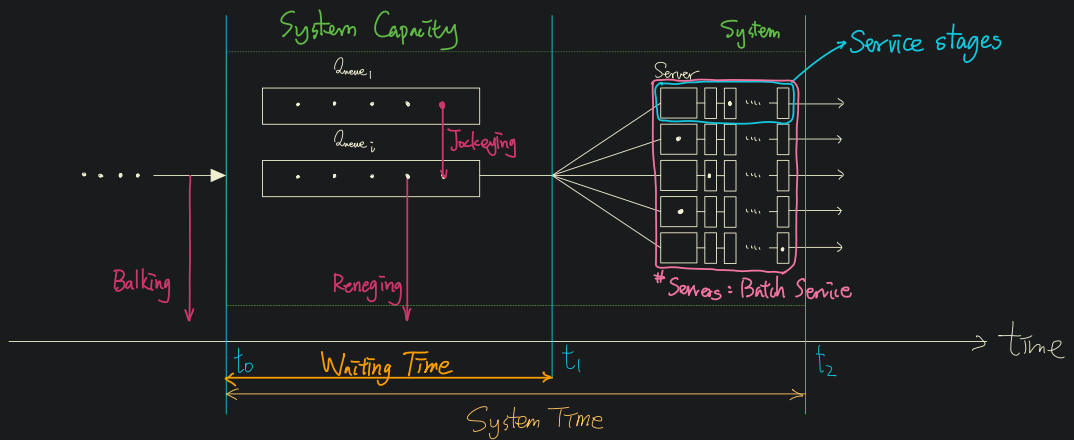
1.1 Measures of System Performance



Time Length

1. Waiting Time : $t_1 - t_0$
2. System Time : $t_2 - t_0$

Customers

1. In Queue
2. In System

Servers Time Length

1. Idle Time
2. Busy Period

Table 1.1 Queueing notation $A/B/X/Y/Z$

default ∞ default FCFS

Characteristic	Symbol	Explanation
Interarrival-time distribution (A)	M	Exponential
	D	Deterministic (Const)
	E_k	Erlang type k ($k = 1, 2, \ldots$)
Service-time distribution (B)	H_k	Mixture of k exponentials
	PH	Phase type
	G	General ⇒ A
Parallel servers (X)	$1, 2, \ldots, \infty$	
System capacity (Y)	$1, 2, \ldots, \infty$	$= \#Q + \#S$
Queue discipline (Z)	FCFS	First come, first served
	LCFS	Last come, first served
	RSS	Random selection for service
	PR	Priority
	GD	General discipline ⇒ Any

E_k (Erlang Type) : ⇒ Exp + Exp + ⋯ + Exp

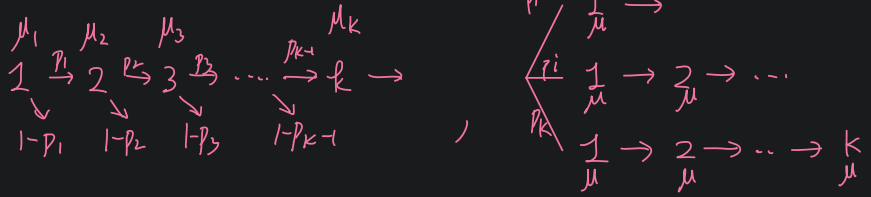
$$E_k: f(x; k, \mu) = \frac{k\mu(k\mu x)^{k-1}}{(k-1)!} e^{-k\mu x}$$

H_k (Mixture of k Exponentials) :

$$H_k: f(x; k) = \sum_{i=1}^{k} p_i \mu_i e^{-\mu_i x}, \quad \sum_{i=1}^{k} p_i = 1$$

(p_i: prob. of state; $\mu_i e^{-\mu_i x}$: state dist)

PH (Phase Type)



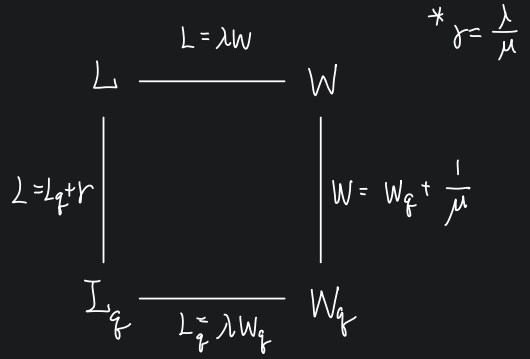
$$\begin{bmatrix} P_{11} & \cdots & P_{1K} \\ \vdots & \ddots & \vdots \\ P_{K1} & \cdots & P_{KK} \end{bmatrix} \quad P^{\infty} \to \emptyset \Rightarrow \text{will leave Markov Chain.}$$

1.4 Little Law

L : ave. # Customers [1]

λ : ave. rate [1/T]

W : ave. time [T]



* $r = \frac{\lambda}{\mu}$

Table 1.2 Summary of notation

1/t	λ	Average arrival rate
t	S	Random service time
1/t	$\mu \equiv 1/\mathrm{E}[S]$	Average service rate
n	c	Number of servers
n	$r \equiv \lambda/\mu$	Offered load
n	$\rho \equiv \lambda/c\mu$	Traffic intensity or utilization
t	T, T_q	Random time a customer spends in the system / queue
t	W, W_q	Average time a customer spends in the system / queue
n	N, N_q	Random number of customers in the system / queue
n	L, L_q	Average number of customers in the system / queue

Table 1.5 Notation and basic relationships

Variable	Definition	Sample Relationship
$A^{(n)}$	Arrival time of cust. n	
$S^{(n)}$	Service time of cust. n	
$T^{(n)}$	Interarrival time cust. n and $n+1$	$T^{(n)} = A^{(n+1)} - A^{(n)}$
$U^{(n)}$	Time cust. n starts service	$U^{(n+1)} = \max\{D^{(n)}, A^{(n+1)}\}$
$D^{(n)}$	Departure time of cust. n	$D^{(n)} = U^{(n)} + S^{(n)}$
$W_q^{(n)}$	Time in queue of cust. n	$W_q^{(n)} = U^{(n)} - A^{(n)}$
$W^{(n)}$	Time in system of cust. n	$W^{(n)} = W_q^{(n)} + S^{(n)}$