading among submarkets that partition these demographics, university administration is forced to think clearly about the proportions of demographics in houses that are most desirable.

10 Extensions

10.1 Linking Groups

We did not write code for linking groups, but here is how we would extend all three mechanisms to consider them.

Linking groups submit a strict preference ordering over all possible pairs of houses. They will be within a separate market from blocking groups. The ordering within each pair does not matter for our purposes. We will make the simplifying assumption that linking groups care more about being together, and being in a specific pair of houses, than which house each block gets within the pair. We feel that this is a good simplification because it reduces the number of preferences that linking groups have to choose over by a factor of 2.

There are 4 neighborhoods, each with $\binom{3}{2} = 3$ pairs of compatible houses, for a total of 12 house pairs.

Linking groups will be assigned pairs of compatible houses using a black box mechanism. Then, they will trade on pairs of compatible houses. We will form a separate submarket for every possible pair of group sizes. In particular, these pairs of possible sizes are

$$(1,1),\ldots,(1,8),(2,2),\ldots,(2,8),(3,3),\ldots,(3,8),(4,4),\ldots,(4,8),$$
$$(5,5),\ldots,(5,8),(6,6),\ldots,(6,8),(7,7),(7,8),(8,8)$$

.

In practice, many of these markets will probably have very few linking groups, since for example, we expect very few linking groups with block sizes $(1, 8), (2, 8)$, and other markets will be more popular. There will be lower liquidity in this setting than in the blocking group market.

10.1.1 Mechanism 1

Consider one submarket. Every pair of houses will assign a random ordering over linking groups within the submarket. Each house pair will keep a ledger that contains the ordering of each linking group. The top linking group (by ordering) in each house pair points to their most desired house pair. If a linking group points to its current house pair, then it exits the market. If a cycle is detected, the linking groups along that cycle will trade on that cycle and then exit the market. Each house pair which was involved in the cycle then moves to the next linking group in the ledger. We will keep looking for cycles and repeat until there are no more cycles.

When there are no more cycles, we pick a house pair uniformly at random from the house pairs that are not currently on the bottom group in their ledger. We move to the next highest linking group by ranking in this house pair, and have it point to its most desired house pair. We then look for cycles and enact trades on these cycles until none remaining. We repeat the above process until we reach the bottom group on the ledger in each house pair.

Then, we have each linking group remaining point to their second most preferred house pair. We run the exact same process as above. We repeat, with all remaining linking groups pointing to their next most preferred house pair until all linking groups exit the market.

10.1.2 Mechanism 2

Each node in our graph is now representative of a pair of compatible houses. Other than that, the mechanism for linking groups works exactly the same way as the mechanism for blocking groups. If we fix a submarket, what happens is that for each pair of compatible house pairs (X, Y), we count the number of requests from link X to link Y and form directed (X, Y) with weight corresponding to the number of such requests. We then find cycles in our graph where each edge in the cycle has positive weight. Suppose we find a cycle with minimum weight w. Whenever we detect a cycle, we trade w linking groups along each edge in the cycle (where we can choose any of the available links for trade to do this with). These linking groups leave the market.

Thus we modify our graph, subtracting out the weights from edges where necessary to represent this change. We continue finding cycles, doing the above until we cannot find any more cycles with positive weight edges. Then, we move on to the next round of TTC, and each linking group points to their most desired house pair still on the market (i.e. there are still linking groups in the market in that house pair). We run the above once again, and finish when everyone has left the market.

10.1.3 Mechanism 3

For each individual submarket, we choose a (uniformly) random linking group, group A from one house pair D. Each linking group then points to their most preferred house pair E. Note that this mechanism chooses one house pair at random first, and then chooses a linking group within that house pair at random, which means that a skew in the distribution would not be accounted for when choosing a house pair uniformly at distribution). Within this linking group's most-preferred house pair, we find whether there exist linking groups that currently prefer house pair D to house pair E, and we find the linking group that most prefers this change (e.g. linking group D is ranked highest among the ones that prefer D to E). Let us denote this linking group as group B. We conduct a swap between these two house pairs between group A and group B. If there are not suitable linking groups, we restart the loop and wait to see whether there are groups that would trade with this house pair's lower preferences in later rounds. We restart this loop, and this loop continues until all individuals have been considered within the loop.

10.2 Relaxed Markets

One possible extension for both of the proposed mechanisms is to relax the assumption that only blocking groups of the same size can trade with each other. For example, we might have a class of blocking groups of large size (e.g. 6,7,8) that form a single submarket, encouraging a larger set of possible trades without running into large problems with house capacity. The flexibility of rooming assignments is the bottleneck for this being feasible. If there is a way to guarantee that groups of similar but not same sizes can trade with each other while receiving feasible rooming assignments, then this becomes possible.

10.3 Matching Students at Harvard

One difference between our simulations and reality is that, as mentioned before, students' preferences are not random. In fact, one would expect a heavy bias towards houses outside of the Quad. As a result, empirically one might see many swaps occurring among groups from the Quad entering the market in an attempt to get out of the area, but only succeeding in making a minor improvement over their current situation. Obtaining an accurate distribution of students' preferences (perhaps via survey or another method), would allow for more accurate simulations in the future accounting for students' true preferences. However, it is good to note that this mechanism remains useful for assigning goods with truly subjective values that different individuals value uniformly.

10.4 Other Applications

We may be able to use these mechanisms for other settings, such as for housing transfers among individual upperclassmen. We would remove the black box housing assignment at the beginning, since the students applying for transfer already have houses, and elicit strict preference orderings over houses by each individual.

This mechanism is also applicable for lotteries into classes: a student who has won a position in a class may trade on cycles with other students in other classes, like the fixed set of Gen-Ed classes offered each fall. The analogy would be that a position in a class has the same role as a blocking group in a particular house, and each Gen-Ed class corresponds to a house. This would allow classes to fill with as many students as intended, while at the same time allowing students to end up in strictly more preferred positions.

10.5 Future Work

We would like to explore more of the theoretical properties of the mechanisms we have here. We suggest that the RSD-based model has properties that mimic other strategy-proof models. Because it encourages students to propose their preference order truthfully, at risk of being assigned a less-desirable house, it appears that this particular mechanism could be seen as strategy-proof. In future work, we would like to prove this conjecture and verify that its theoretical properties hold with our empirical results in testing strategy-proofness. We also hope to better tune parameters of our models to change the liquidity, further confirming our observations from the data collected in this paper.

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