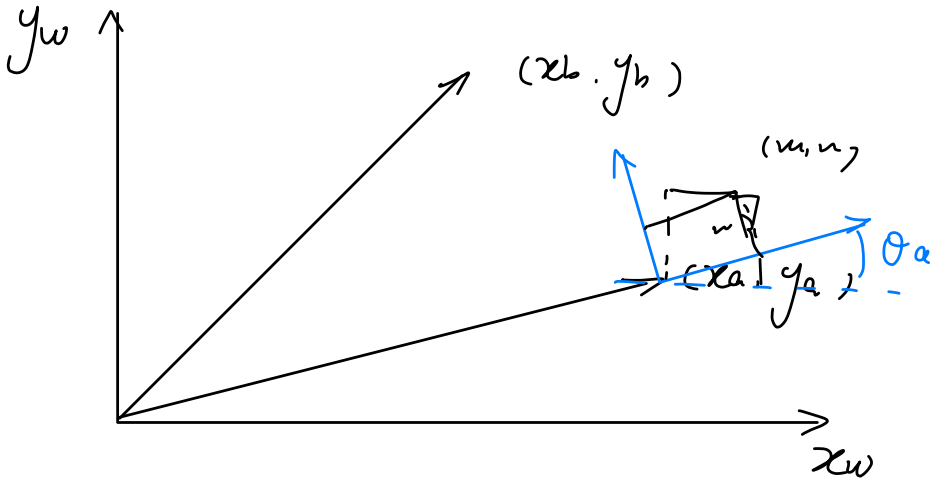
Task 3

已知 $\underline{x}_a^w = \begin{pmatrix} x_a \\ y_a \\ \theta_a \end{pmatrix}$

$\underline{x}_b^w = \begin{pmatrix} x_b \\ y_b \\ \theta_b \end{pmatrix}$



-(1.) 求 $\underline{x}_b^a$

解：由 $\underline{x}_a^w$ 得 $T_a^w = \begin{pmatrix} \cos\theta_a & -\sin\theta_a & x_a \\ \sin\theta_a & \cos\theta_a & y_a \\ 0 & 0 & 1 \end{pmatrix}$

由 $\underline{x}_b^w$ 得 $T_b^w = \begin{pmatrix} \cos\theta_b & -\sin\theta_b & x_b \\ \sin\theta_b & \cos\theta_b & y_b \\ 0 & 0 & 1 \end{pmatrix}$

b 在 a 下.

则 $T_b^a = T_w^a \, T_b^w = (T_a^w)^{-1} \cdot T_b^w$

得到 $T_b^a = \begin{pmatrix} \gamma & * & \alpha \\ * & * & \beta \\ 0 & 0 & 1 \end{pmatrix}$

可求 $\underline{x}_b^a = (\alpha, \beta, \arccos(\gamma))^T$

$$\theta_b^a = \theta_a^w + \theta_b^w$$

(2).

$k=0$ 时 $\quad \underline{x}_{a,0}^w = \begin{pmatrix} x_a \\ y_a \\ \theta_a \end{pmatrix}$

$k=1$ 时 求 $\underline{x}_{a,1}^w = T_1^{w,0} \cdot \underline{x}_{a,0}^w$

其中 T_1^0 为 $k=1$ 时刻相对 $k=0$ 时刻以 w 坐标系运动

$$T_{a1}^{a0} = \begin{pmatrix} \cos\theta_d & -\sin\theta_d & d \\ \sin\theta_d & \cos\theta_d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

现在求物体 b 相对 a_1 的坐标

可求 $\quad T_b^{a1} = T_{a0}^{a1} \; T_w^{a0} \; T_b^w$

$$= (T_{a1}^{a0})^{-1} \cdot (T_{a0}^w)^{-1} \cdot T_b^w$$

得到 $\quad T_b^{a0} = \begin{pmatrix} \gamma & * & \alpha \\ * & * & \beta \\ 0 & 0 & 1 \end{pmatrix}$

可求 $\quad \underline{x}_b^{a1} = (\alpha, \beta, \arccos(\gamma))^T$