AS & A-Level Physics Lecture Notes

(Year 1)

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Contents

Practical Issues

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The latest update can be found via: https://github.com/yuhao-yang-cy/asphysics

About the Notes

The contents of the notes are consistent with the CIE A-Level physics syllabus. I attempt to systematically cover all the key points in the syllabus with brief but sufficient explanations. The notes should be able to serve as a self-contained study guide for the AS CIE course.

I am still working on the notes. The latest release is far from complete as it only covers the first few chapters. I hope I will follow up the other chapters before the end of this year.

If you spot any errors, please let me know.

Literature

I borrow heavily from the following resources:

- Cambridge International AS and A Level Physics Coursebook, by *David Sang, Graham Jones, Richard Woodside* and *Gurinder Chadha*, Cambridge University Press
- International A Level Physics Revision Guide, by *Richard Woodside*, Hodder Education
- Longman Advanced Level Physics, by *Kwok Wai Loo*, Pearson Education South Asia
- Conceptual Physics (10^{th} Edition), by *Paul G. Hewitt*, Pearson International Education
- Physics (5^{th} Edition), by *Robert Resnick, David Halliday* and *Kenneth S. Krane*, John Wiley & Sons 2002
- Past Papers of Cambridge Internation A-Level Physics Examinations
- HyperPhysics Website: http://hyperphysics.phy-astr.gsu.edu/hbase/index.html
- Wikipedia Website: https://en.wikipedia.org

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Recommended Reading

Chapter 1

Physical Quantities

1.1 units of measurement

any physical quantity contains a numerical value and its associated unit

a system of units of measurement used throughout the scientific world is the **SI units**[1]

1.1.1 SI base units

SI defines seven units of measure as a basic set, known as the **SI base units**

base quantity	base unit	symbol
mass	kilogram	kg
length	metre	m
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

1.1.2 derived units

the seven[2] SI base units are building blocks of the SI system

all other quantities are derived from the base units

Example 1.1 Give the SI base units of (a) speed, (b) acceleration, (c) force, (d) work done.

[1]SI units, abbreviated from the French *Système Internationale d'Unités*, means the International System of Units. Those who are interested in the history and evolution of the SI can check out the Wikipedia article: https://en.wikipedia.org/wiki/International_System_of_Units

[2]Luminous intensity is beyond the scope of the AS-Level syllabus. You are only required to know the other six SI base quantities and their units.

$$\text{speed} = \frac{\text{distance}}{\text{time}} \quad \Rightarrow \quad [v] = \frac{[s]}{[t]} = \frac{\text{m}}{\text{s}} = \text{m s}^{-1}$$
$$\text{acceleration} = \frac{\text{speed}}{\text{time}} \quad \Rightarrow \quad [a] = \frac{[v]}{[t]} = \frac{\text{m s}^{-1}}{\text{s}} = \text{m s}^{-2}$$
$$\text{force} = \text{mass} \times \text{acceleration} \quad \Rightarrow \quad [F] = [m][a] = \text{kg m s}^{-2}$$
$$\text{work} = \text{force} \times \text{distance} \quad \Rightarrow \quad [W] = [F][s] = \text{kg m s}^{-2} \times \text{m} = \text{kg m}^2\text{s}^{-2}$$

□

1.1.3 metric prefixes

prefixes are used to indicate multiples and sub-multiples of original units

name	symbol	meaning	name	symbol	meaning
pico	p	10^{-12}	hecto	h	10^2
nano	n	10^{-9}	kilo	k	10^3
micro	μ	10^{-6}	mega	M	10^6
milli	m	10^{-3}	giga	G	10^9
centi	c	10^{-2}	tera	T	10^{12}
deci	d	10^{-1}			

1.1.4 dimensional analysis

if an equation is correct, then the units on both sides must be the same. Such an equation with consistent units is said to be **homogeneous**.

dimensional analysis is widely used as a rough guide to check for the correctness of equations

there are times when the dependence of one physical quantity on various other quantities cannot not be seen easily, but it might give us helpful hints by merely investigating their units

- there are *unit free*, or *dimensionless* quantities that do not have units
  examples are real numbers (2, $\frac{4}{3}$, π, etc.), coefficient of friction (μ), refraction index (n), etc.
- a correct equation must be homogeneous, but the converse may not be true
  possible problems include an incorrect coefficient, extra term, an incorrect sign, etc.

Example 1.2 A ball falls in vacuum, all its gravitational potential energy converts into kinetic energy. This is expressed by the equation: $mgh = \frac{1}{2}mv^2$. Show that this equation is homogeneous.

✍ LHS: $[mgh] = [m][g][h] = \text{kg} \times \text{ m s}^{-2} \times \text{m} = \text{kg m}^2\text{s}^{-2}$

RHS: $\left[\frac{1}{2}mv^2\right] = [m][v]^2 = \text{kg} \times (\text{ m s}^{-1})^2 = \text{kg m}^2\text{s}^{-2}$

so we see the equation $mgh = \dfrac{1}{2}mv^2$ is homogeneous □

Example 1.3 The speed of a wave travelling along an elastic string is determined by three things: the tension T in the string, the length L of the string, and the mass m of the string. Let's assume $v = T^a L^b m^c$, where a, b, c are some numerical constants. Find the values of a, b and c.

✍ RHS: $[T]^a[L]^b[m]^c = (\text{kg m s}^{-2})^a \text{m}^b \text{kg}^c = \text{kg}^{a+c}\text{m}^{a+b}\text{s}^{-2a}$

for the equation to be homogeneous, we must have:

$$\text{kg}^{a+c}\text{m}^{a+b}\text{s}^{-2a} = \text{m s}^{-1} \quad \Rightarrow \quad \begin{cases} \text{kg}: & a+c=0 \\ \text{m}: & a+b=1 \\ \text{s}: & -2a=-1 \end{cases} \quad \Rightarrow \quad \begin{cases} a = \frac{1}{2} \\ b = \frac{1}{2} \\ c = -\frac{1}{2} \end{cases}$$

so wave speed is given by: $v = T^{1/2}L^{1/2}m^{-1/2}$, or $v = \sqrt{\dfrac{TL}{m}}$

this happens to be the correct formula for the wave speed on a string □

1.2 scalars & vectors

physical quantities come in two types: scalars and vectors

> a **scalar** quantity has magnitude only
>
> a **vector** quantity has magnitude and direction

➢ a scalar can be described by a single number

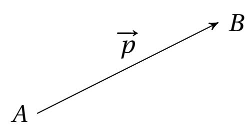
examples of scalars are time, distance, speed, mass, temperature, energy, density, volume, etc.

➢ a vector is usually represented by an arrow in a specific direction

a vector $\vec{p}$ pointing from A to B is shown

length of the arrow shows the magnitude of the vector

direction of the arrow gives the direction of the vector



examples of vectors are displacement, velocity, acceleration, force, field strength, etc.

➢ scalar algebra is just ordinary algebra

one can add and subtract scalar quantities in the same way as if they were ordinary numbers

for example, a set of objects with mass $m_1, m_2, \cdots, m_n$ have a total mass of $M = m_1 + m_2 + \cdots + m_n$

➢ vector algebra is more complicated, since we need keep track of the direction

1.2.1 multiplication of vectors

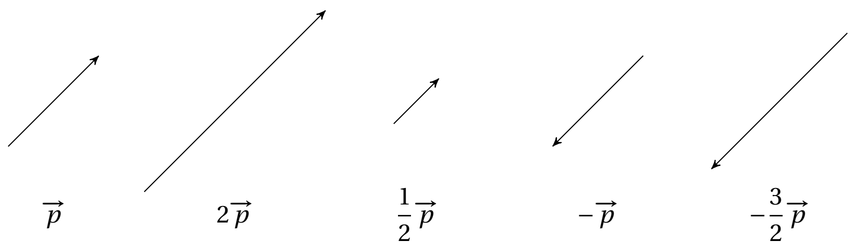
vectors can be multiplied by scalars [3]

when being multiplied by a scalar number, magnitude of the vector changes

if this number is positive, the vector becomes longer or shorter, but still points in same direction

if the number to be multiplied is negative, the operation reverses the vector's direction

Example 1.4 Given a vector $\vec{p}$, the graphical representations of $2\vec{p}$, $\frac{1}{2}\vec{p}$, $-\vec{p}$, $-\frac{3}{2}\vec{p}$ are:



1.2.2 addition of vectors

vectors can be added to form a **resultant** vector

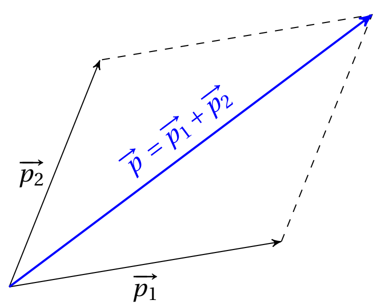
to deal with vector sums, we need take the directions of vectors into account

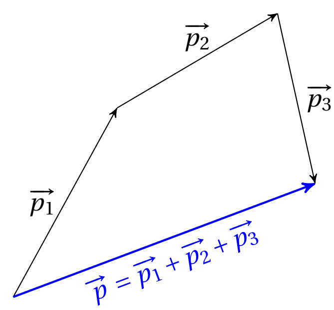
let's consider the sum of two vectors $\vec{p_1}$ and $\vec{p_2}$

resultant vector $\vec{p} = \vec{p_1} + \vec{p_2}$ lies on the diagonal of the parallelogram subtended by $\vec{p_1}$ and $\vec{p_2}$

this is called the **parallelogram rule** for vector addition

if the resultant of several vectors $\vec{p} = \vec{p_1} + \vec{p_2} + \cdots \vec{p_n}$ is to be found, one can join these vectors head-to-tail, the resultant is given by the arrow connecting the tail of $\vec{p_1}$ to the head of $\vec{p_n}$

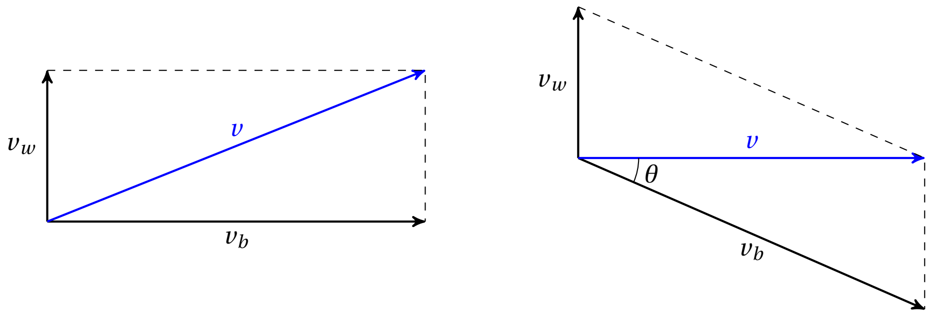




[3]It is also possible to multiply vectors with vectors, and there are basically two ways of doing vector multiplication: the *dot product* and the *cross product*. Both vector products are useful in physics, but we will not go into the details. You may learn more about vector multiplication in the A-Level mathematics course.

Example 1.5 A river flows from south to north with a speed of 2.0 m s^{-1} and the speed of a boat with respect to the water flow is 5.0 m s^{-1}. (a) Suppose the boat leaves the west bank heading due east, what is the resultant velocity of the boat? (b) If the boat is to reach the exact opposite bank across the river, what is the resultant velocity and in what direction should the boat be headed?

✍ vector diagrams for resultant velocity of the boat is illustrated below



(a) boat heading due south (b) boat aiming at exact opposite bank

(a) magnitude of resultant velocity: $v = \sqrt{v_b^2 + v_w^2} = \sqrt{5.0^2 + 2.0^2} \approx 5.4$ m s^{-1}
 in this case, the boat reaches the opposite bank in shortest time but will drift downstream

(b) magnitude of resultant velocity: $v = \sqrt{v_b^2 + v_w^2} = \sqrt{5.0^2 - 2.0^2} \approx 4.6$ m s^{-1}
 in this case, the boat reaches the opposite bank in shortest distance
 but the boat is headed slightly upstream: $\theta = \sin^{-1}\dfrac{v_w}{v_b} = \sin^{-1}\dfrac{2.0}{5.0} \approx 24°$ □

1.2.3 resolving vectors

one can also resolve a single vector into two (or more) components [4]

let's place a general 2D vector $\vec{p}$ in Cartesian coordinates

vector $\vec{p}$ can be split into two perpendicular components

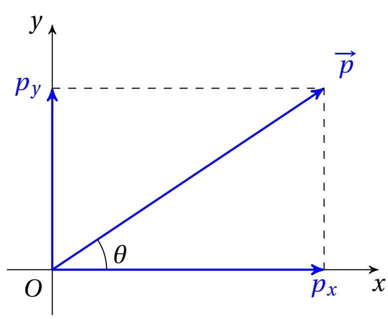
– a horizontal component p_x

– a vertical component p_y

if $\vec{p}$ forms an angle θ to the x-axis, then:

$$p_x = p\cos\theta, \quad p_y = p\sin\theta$$

$$p = |\vec{p}| = \sqrt{p_x^2 + p_y^2}, \quad \tan\theta = \frac{p_y}{p_x}$$



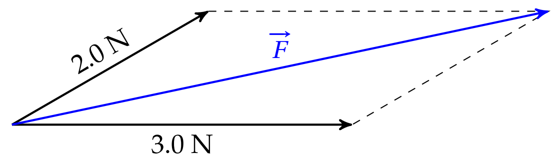
Example 1.6 A force of 3.0 N towards east and a force of 2.0 N towards 30° north of east act on

[4]This depends on the number of dimensions of space we are working with.

an object. Find the magnitude and the direction of the resultant force.

✍ suppose an arrow of length 1 cm represents a force of 1 N

one can draw a *scale diagram* with a ruler and a protractor as shown



one can find length of the resultant vector is about 4.8 cm

also it forms an angle of about 12° to the 3.0 N force

so resultant force is of 4.8 N acting towards 12° north of east

alternatively, one can find components of the resultant as the sum of individual components

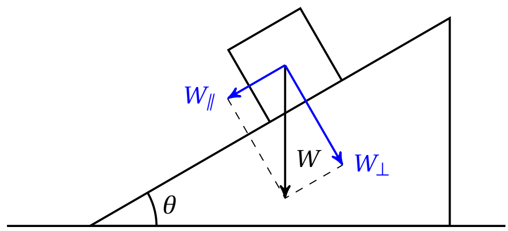
$$F_x = 3.0 + 2.0\cos 30^\circ \approx 4.73 \text{ N}, \quad F_y = 2.0\sin 30^\circ = 1.0 \text{ N}$$

magnitude and direction of the resultant can then be found from its components

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{4.73^2 + 1.0^2} \approx 4.84 \text{ N}, \theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{1.0}{4.73} \approx 11.9^\circ$$

this of course agrees with scale diagram method, but resolving gives more precise results □

Example 1.7 A box of weight $W = 20.0$ N is resting on an inclined slope at 30° to the horizontal. Find the components of weight parallel to the slope and normal to the slope.



✍ the vector diagram is shown

component of weight parallel to slope: $W_{\parallel} = W\sin\theta = 20.0 \times \sin 30^\circ = 10.0$ N

component of weight normal to slope: $W_{\perp} = W\cos\theta = 20.0 \times \cos 30^\circ \approx 17.3$ N □

1.3 uncertainties

physics is a practical science, any law of physics must be evidenced by experimental facts

any meaningful physical quantity is measured either directly or indirectly

but repeated readings may not give a consistent value, instead they show a *spread* of data

uncertainty gives the *range* of values in which *true value* of the measurement is asserted to lie

measurement of a particular quantity is usually reported as $x \pm \Delta x$, where reported value x is the average of repeated readings, and Δx is its uncertainty

1.3.1 absolute uncertainty

Δx measures the size of the range of values where true value probably lies

therefore Δx is called the **absolute uncertainty**

- absolute uncertainty Δx carries the same unit as quantity x
- absolute uncertainty can be worked out from *range* of readings

  range of a data set $x_1, x_2, x_3, \cdots$ is the difference between greatest and smallest value

  absolute uncertainty is given by: $\boxed{\Delta x = \frac{1}{2}(x_{\text{max}} - x_{\text{min}})}$
- absolute uncertainty is usually kept to one significant figure only [5]

  since Δx indicates where the readings start to get problematic

  measured quantity x is kept to the same decimal place as Δx

for example, if value for the speed of an athlete is found to be $v = (8.16 \pm 0.27)$ m s^{-1}, the result, to an appropriate number of significant figures, should be kept as: $v = (8.2 \pm 0.3)$ m s^{-1}.

1.3.2 fractional & percentage uncertainty

ratio of absolute uncertainty to reported value, i.e., $\frac{\Delta x}{x}$, gives the **fractional uncertainty**

recording this ratio as a percentage number, this is known as the **percentage uncertainty**

- fractional and percentage uncertainty have no unit
- $\frac{\Delta x}{x}$ gives relative measure of spread of data, so it is also called the relative uncertainty

Example 1.8 A students measures the diameter of a cylindrical bottle with a vernier calliper. The measurements are taken from several different positions and along different directions. The read-

[5]In some cases where the uncertainty of a quantity is not stated explicitly, the uncertainty is indicated by the number of significant figures in the stated value. If the height of a person is measured to be 1.75 m, this means the first two digits (1 and 7) are certain, while the last digit (5) is uncertain.

When you add or subtract numbers, the number of significant figures is determined by the location of the decimal place. For example, $1.11 + \underline{4.2} + 0.563 = 5.873$, the result should be written as 5.9. When you multiply or divided numbers, the result can have no more significant figures than the term with the fewest significant figures. For example, $1.35 \times 462 \times \underline{0.27} = 168.399$, the result should be written as 170.

However, in AS & A-Level physics, apart from the problems regarding uncertainties, it is allowed to give one more significant figure that what is required. So in other sections of my notes where we do not keep track of the uncertainties, I could be a bit sloppy with the issue of significant figures when numerical values are worked out.

ings she obtained are: 4.351 cm, 4.387 cm, 4.382 cm, 4.372 cm, 4.363 cm. What is the percentage uncertainty of her measurements?

✍ average value: $d = \frac{1}{5}(4.351 + 4.387 + 4.382 + 4.372 + 4.363) = 4.371$ cm

absolute uncertainty: $\Delta d = \frac{1}{2}(d_{\text{max}} - d_{\text{min}}) = \frac{1}{2}(4.387 - 4.351) = 0.036$ cm

result of measurement should be recorded as: $d = 4.37 \pm 0.04$ cm

percentage uncertainty: $\frac{\Delta d}{d} = \frac{0.036}{4.371} \approx 0.082\%$ □

1.3.3 propagation of uncertainties

in many situations, the quantity that we want to find cannot be measured directly

the quantity of interest has to be computed from other quantities

uncertainty of this calculated quantity would depend on two things:

– uncertainties of the raw data from which it is calculated,

– how calculated quantity is related to those original quantities

suppose quantities A and B are two measurables with uncertainty ΔA and ΔB

X is a quantity to be computed by taking their sum, difference, product or quotient

to evaluate uncertainty in X, we estimate the worst scenario, i.e., the greatest deviation from its reported value

addition: $S = A + B$

$$S_{\text{max}} = A_{\text{max}} + B_{\text{max}} = (A+\Delta A) + (B+\Delta B) = (A+B) + (\Delta A+\Delta B) = S + (\Delta A+\Delta B) \quad \Rightarrow \quad \boxed{\Delta S = \Delta A + \Delta B}$$

subtraction: $D = A - B$

$$D_{\text{max}} = A_{\text{max}} - B_{\text{min}} = (A+\Delta A) - (B-\Delta B) = (A-B) + (\Delta A+\Delta B) = D + (\Delta A+\Delta B) \quad \Rightarrow \quad \boxed{\Delta D = \Delta A + \Delta B}$$

multiplication: $P = AB$

$$P_{\text{max}} = A_{\text{max}} B_{\text{max}} = (A + \Delta A)(B + \Delta B) = AB + B\Delta A + A\Delta B + \Delta A\Delta B \quad \Rightarrow \quad \Delta P = B\Delta A + A\Delta B + \Delta A\Delta B$$

divide both sides by $P = AB$, we get

$$\frac{\Delta P}{P} = \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta A}{A} \cdot \frac{\Delta B}{B} \quad \Rightarrow \quad \boxed{\frac{\Delta P}{P} = \frac{\Delta A}{A} + \frac{\Delta B}{B}}$$

percentage uncertainty of a measurable is usually a few percent, so $\frac{\Delta A}{A} \cdot \frac{\Delta B}{B} \approx 0$

so this piece is dropped from the last expression

division: $Q = \dfrac{A}{B}$

one can show that $\boxed{\dfrac{\Delta Q}{Q} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}}$

the derivation is left as an exercise for the reader

power & roots: $Q = A^l B^m C^n \cdots$

percentage uncertainty in Q is: $\dfrac{\Delta Q}{Q} = l\dfrac{\Delta A}{A} + m\dfrac{\Delta B}{B} + n\dfrac{\Delta C}{C} + \cdots$

this can be thought of as a generalization for multiplication and division operations

the proof is also left as an exercise to the reader

brief summary

– for addition and subtraction, *absolute uncertainties* add up

– for multiplication, division and powers, *percentage uncertainties* add up

➢ notice that uncertainties always add

Example 1.9 The resistance of a resistor is measured in an experiment. The current through the resistor is 1.8 ± 0.1 A and the potential difference across is 7.5 ± 0.2 V. What is the resistance of the resistor and its uncertainty?

value of resistance: $R = \dfrac{V}{I} = \dfrac{7.5}{1.8} \approx 4.17\ \Omega$

percentage uncertainty in resistance: $\dfrac{\Delta R}{R} = \dfrac{\Delta V}{V} + \dfrac{\Delta I}{I} = \dfrac{0.2}{7.5} + \dfrac{0.1}{1.8} \approx 8.2\%$

absolute uncertainty in resistance: $\Delta R = 8.2\% \times 4.17 \approx 0.34\ \Omega$

so we find resistance of the resistor: $R = 4.2 \pm 0.3\ \Omega$ □

Example 1.10 The density of a liquid is found by measuring its mass and its volume. The following is a summary of the measurements: mass of empty beaker = (20±1)g, mass of beaker and liquid = (100 ± 1)g, and volume of liquid = $(10.0 \pm 0.5)\text{cm}^3$. What is the density of this liquid and the uncertainty in this value?

✍ mass of liquid: $m = m_2 - m_1 = 100 - 20 = 80$ g

uncertainty in mass: $\Delta m = \Delta m_2 + \Delta m_1 = 1 + 1 = 2$ g

density of liquid: $\rho = \dfrac{m}{V} = \dfrac{80}{10.0} = 8.00\ \text{g cm}^{-3}$

percentage uncertainty in density: $\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V} = \frac{2}{80} + \frac{0.5}{10.0} = 7.5\%$

absolute uncertainty in density: $\Delta\rho = 7.5\% \times 8.00 = 0.60 \text{ g cm}^{-3}$

so density of this liquid is recorded as: $\rho = 8.0 \pm 0.6 \text{ g cm}^{-3}$ □

Example 1.11 The period of simple pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$. In an experiment, the length of string is measured to be $L = 100.0 \pm 0.5$ cm, and the time taken for 10 full oscillations is $t = 20.0 \pm 0.2$ s. What is the value for acceleration of free fall g and its uncertainty?

✍ period of one oscillation: $T = \frac{1}{10}t = 2.00 \pm 0.02$ s

let's rearrange $T = 2\pi\sqrt{\frac{L}{g}}$ into $g = \frac{4\pi^2 L}{T^2}$

acceleration of free fall: $g = \frac{4\pi^2 \times 100.0}{2.00^2} \approx 987 \text{ cm s}^{-2}$

percentage uncertainty: $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T} = \frac{0.5}{100.0} + 2 \times \frac{0.02}{2.00} = 2.5\%$ [6]

absolute uncertainty: $\Delta g = 2.5\% \times 987 \approx 24.7 \text{ cm s}^{-2}$

so result of this measurement is: $g = 990 \pm 20 \text{ cm s}^{-2}$ □

1.4 measurement errors

difference between the measured value and the true value is called **error**

total error is usually a combination of two components: systematic error and random error

1.4.1 systematic & random errors

> **systematic errors** cause the readings to be greater or smaller than the true value by the same amount

➢ faulty equipments, biased observers, calibration errors could produce systematic errors

examples of systematic errors include:

– a vernier calliper does not read zero when fully closed, this introduces **zero error**

– one always reads a measuring cylinder from a higher angle, this introduces **parallax error**

[6]Starting from the formula $T = 2\pi\sqrt{\frac{L}{g}}$, it is attempting to write $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta L}{L} + \frac{1}{2}\frac{\Delta g}{g}$. But this would mean that T is a calculated quantity whose uncertainty is determined by the uncertainty in L and the uncertainty in g, which is incorrect. The right way to do it is to rearrange the formula so that calculated quantity of interest is made the subject of the working equation, the propagation of uncertainties then becomes explicit.

– spring of force meter becomes weaker over time, so force meter always gives overestimates

➢ systematic errors can be reduced by using better equipments or methods

– one can check for zero error before taking readings with a micrometer

– one can *calibrate* a balance with a known mass before using it to measure mass of an object

random errors cause the readings to fluctuate above or below the actual value

➢ deviations caused by random error are unpredictable

➢ insensitive equipments, lack of observer precision, changes in environment, imprecise definitions could produce random errors

examples of causes of random errors include:

– **human reaction errors** when measuring a time quantity on a stop-watch

– electronic noise due to thermal vibrations of atoms when measuring an electric current

– when measuring length of a crack, different people could pick different end points

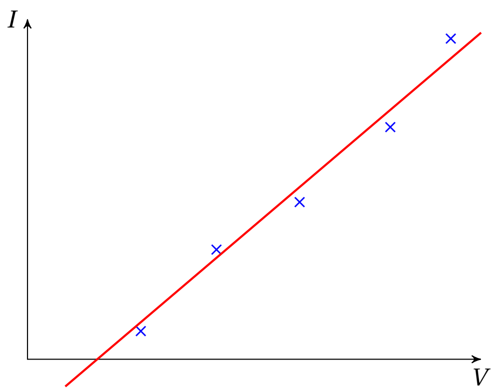
➢ random errors can be reduced by averaging the results from repeated measurements

for example, diameter of a sphere can be measured along different directions and averaged

➢ random errors can also be reduced by using better equipment and better technique

for example, time for an object to fall can be measured with a light gate, instead of a stopwatch

Example 1.12 An experiment is carried out to measure the resistance of a metallic resistor, which is known to be constant throughout the experiment. A set of readings for voltage V across the resistor and the corresponding current I are obtained. A graph of I against V is plotted as shown. What can you say about the errors of the experiment?



✍ one can first draw a *best fit line* to see the distribution of data points

constant resistance means I should be directly proportional to V

so the best fit should be a straight line through the origin

but the best fit does not pass through origin, so there exists systematic error

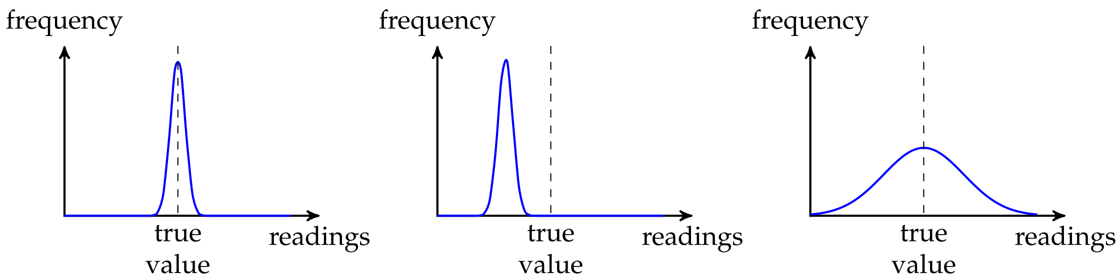
also data points scatter above and below the best fit, so random errors are present □

1.4.2 accuracy & precision

to analyse the result of an experiment, two important aspects are accuracy and precision

measurement is said to be **accurate** if the result is close to the true value

measurement is said to be **precise** if repeated readings are close to each other



(a) precise and accurate (b) precise but not accurate (c) accurate but not precise

distribution of readings with different precision and accuracy

- accuracy of a measurement is closely related to systematic errors
  large systematic errors mean the results must be inaccurate
- precision of a measurement is closely related to random errors
  large random errors cause repeated readings to spread, so the result must be imprecise
- precision is usually indicated by the percentage uncertainty of the measurement
  similarly, precision is also indicated by the number of significant figures in a measurement
  for example, metre rule has a precision of 0.1 cm, while micrometer has a precision of 0.001 cm

Example 1.13 The value for the acceleration of free fall is determined in an experiment. The result is reported to be $g = 14 \pm 5$ m s^{-2}. Is this result accurate? Is it precise?

✍ true value for g is around 9.8 m s^{-2}

stated value is not close to the true value, so the result is not accurate

percentage uncertainty in this result is $\frac{5}{14} \approx 36\%$, which is quite large

so the result is not precise either □

1.5 end-of-chapter questions

SI units

Question 1.1 What are the SI base units of (a) density, (b) pressure, (c) energy, (d) electric charge?

Question 1.2 For a substance of mass m, the heat energy Q needed to change its temperature by ΔT is given by: $Q = cm\Delta T$. Find the SI base units of the constant c.

dimensional analysis

Question 1.3 The resistive force F on a metal ball falling at low speeds in water is given by the equation $F = krv$, where r is the radius of the metal ball, v is its speed and k is a constant. Find the base units of k in the SI system.

Question 1.4 The speed of sound in air can be given by $c = \sqrt{\frac{\gamma p}{\rho}}$, where p is the pressure of the air and ρ is the air density. Show that γ is unit free.

Question 1.5 The effective power output from a wind turbine is given by the equation $P = \frac{1}{2}\eta\rho A v^n$, where ρ is the air density, A is the area of the turbine blades, and v is the wind speed. Given that η is a constant with no units, what is the value of n?

vector algebra

Question 1.6 An aircraft, which has a speed of 35 m s^{-1} in still air, is flying from south to north at a speed of 32 m s^{-1} with respect to a stationary observer on the ground. Find the magnitude and the possible directions of wind velocity.

Question 1.7 Three forces of 5.0 N, 12 N and 13 N act at one point on an object. The angles at which the forces act can vary. What is the maximum and the minimum resultant force?

propagation of uncertainties

Question 1.8 A thermometer can measure to a precision of 0.5°C , what is the temperature rise from 20.0°C to 50.0°C and its uncertainty?

Question 1.9 The radius of a sphere is measured to be 5.0 cm with an uncertainty of 1%. What is the volume of this sphere and the uncertainty? (Volume of a sphere is given by: $V = \frac{4}{3}\pi R^3$.)

Question 1.10 The power radiated from a star of radius R and surface temperature T is given by the formula: $P = 4\pi\sigma R^2 T^4$, where σ is the Stefan–Boltzmann constant known to have the value of 5.67×10^{-8} W m^{-2} K^{-4}. If the sun is measured to have a surface temperature of 5800 ± 200 K and a diameter of $(1.40 \pm 0.03) \times 10^9$ m. (a) Find the radiation power P of the sun and its absolute uncertainty. (b) Suggest which measurement has a larger effect on the uncertainty in P.

Chapter 2

Kinematics

Kinematics is the study of motion. In this chapter, we define three useful kinematic quantities, displacement, velocity and acceleration, and use these terms to discuss the motion of an object.

2.1 kinematic quantities

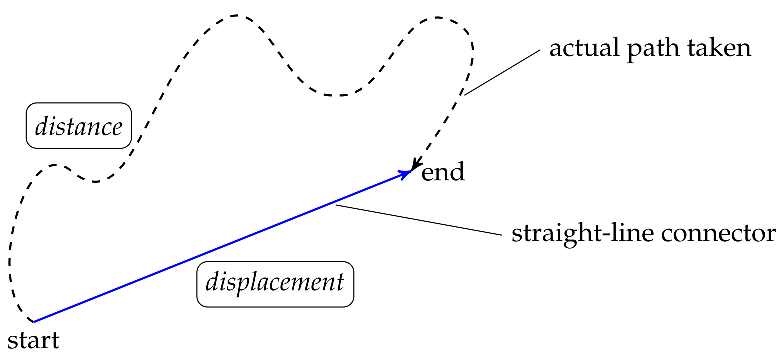
2.1.1 displacement & distance

in everyday language, we talk about the **distance** travelled by an object, which usually refers to the length travelled by an object without considering in what direction it moves

to fully describe position of an object, we also need specify where it moves

> we define **displacement** as the distance moved by an object in a specific direction

- displacement and distance are measured in metres, or any reasonable length units
- displacement is a vector quantity, while distance is a scalar
- displacement is the *straight-line* distance pointing from starting point towards end point
  even if actual path taken is curved, displacement is always the straight-line distance



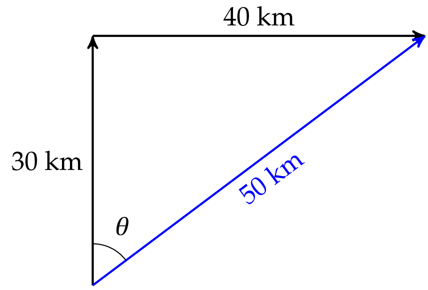
difference between displacement and distance

Example 2.1 An athlete is running around a circular track of radius 60 m. When he completes one lap, what is the distance moved out? What about his displacement?

✍ distance moved is the perimeter of the circle: $s = 2\pi r = 2\pi \times 60 \approx 380$ m

athlete returns to same starting point after one lap, so displacement is zero □

Example 2.2 A ship travels 30 km north, takes a right, and then travels 40 km east to reach its destination. Compare the distance and the displacement travelled.



✍ sum of all lengths gives distance: $30 + 40 = 70$ km

displacement vector is shown on the graph

magnitude of displacement $= \sqrt{30^2 + 40^2} = 50$ km

it is at an angle of $\theta = \tan^{-1}\frac{40}{30} \approx 53°$ east of north □

2.1.2 velocity & speed

displacement of a moving object may change with respect to time

an object is moving fast if it has a large change in displacement during a given time interval

change in displacement per unit time is called the **velocity** of the object: $\boxed{v = \frac{\Delta s}{\Delta t}}$

- SI unit of measurement for velocity: $[v] =$ m s^{-1}
- velocity is a vector quantity

  this follows from the fact that displacement is a vector quantity
- for *linear* motion, one shall pick a specific direction as the positive direction

  then a negative velocity implies motion in the opposite direction
- it is also common to use **speed** to describe how fast an object moves

  speed is defined as the change of the distance travelled per unit time

  velocity can be thought as speed in a particular direction
- defining equation $v = \frac{\Delta s}{\Delta t}$ gives the *average* value for velocity or speed over an interval Δt

  more precisely: $\boxed{\text{average velocity} = \frac{\text{total displacement}}{\text{time taken}}}$, and $\boxed{\text{average speed} = \frac{\text{total distance}}{\text{time taken}}}$

  this should be distinguished from the notion of *instantaneous velocity*

  instantaneous velocity is defined as the rate of change in displacement at a particular instant if we take a very short interval Δt, as $\Delta t \to 0$, average velocity tends to instantaneous velocity

this is expressed in a compact differential form: $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \Rightarrow \boxed{v = \frac{\mathrm{d}s}{\mathrm{d}t}}$

Example 2.3 A cyclist travels a distance of 3.0 km in 20 minutes. She rests for 15 minutes. She then covers a further distance of 5.1 km in a time of 40 minutes. Calculate the average speed of the cyclist in m s^{-1}: (a) during the first 20 minutes of the journey, (b) for the whole journey.

✍ for the first 20 minutes: $v = \frac{3.0 \times 10^3}{20 \times 60} = 2.5$ m s^{-1}

for whole journey: $v = \frac{(3.0 + 0 + 5.1) \times 10^3}{(20 + 15 + 40) \times 60} = 1.8$ m s^{-1} □

Example 2.4 A man walks along a straight road for a distance of 800 m in 5.0 minutes. He then turns around, and walks along the same road for a distance of 280 m in 3.0 minutes. What is the average speed and the average velocity of this man during the 8.0 minutes?

✍ total distance travelled $= 800 + 280 = 1080$m, so average speed: $v = \frac{1080}{8.0 \times 60} = 2.25$ m s^{-1}

change of displacement $= 800 + (-280) = 520$m, so average velocity: $v = \frac{520}{8.0 \times 60} \approx 1.08$ m s^{-1} □

Example 2.5 A maglev train travels at an average speed of 60 m s^{-1} from the city centre to the airport, and at 40 m s^{-1} on its return journey over the same distance. What is the average speed of the round-trip? What about the average velocity?

✍ suppose the distance between airport and city centre is S

average speed: $v = \frac{2S}{t_1 + t_2} = \frac{2S}{\frac{S}{60} + \frac{S}{40}} = 48$ m s^{-1}

for a round-trip, train returns to same staring position

change in displacement is zero, so average velocity is zero □

2.1.3 acceleration

velocity of a moving object may change as well, i.e., objects can speed up or slow down

> change in velocity per unit time is defined as the **acceleration**: $\boxed{a = \frac{\Delta v}{\Delta t}}$

- unit of measurement for acceleration: $[a] =$ m s^{-2}
- acceleration is a vector quantity, it has both magnitude and direction

  this is because of vector nature of velocity, change in velocity must also have direction
- for *linear* motion, one usually pick direction of initial velocity as positive direction

  $a > 0$ would imply acceleration in the normal sense, i.e., motion with an increasing speed

$a < 0$ would imply deceleration, i.e., motion with a decreasing speed

➢ when velocity changes, it could be change in magnitude or/and change in direction [7]

for example, for an object moving along a curved path, its velocity is constantly changing direction, so it must have a non-zero acceleration

no acceleration would imply no change in speed and no change in direction of motion

➢ defining equation $a = \frac{\Delta v}{\Delta t}$ gives average acceleration over time interval Δt

we can likewise introduce *instantaneous acceleration* as the rate of change in velocity

taking the limit where the time interval $\Delta t \to 0$. we have: $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \Rightarrow \boxed{a = \frac{\mathrm{d}v}{\mathrm{d}t}}$

Example 2.6 A ball hits a barrier at right angles with a speed of 15 m s^{-1}. It makes contact with the barrier for 30 ms and then rebounds with a speed of 12 m s^{-1}. What is the average acceleration during the time of contact?

✍ note that direction of velocity changed during rebound, so $\Delta v = 15 - (-12) = 27$ m s^{-1}

average acceleration: $a = \frac{\Delta v}{\Delta t} = \frac{27}{30 \times 10^{-3}} = 900$ m s^{-2} □

2.2 motion graphs

how one physical quantity changes with another quantity can be visually shown on a *graph*

changes in displacement, velocity or acceleration over time can be shown on *motion graphs*

as we will see, s-t graph, v-t graph and a-t graphs are closely interrelated to one another

2.2.1 displacement-time graphs

a displacement-time graph shows an object's position at any given time

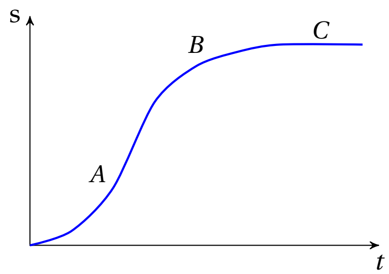
➢ gradient of tangent gives rate of change in displacement

but this is instantaneous velocity, which can be given by $v = \frac{\mathrm{d}s}{\mathrm{d}t}$, so we have:

$$\boxed{\text{velocity} = \text{gradient of } s\text{-}t \text{ graph}}$$

[7] Acceleration of an object can be considered as the combination of two components. One component is known as the *normal* acceleration or the *centripetal* acceleration, which is at right angle to the velocity and is responsible for the change in direction of motion. The other component is called the *tangential* acceleration, which is parallel to the direction of motion and causes change in magnitude of object's velocity. You will learn more about these in further mechanics.

Example 2.7 Describing the motion from the displacement-time graph shown.

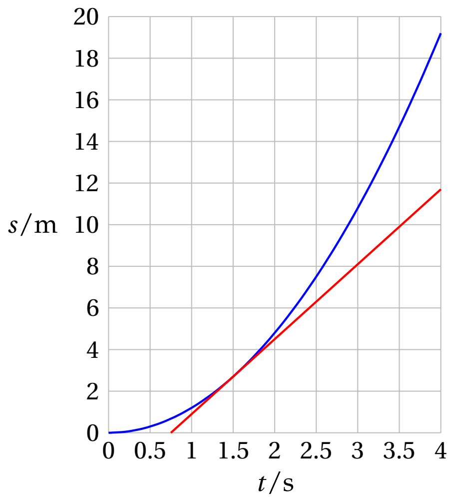


stage A: gradient of the graph is increasing, showing that the object is speeding up

stage B: gradient starts to decrease, so the object gradually slows down

stage C: curve becomes horizontal, gradient becomes zero, means that the object eventually comes to a stop □

Example 2.8 The diagram shows the displacement-time graph for a vehicle travelling along a straight road. Use the graph to find, (a) the average velocity during the first 4.0 s of the motion, (b) the velocity of the vehicle *at* time $t = 1.5$ s.



✍ during first 4.0 s, average velocity is

$$v = \frac{\Delta s}{\Delta t} = \frac{19.2}{4.0} \approx 4.8 \text{ m s}^{-1}$$

to find velocity at $t = 1.5$ s, a tangent is drawn

gradient of tangent gives instantaneous velocity:

$$v = \frac{11.6 - 0}{4.0 - 0.75} \approx 3.6 \text{ m s}^{-1}$$ □

2.2.2 velocity-time graphs

a velocity-time graph shows the velocity of a moving object at any instant

➢ since the rate of change of velocity gives the acceleration, so

$$\boxed{\text{acceleration} = \text{gradient of } v\text{-}t \text{ graph}}$$

➢ v-t graph also gives information about the change in displacement

$$\boxed{\text{change in displacement} = \text{area under } v\text{-}t \text{ graph}}$$

in very short time interval Δt_i, change in velocity is small so $v(t_i) \approx$ constant during this time

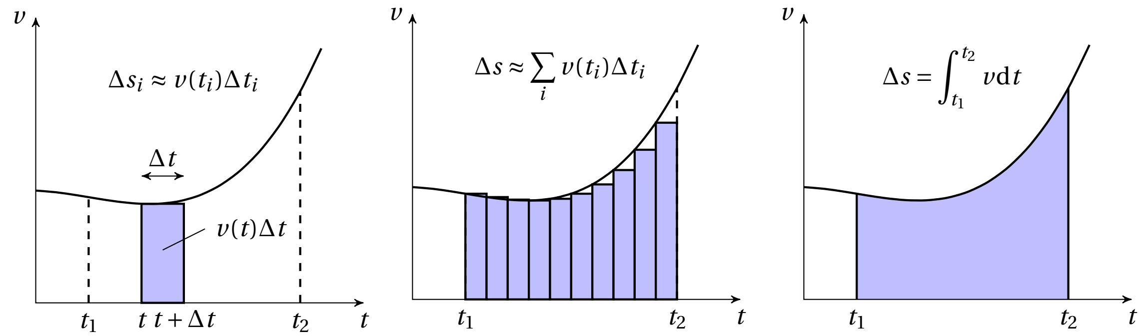
displacement moved out $\Delta s_i \approx v(t_i)\Delta t_i$, which corresponds to area of a thin rectangle

sum of all these small Δs_i's gives total change in displacement over a period of time

now consider the limit where each of the time interval $\Delta t_i \to 0$

total area of these rectangles approximates area bounded by the v-t curve and time axis [8]

[8] Mathematically, integration is the inverse operation of taking derivatives. By definition $v = \frac{\mathrm{d}s}{\mathrm{d}t}$, then it follows naturally that $\Delta s = \int v \mathrm{d}t$. While the derivative of a given function gives the gradient of tangent at

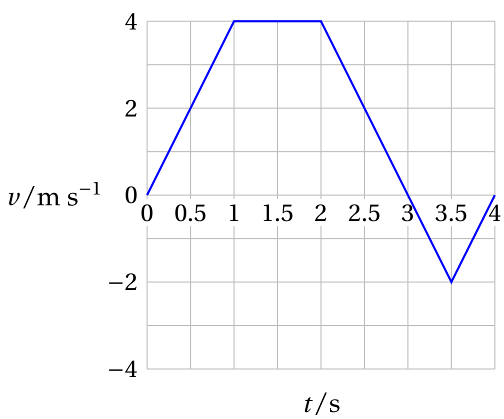


(a) displacement Δs in short interval Δt

(b) total displacement estimated by summing the many Δs_i's

(c) total displacement as area under v-t graph

calculating change in displacement by finding the area under velocity-time graph



Example 2.9 The velocity of a toy car is shown. For the journey shown on the graph, use the graph to find (a) the total distance travelled, and (b) the total displacement travelled.

✍ distance is estimated using area under v-t graph

0~3.0 s: $s_1 = \frac{1}{2} \times 1.0 + 3.0 \times 4.0 = 8.0$ m

3.0~4.0 s: $s_2 = \frac{1}{2} \times 1.0 \times 2.0 = 1.0$ m

total distance $= 8.0 + 1.0 = 9.0$ m

total displacement $= (+8.0) + (-1.0) = 7.0$ m □

2.2.3 acceleration-time graphs

one can similarly plot an acceleration-time graph to give the changes in acceleration

➢ a-t graphs can give information about changes in velocity

similar discussions lead to the following conclusion:[9]

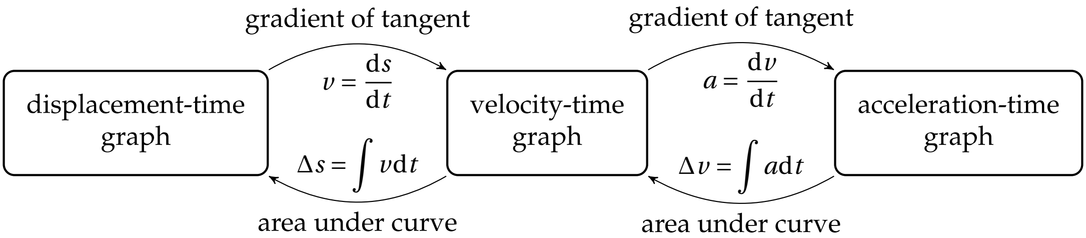
$$\boxed{\text{change in velocity} = \text{area under } a\text{-}t \text{ graph}}$$

---

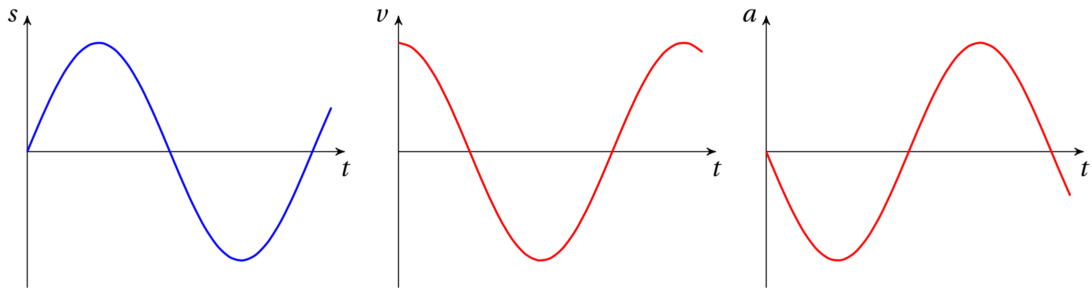
each point on its graph, integrating a function gives the signed area bounded by the graph. The reader may find the formal treatment of this relationship in any calculus textbook.

[9]Using area under a-t graph to find changes in velocity is not required in the AS-Level physics syllabus. I am putting this in the notes mainly for the completeness of the discussions on motion graphs.

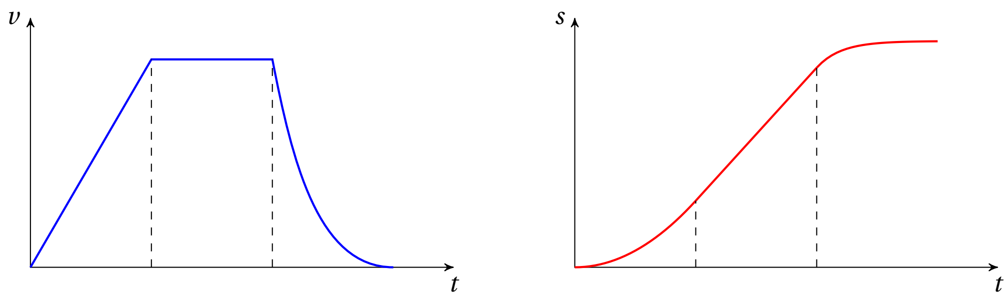
relationships between displacement, velocity and acceleration graphs are summarised below



Example 2.10 Given the displacement-time graph as shown, check yourself that this s-t graph leads to the velocity-time graph and the acceleration-time graph shown.



Example 2.11 Given the velocity-time graph as shown, check yourself that this v-t graph leads to the displacement-time graph as shown.



2.3 linear motion with constant velocity

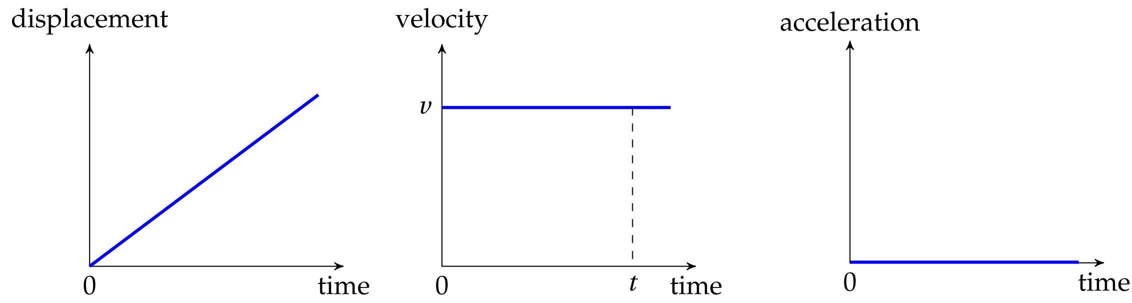
let's look at the simplest kind of motion

that is, an object moving at constant speed in a straight line: $v = \text{constant}$

the equations of motion are straightforward:

$$a = 0 \qquad \boxed{s = vt}^{[10]} \tag{2.1}$$

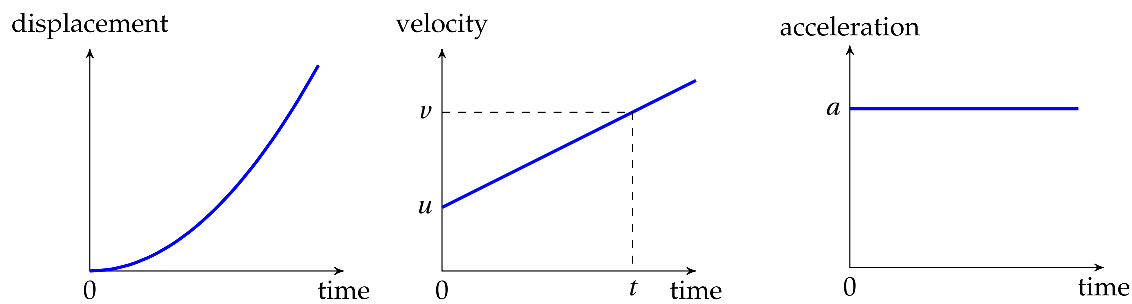
[10] It is implicitly assumed that the motion starts from the origin with respect to which displacement is



motion graphs for linear motion at constant velocity

2.4 linear motion with constant acceleration

the second simplest type of motion is a linear motion with acceleration $a = \text{constant}$



motion graphs for linear motion at constant acceleration

2.4.1 equations of motion

during a time interval t, suppose velocity changes from initial value u to final value v

from the defining equation of acceleration $a = \dfrac{\Delta v}{\Delta t} = \dfrac{v-u}{t}$, we get

$$\boxed{v = u + at}\text{[11]} \tag{2.2}$$

to find total displacement travelled, we compute the area under the v-t graph

$$\boxed{s = \frac{1}{2}(u+v)t} \tag{2.3}$$

for which we can interpret $\bar{v} = \dfrac{1}{2}(u+v)$ as the average velocity during that time

defined. More generically, we should write: $s = s_0 + vt$, where s_0 is the initial displacement.

[11]Proof with calculus: $\mathrm{d}v = a\mathrm{d}t \Rightarrow \Delta v = \int_u^v \mathrm{d}v = \int_0^t a\mathrm{d}t \Rightarrow v - u = at \Rightarrow v = u + at$

plug (2.2) into (2.3), we find an expression for the displacement travelled in terms of u and a:

$$\boxed{s = ut + \frac{1}{2}at^2} \tag{2.4}$$ [12][13]

this shows the displacement s is a quadratic function in time t

this is consistent with the parabolic shape of the s-t graph shown

we can also eliminate the time variable t to derive one last equation

from (2.2) we have $t = \frac{v-u}{a}$, substitute this into (2.3), we find

$$s = \frac{1}{2}(u+v) \times \frac{v-u}{a} \quad \Rightarrow \quad \boxed{2as = v^2 - u^2} \tag{2.5}$$

Example 2.12 A car starts from rest and accelerates uniformly at 5.0 m s^{-2}for 6.0 s. (a) How fast is the car travelling at $t = 8.0$ s? (b) What is the distance travelled by the car in this time?

✍ $$v = u + at \quad \Rightarrow \quad v = 0 + 5.0 \times 6.0 = 30 \text{ m s}^{-1}$$
$$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad s = 0 + \frac{1}{2} \times 5.0 \times 6.0^2 = 90 \text{ m}$$ □

Example 2.13 A car is travelling at 30 m s^{-1}. A hazard appears in front of the car, and the driver takes immediate action to stop the car. When brakes are applied, deceleration of the car is 5.0 m s^{-2}. What is the braking distance?

✍ $$2as = v^2 - u^2 \quad \Rightarrow \quad s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 30^2}{2 \times (-5.0)} = 90 \text{ m}$$ □

Example 2.14 At the instant the traffic light turns green, a motorcycle waiting at the stop line starts with a constant acceleration of 2.0 m s^{-2}. At the same instant, a truck at a constant speed of 16 m s^{-1} overtakes and passes the motorcycle. How far beyond the stop line will the motorcycle overtake the truck?

✍ suppose overtake occurs at time t after motorcycle starts to accelerate

distance travelled by motorcycle: $s_m = u_m t + \frac{1}{2}at^2 \quad \Rightarrow \quad s_m = 0 + \frac{1}{2} \times 2.0 \times t^2$

distance travelled by truck: $s_t = v_t t \quad \Rightarrow \quad s_t = 16t$

overtake when $s_m = s_t \quad \Rightarrow \quad \frac{1}{2} \times 2.0 \times t^2 = 16t \quad \Rightarrow \quad t = 16$ s

substitute t into either s_m or s_t, one finds distance travelled: $s = 256$ m □

[12]Proof with calculus: $\mathrm{d}s = v\mathrm{d}t \Rightarrow \Delta s = \int_0^s \mathrm{d}s = \int_0^t v\mathrm{d}t \Rightarrow s = \int_0^t (u+at)\mathrm{d}t = \left(ut + \frac{1}{2}at^2\right)\Big|_0^t \Rightarrow s = ut + \frac{1}{2}at^2$

[13]Equation (2.4) assumes a zero initial displacement at $t = 0$. If there is a non-zero initial displacement, one should write $s = s_0 + ut + \frac{1}{2}at^2$. Similar discussion applies to equation (2.3).

2.4.2 free fall

a typical example of uniformly accelerated motion is the free fall

everything has the tendency to fall towards ground due to earth's gravity

in this section, we assume that effects of air resistance are negligible

acceleration of free fall is then a constant $a = g$, regardless of mass of falling object[14]

- near surface of earth, value of acceleration of free fall: $g \approx 9.81 \text{ m s}^{-2}$

  value of g could be different on a different planet

- for a freely-falling object released from rest, its velocity increases with time as

$$v = u + at = 0 + gt \quad \Rightarrow \quad v = gt$$

the distance it has fallen from the point of release is

$$s = ut + \frac{1}{2}at^2 = 0 \cdot t + \frac{1}{2}gt^2 \quad \Rightarrow \quad s = \frac{1}{2}gt^2$$

Example 2.15 An object is released from rest from a height of $h = 24$ m and falls freely under gravity. Air resistance is negligible. (a) How long does it take to hit the ground? (b) What is its speed when hitting the ground?

✍
$$h = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 24}{9.81}} \approx 2.21 \text{ s}$$
$$v = gt = 9.81 \times 2.21 \approx 21.7 \text{ m s}^{-1}$$ □

Example 2.16 A photograph is taken for a small particle falling from rest. The photograph is taken at 0.400 s after the object is released. Since the particle is still moving when the photograph is being taken, the image is blurred. The blurred part is found to have a length of 20.8 cm. What is time of exposure for the photograph?

✍ from $t = 0$ to right before photo is taken:

$$s_1 = \frac{1}{2}gt_1^2 = \frac{1}{2} \times 9.81 \times 0.400^2 \approx 0.785 \text{ m}$$

from $t = 0$ to right after photo has been taken:

$$s_2 = s_1 + \Delta s = \frac{1}{2}gt_2^2 \quad \Rightarrow \quad t_2 = \sqrt{\frac{2(s_1 + \Delta s)}{g}} = \sqrt{\frac{2 \times (0.785 + 0.208)}{9.81}} \approx 0.450 \text{ s}$$

time of exposure: $\Delta t = t_2 - t_1 = 0.450 - 0.400 \approx 0.050$ s □

[14]The reason for this constant acceleration of free fall will be elaborated in §3.3.1.

2.4.3 upward projection

like a freely-falling object, an object tossed upwards experiences the same constant downward acceleration $a = g \approx 9.81\ \mathrm{m\ s^{-2}}$ as long as resistive forces can be ignored

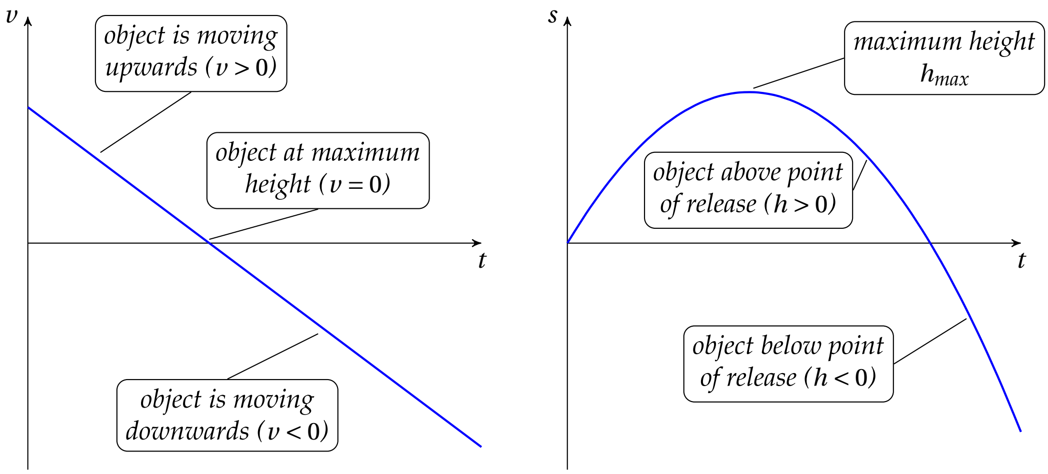
note that initial velocity u is upwards, but acceleration a is downwards so we will have different signs for u and a in the equations

conventionally, positive direction is taken as same direction as initial velocity

in our case, positive direction is upwards, the acceleration is then negative $a = -g$

so the velocity-time relation and displacement-time relation are

$$v = u - gt \qquad s = ut - \frac{1}{2}gt^2$$



v-t graph and s-t graph for upward projectile motion

- sign of v now gives direction of motion

  $v > 0$ means object is moving upwards, $v < 0$ means it has reversed direction and starts falling

  in particular, object attains greatest height when $v = 0$

- sign of s gives whether object is at a higher or lower position with respect to point of release

  $s > 0$ means the object is above the position from which it is projected

  $s < 0$ means it is now below the point of release

Example 2.17 A ball is projected vertically upwards at $12\ \mathrm{m\ s^{-1}}$. Air resistance is negligible. (a) Find the time taken for the ball to reach the highest position. (b) Find the greatest height.

✍ maximum height is reached when $v = 0$, so

$$v = u - gt = 0 \quad \Rightarrow \quad t = \frac{u}{g} = \frac{12}{9.81} \approx 1.22\ \mathrm{s}$$

$$H_{\max} = ut - \frac{1}{2}gt^2 = 12 \times 1.22 - \frac{1}{2} \times 9.81 \times 1.22^2 \approx 7.34 \text{ m}$$

it is also possible to use $v^2 - u^2 = 2as$ to find $H_{\max}$, this is:

$$0^2 - u^2 = 2(-g)H_{\max} \quad \Rightarrow \quad H_{\max} = \frac{u^2}{2g} = \frac{12^2}{2 \times 9.81} \approx 7.34 \text{ m}$$ □

Example 2.18 A stone is thrown vertically upwards with an initial velocity of 14.0 m s^{-1} from the edge of a cliff that is 35 m from the sea below. (a) Find the speed at which it hits the sea. (b) Find the time taken for the stone to hit the sea.

✍ take positive direction to point upwards, we use $v^2 - u^2 = 2as$ to find

$$v^2 = 14.0^2 + 2 \times (-9.81) \times (-35) \approx 883 \text{ m}^2 \text{ s}^{-2} \quad \Rightarrow \quad v \approx -29.7 \text{ m s}^{-1}$$ [15]

to find time, we can use $v = u - gt$, hence: $t = \dfrac{v-u}{-g} = \dfrac{-29.7 - 14.0}{-9.81} \approx 4.46$ s

one can also attempt $s = ut - \frac{1}{2}gt^2$, this leads to the equation: $-35 = 14.0t - \frac{1}{2} \times 9.81t^2$

this quadratic equation in t gives two roots: $t_1 \approx 4.46$ s, and $t_2 \approx -1.60$ s

negative root should be discarded since it means stones hits the sea below it is thrown

so time taken for stone to hit the sea is $t \approx 4.46$ s □

2.5 motion in two dimensions – projectile motion

a **projectile** is an object whose motion is only affected by gravity

for projectile motion, we assume no air resistance and no other forces

gravity causes a constant acceleration of free fall that acts vertically downwards

➢ curved path of a projectile is the combination of its *horizontal* and *vertical* motion

– horizontally: no acceleration, so horizontal component of velocity $v_x =$ constant

– vertically: constant acceleration, vertical component of velocity v_y varies over time

as a consequence, a projectile would follows a *parabolic* path as it travels[16]

let's consider a projectile launched at initial velocity u at angle θ to the horizontal

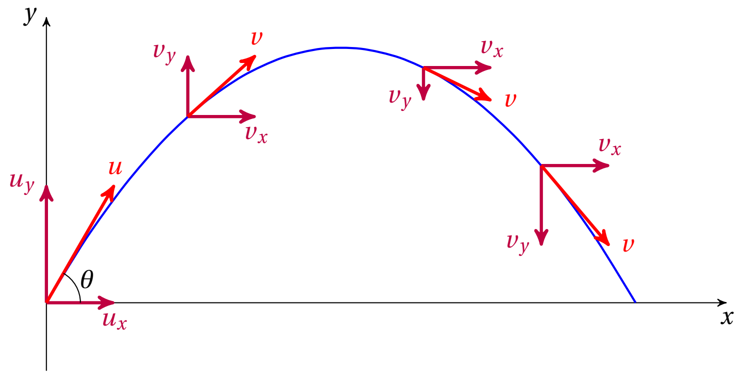
➢ horizontally, projectile maintains a constant velocity, so

$$v_x = u_x \qquad x = u_x t$$

where $u_x = u\cos\theta$ is horizontal component of initial velocity

[15]Note that we have substituted $a = -g$ since acceleration of free fall always points downwards, and $s = -35$ m since sea is below point of release. Also final velocity when hitting water is downwards, which should take a negative sign, so we discarded the positive solution for v.

[16]You may be able to prove this statement in Question 2.24.



components of the velocity of a projectile at different points along its path

- vertically, if upward direction is taken to be positive, then acceleration $a = -g$, so
$$v_y = u_y + at = u_y - gt \qquad y = u_y t + \frac{1}{2}at^2 = u_y t - \frac{1}{2}gt^2$$
where $u_y = u\sin\theta$ is vertical component of initial velocity
- components can be combined to give resultant velocity or resultant displacement:
$$v = \sqrt{v_x^2 + v_y^2} \qquad s = \sqrt{x^2 + y^2}$$

maximum height reached by an projectile

when a projectile reaches the highest position, its instantaneous vertical velocity becomes zero

we can then find the time it takes to attain this maximum height.
$$v_y = u\sin\theta - gt = 0 \quad \Rightarrow \quad t = \frac{u\sin\theta}{g}$$
to find $H_{\max}$, one can use either equation (2.3) or (2.4)
$$H_{\max} = \frac{1}{2}(u_y + v_y)t = \frac{1}{2}u_y t = \frac{1}{2} \times u\sin\theta \times \frac{u\sin\theta}{g} = \frac{u^2\sin^2\theta}{2g}$$
$$H_{\max} = u_y t - \frac{1}{2}gt^2 = u\sin\theta \times \frac{u\sin\theta}{g} - \frac{g}{2} \times \left(\frac{u\sin\theta}{g}\right)^2 = \frac{u^2\sin^2\theta}{2g}$$
- for the same initial speed u, the greater the angle of projection, the higher the object can get

  in the extremal case where $\theta = 90^\circ$, it simply becomes an upward projection motion

airborne time and horizontal range of an projectile

a ball projected from the ground will first rise in height

but it will eventually fall to the ground due to the gravitational pull after a period of time T

when it lands, its vertical displacement is zero, so
$$Y = u_y T - \frac{1}{2}gT^2 = u\sin\theta T - \frac{1}{2}gT^2 = 0 \quad \Rightarrow \quad T = \frac{2u\sin\theta}{g}$$

the horizontal range is given by

$$X = u_x T = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g} \quad\Rightarrow\quad X = \frac{u^2\sin 2\theta}{g}$$

where in the last step the trigonometric identity $\sin 2\alpha = 2\sin\alpha\cos\alpha$ has been used

➢ for same initial speed u, projectile launcher at greater angle stays in air for longer time

greatest airborne time is obtained if object is projected straight up, i.e., $\theta = 90^\circ$

➢ for same initial speed, horizontal range of projectile depends on angle θ of projection

to obtain the greatest horizontal range, two things are required

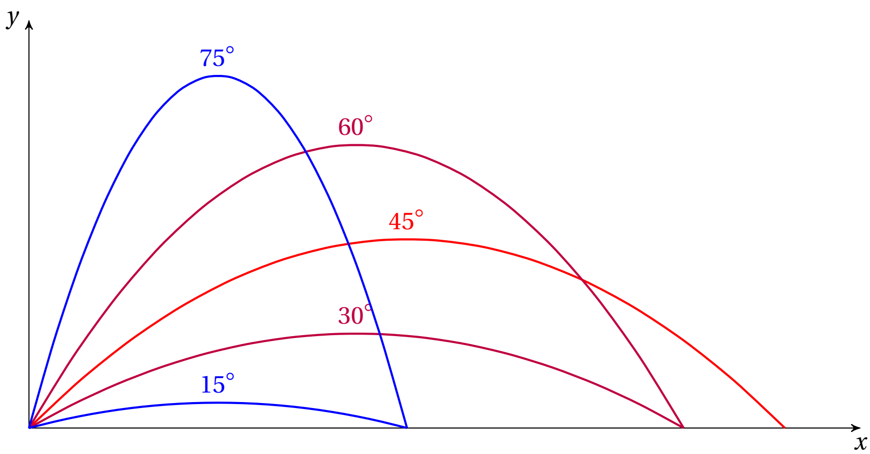
– sufficiently large horizontal velocity v_x

– sufficiently long time T staying in the air

however, a larger v_x requires a smaller θ, hence a shorter airborne time T

therefore, there is a compromise between the two

optimal angle should be neither be too large nor too small, which can be shown to be 45°



trajectories of projectiles launched at the same speed but different angles

Example 2.19 A ball is thrown from a point O at $15\ \mathrm{m\,s^{-1}}$ at an angle of 40° to the horizontal. The ball reaches its highest position at point P. Ignore the effects of air resistance. (a) How long does it take to reach P? (b) What is the magnitude of the displacement OP?

✍ at highest point: $v_y = u_y - gt = 0 \quad\Rightarrow\quad t = \frac{u\sin\theta}{g} = \frac{15\times\sin 40^\circ}{9.81} \approx 0.983\ \mathrm{s}$

vertical displacement: $y = u_y t - \frac{1}{2}gt^2 = 15\sin 40^\circ \times 0.983 - \frac{1}{2}\times 9.81\times 0.983^2 \approx 4.74\ \mathrm{m}$

horizontal displacement: $x = u_x t = 15\cos 40^\circ \times 0.983 \approx 11.3\ \mathrm{m}$

resultant displacement: $|OP| = \sqrt{x^2+y^2} = \sqrt{11.3^2+4.74^2} \approx 12.2\ \mathrm{m}$ □

Example 2.20 A small object is horizontally projected at 7.20 ms^{-1} from a surface at a height of $h = 1.2$ m above the ground. Assume there is no air resistance. (a) What is the time taken for the object to hit the ground? (b) What is the horizontal range? (c) Find the velocity at which the object hits the ground.

✍ vertically, take downward as positive: $h = \cancelto{0}{u_y t} + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1.2}{9.81}} \approx 0.495$ s

horizontal range: $x = u_x t = 7.20 \times 0.495 \approx 3.56$ m

final vertical velocity: $v_y = \cancelto{0}{u_y} + gt = 9.81 \times 0.495 \approx 4.85$ m s^{-1}

magnitude of resultant velocity: $v = \sqrt{v_x^2 + v_y^2} = \sqrt{7.20^2 + 4.85^2} \approx 8.68$ m s^{-1}

angle to which resultant velocity makes with horizontal: $\phi = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{4.85}{7.20} \approx 34°$ □

2.6 end-of-chapter questions

kinematic quantities

Question 2.1 What is the distance covered for a car that travels half a lap along a circular path of radius of 200 m. What about the displacement?

Question 2.2 A ball is released from a height of 2.0 m above the ground. It bounces vertically for quite a number of times before coming to rest. (a) State the change of displacement for the ball. (b) Explain how the distance travelled is different from the change in displacement.

Question 2.3 For an athlete running around a track for many laps, suggest how his average velocity could be zero?

Question 2.4 A car travels 2400 m east in 3.0 minutes, then takes a left turn, and then travels 700 m north in 1.5 minutes. What is the average speed and the average velocity for this journey?

Question 2.5 Is it possible for an object moving at a steady speed to have acceleration?

motion graphs

Question 2.6 For the v-t graph given in Example 2.11, sketch the a-t graph for this motion.

Question 2.7 If the tangent of a displacement-time graph at one particular instant is sloping downwards, what does that imply about the velocity at that instant?

Question 2.8 A vehicle initially travels at a steady speed of 15 m s^{-1}. It accelerates uniformly for 10 s to reach a higher speed of 20 m s^{-1}. It maintains at this speed for 20 s, and then decelerates uniformly to a stop in the last 10 s. (a) Sketch the velocity-time graph for this motion. (b) Sketch the acceleration-time graph. (c) Find the distance travelled during the 40 s.

linear motion with constant velocity

Question 2.9 Sonar is a technique that uses sound waves to detect objects. It can be used to measure the depth of the seabed. Given that speed of sound in water is 1500 m s^{-1}, and reflected waves sent from a submarine are detected 0.50s after they are transmitted. How deep is the water below the submarine?

Question 2.10 Given that the speed of sound in air is 340 m s^{-1} and the speed of light in air is 3.0×10^8 m s^{-1}. If a person hears the sound of a thunder 5.0 seconds after seeing a lightning flash, how far away from this person is did the lightning strike?

linear motion with constant acceleration

Question 2.11 A train initially travels at a speed of 40 m s^{-1}. It starts to decelerate at 0.50 m s^{-2}. (a) What is the distance travelled in 50 s? (b) When it comes to a stop, how far out has it travelled?

Question 2.12 A vehicle moving at 14 m s^{-1} accelerate uniformly to 26 m s^{-1} in 6.0 s. (a) What is the average velocity during this time? (b) What is the acceleration during this time (c) What is the distance travelled by the vehicle? (d) The vehicle then braked with constant deceleration to stop in another 8.0 s. What is the distance travelled during the time when brakes are applied?

free fall

Question 2.13 Two balls are dropped from rest from the same height. The second ball is released 0.80 s after the first one. What is their separation 1.5 s after the second ball is dropped?

Question 2.14 A golf ball is dropped from the top of a tower of height 30 m. The ball falls from rest and air resistance is negligible. What time is taken for the ball to fall (a) the first 10 m from rest, (b) the last 10 m to the ground?

Question 2.15 The acceleration of free fall on Pluto is about one-fifteenth of that on Earth. If it takes a time of T for a rock to fall from rest a distance of S, what is the time taken, in terms of T, for a rock to fall from rest through the same distance S on Pluto?

Question 2.16 In an experiment is carried out to determine the acceleration of free fall g using a falling body. The body is released from rest from a height of h, the time taken t for it to hit the floor is measured. (a) Find the expression that can be used to calculate the value of g? (b) Suggest what could lead to an overestimation for the value of g.

upward projection

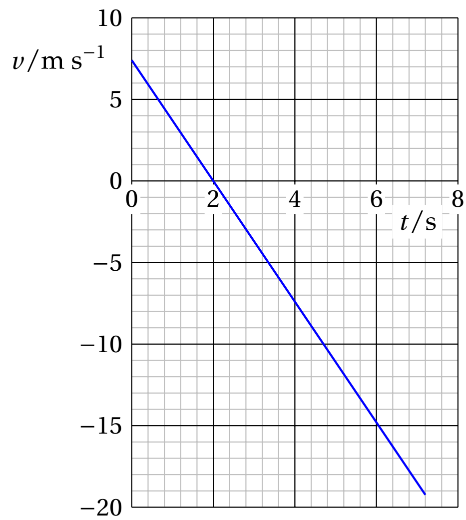
Question 2.17 A ball is tossed upwards with a speed of 9.0 m s^{-1}. (a) How long does it take to return to the same point if air resistance is negligible? (b) How does the return velocity compare with its initial velocity?

Question 2.18 Someone wants to toss a ball onto a platform that is at a height of 20 m above him. What is the minimum initial velocity needed to launch the ball?

Question 2.19 Someone standing at the top of a high building throws a ball straight up and another ball straight down with the same initial speed. Assume that air drag is negligible, which ball will have a greater speed when it hits the ground?

Question 2.20 In basketball games, hang time refers to the length of time a player stays in the air after jumping from the floor. (a) It is reported that Michael Jordan is able to stay in the air for $T = 1.0$ s to do his slam-dunk tricks, estimate how high he can jump. (b) The acceleration of fee fall on the Moon is about one-sixth of that on Earth. What is the hang time of Michael Jordan if he takes off from the surface of the Moon?

Question 2.21 Mark Watney[17] stands at the edge of a cliff on the Mars and throws a rock vertically upwards with a speed of 7.4 m s^{-1}. The graph shows the variation with the time t of the rock's velocity v. (a) What is the acceleration of free fall on the Mars? (b) When does the rock reach the maximum height? (c) What is the height above the base of the cliff the moment when the rock is thrown? (d) What is the maximum height above the base of the cliff to which the rock rises? (e) What is the total distance travelled by the rock before it strikes the ground?



[17] A fictional character in the science fiction movie *The Martian* (2015) based on the novel of the same name written by *Andy Weir*.

projectile motion

Question 2.22 A ball rolls off a table and lands at a position of a horizontal distance of 1.2 m from the table. The table is 0.95 m high. Find the speed at which the ball leaves the table.

Question 2.23 A ball is kicked from the ground towards a vertical barrier. The barrier is at a horizontal distance of 18 m from the initial position of the ball. The ball strikes the barrier after 1.5 s at a height of 2.5 m above the ground. (a) Find the magnitude and the direction of the initial velocity. (b) Find the magnitude and the direction of the velocity at which the ball hits the barrier.

Question 2.24 When a particle is launched from the origin at an angle θ with the horizontal at a speed of u, show that its trajectory is a parabola given by the equation: $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$.

Question 2.25 Two golf players each hit a ball at the same speed. One at 30° with the horizontal, the other at 60°. Which ball hits the ground first? Which ball goes farther?

Question 2.26 (a) State the difference between the displacement of a projectile and the distance it travels. (b) Suggest in what situation a projectile's displacement could have the same magnitude as the distance.

Question 2.27 An archer always aims slight higher than the distant target that she wants to hit. Why isn't the bow lined up such that it points exactly at the target?

Chapter 3

Force & Motion

3.1 force & motion: an introduction

in physics, a force appears when two bodies interact with one another

- you will encounter various types of forces in this course, some of which are
  - *weight*: the gravitational attraction acting on any object exerted by the earth
  - *tension*: a force in a string, a rope, a chain, etc. when it is being pulled
  - *normal contact*: when a body's surface is compressed, there reacts a normal contact force[18]
  - *friction*: a force that resists relative motion when two surfaces tend to slide over one another
  - *resistance*: also called drag force, this is experienced when a body travels through a medium
  - *upthrust*: an upward force acting on an object immersed in a fluid
  - *electric force*: an attractive or repulsive interaction between electrically charged objects

  detailed features of these forces follow later in the notes.

- a force can produce various effects to the object, the effect could be
  - an increase/decrease in speed
  - a change in the direction of motion
  - causing the object to rotate
  - a change in shape of the object

  in this and the next few chapters, we will be looking into each of these aspects

- when more than one force act on a body, it is useful to find their *resultant*, or the *net force*

> **resultant force**, or **net force**, is a single force that has the same effect as all forces acting on an object combined

vector sum of all of the individual forces gives the resultant force

[18]Examples of normal contact force are support force that stops a desk from sinking into the ground, and the impact on a football when you kick it, etc.

➢ in this chapter, we will study the dynamics of *point masses*

point mass is an idealization that the object has a mass but does not take up any space

position of an object treated as a point mass is specified with a geometric point in space

this is a simplification when size, shape, rotation, or structure of object are not important

3.2 Newton's laws of motion

Newton's laws of motion[19] are three laws that form the basis of classical mechanics

they describe the relationships between motion of an object and forces acting on it

3.2.1 first law

> **Newton's first law** states that an object continues in its state of rest or uniform motion at constant velocity if there is no resultant force acting

➢ any object 'dislikes' any change to its state of motion, uniform motion tends to persist forever

this tendency to resist changes in motion is called the **inertia**

Newton's first law is also called *the law of inertia*

➢ if there is no change in state of motion, the object is said to be in **equilibrium**

equilibrium could be either *static* (being at rest) or *dynamic* (steadily moving in a straight line)

both cases require zero resultant force

➢ Newton's law of inertia is placed to establish frames of reference

it is in an reference frame that notions of displacement, velocity and acceleration can be defined

an **inertial reference frame** is one in which Newton's laws hold [20]

[19]These three laws were first addressed by *Isaac Newton* in his famous work *Mathematical Principles of Natural Philosophy*, or simply the *Principia*. The three-volume work was first published in 1687, and was soon recognised as one of the most important works in the history of science. Apart the from the three laws that laid the foundations for classical mechanics, the *Principia* also stated *the law of gravitation*, and accounted for planetary orbits and tides and other phenomena.

[20]Inertial frame is not unique. An observer moving at constant velocity to an inertial observer is in a different inertial frame, since constant velocity of object added to a constant relative velocity is still a constant velocity. Two inertial observers would disagree on a body's velocity, but they would agree that the body maintains its velocity in absence of net force, i.e., they will observe the same physics phenomena. This is

3.2.2 second law

if resultant force is non-zero, velocity of the object will change, i.e., force produces acceleration

Newton's second law states that the acceleration is proportional to the resultant force and inversely proportional to the mass of the object

- symbolically, we write $a \propto \dfrac{F_{\text{net}}}{m}$
  with consistent units of measure, this proportionality can be written as an exact equation:
$$a = \frac{F_{\text{net}}}{m} \quad \text{or} \quad \boxed{F_{\text{net}} = ma} \tag{3.1}$$
- SI unit of measurement for force F is **newton** (N)
  a force of one newton acting a body of 1 kg produces an acceleration of 1 m s^{-2}
- note that the force in the equation $F = ma$ is the resultant force
  to determine change in motion for a body, you should always ask what the resultant force is
- acceleration produced is always in same direction of the net force
- for same force, an object of greater mass has a smaller acceleration
  hence mass is a measure of the *inertia* of this object in response to a net force
  a definition for mass of an object from the point of view of Newton's laws can be stated as[21]:

mass is an intrinsic property of a body to resist any change in its state of motion

Example 3.1 A box of 6.0 kg is being pushed along a horizontal surface with a force of 30 N. The resistive force acting is 21 N. What is the acceleration of the box?

✍ $$F_{\text{net}} = F - f = ma \quad \Rightarrow \quad a = \frac{F - f}{m} = \frac{30 - 21}{6.0} = 1.5 \text{ m s}^{-2}$$ □

Example 3.2 A car of mass 800 kg is travelling at a speed of 20 m s^{-1}. The driver then operates the brake pedal so a braking force of 2000 N gradually brings the car to stop. (a) What is the deceleration for the car? (b) What is the stopping distance?

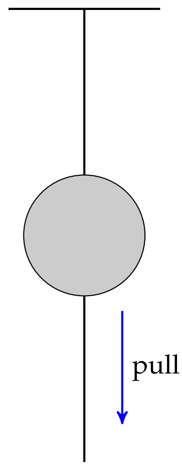
known as the *equivalence principle*.

[21]The concept of mass can be defined in many different ways. You might be familiar with the definition for mass as the amount of matter an object possesses. I personally think this definition is a bit vague and does not tell you anything new. Thinking of mass as a measure of inertia surely brings more insights. Mass also tells the strength at which an object interacts with other bodies through the gravitational attraction. As you will see later, from the view of Albert Einstein, it is also possible to think of mass as a form of energy, which is my favourite definition for mass.

✍ using Newton's second law and noticing braking force acts opposite to direction of motion:

$$F_{\text{net}} = ma \quad \Rightarrow \quad -2000 = 800 \times a \quad \Rightarrow \quad a = -2.5 \text{ m s}^{-2}$$
$$2as = v^2 - u^2 \quad \Rightarrow \quad s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 20^2}{2 \times (-2.5)} = 80 \text{ m}$$ □

Example 3.3 A massive ball is suspended on a string. A second string is attached to the bottom of the ball. If one pulls the bottom string with a gradually increasing force, does the top string or the bottom string break first? What if the bottom string is jerked, which string breaks?



✍ when tension gradually increases, system is always in equilibrium

tensions in strings must have $T_{\text{top}} = T_{\text{bottom}} + mg$

top string suffers a greater force, so it breaks first

however, when bottom string is jerked, the ball tends to remain at rest due to its large mass, preventing sudden change to the tension in top string

so in this case bottom string is more likely to snap □

3.2.3 third law

every force is part of a pair of interactions between one body and another

when one body exerts a force on another, the second body also exerts a reaction on the first

> **Newton's third law**, also called the **action-reaction principle**, states that action and reaction are always equal in magnitude, opposite in directions and of the same type

Example 3.4 Suggest the action and reaction force in the following cases: (a) A man stands on a bathroom scale. (b) A helicopter hovers in air. (c) The earth orbits around the sun.

✍ (a) man exerts downward force on scale, scale exerts an upward reaction on man

(b) rotors of helicopter push air downwards, air exerts an upward force on helicopter

(c) sun pulls the earth through gravitational attraction, earth also attracts the sun in return □

3.2.4 force analysis & free-body diagrams

when doing mechanics problems, it is necessary to find all forces applied upon an object

to visualise all these forces, it is helpful to draw a **free-body diagram** (FBD)

an FBD shows a simplified version of the body with arrows indicating forces applied it is recommended to follow the routine stated below when solving a mechanics problem

(1) draw a FBD for the object in the problem

(2) resolve and find the resultant force with aid of the FBD

(3) apply Newton's laws to write down the equation of motion for the object

(4) solve the equation(s) to find acceleration

(5) use kinematic relations to deduce information about motion of the object

3.3 types of forces

3.3.1 weight

all objects exert attractive forces of gravity upon each other[22]

weight of a body is due to the gravitational pull from our planet – the earth

weight W of any object is proportional to its mass m: $\boxed{W = mg}$

g is called the gravitational field strength, or the gravitational acceleration constant

➢ at vicinity of earth's surface, gravitational field is almost uniform: $g \approx 9.81\ \mathrm{N\ kg^{-1}}$

but this value for g does not hold in a satellite orbit, on Mars, near a black hole, etc.

➢ the concept of weight is different from mass in many aspects

- weight is a force, so it is a vector (always acting downwards still makes a direction)

  mass is a scalar, it has magnitude only
- weight is measured in newtons, mass is measured in kilograms
- weight of object depends on its mass but also strength of gravitational field

  mass is an intrinsic property of object, so does not depend on force fields

  same object can have different weights on different planets, but its mass will be the same[23]

Example 3.5 An astronaut finds that he weighs 300 N on the surface of Mars, where the gravitational field strength is known to be 3.7 N kg^{-1}. Find his mass and hence estimate his weight if he returns to his home on the Earth.

✍ mass of astronaut: $m = \dfrac{W_M}{g_M} = \dfrac{300}{3.7} \approx 81.1\ \mathrm{kg}$

weight on earth: $W_E = mg_E = 81.1 \times 9.81 \approx 795\ \mathrm{N}$ □

[22]You will learn more about gravitational forces at A2 Level.

[23]Here we do not take into account the effects of *relativity*. A clever student who has learned Einstein's theories might suggest the mass of the same object increases with its velocity.

free fall

all things on the earth fall because of the force of gravity

if we ignore the restraints such as air resistance and upthrust force on a falling object, say the object is under the influence of gravity only, then the object is in a state called **free fall**

assuming the object is subject to gravity only, the resultant force is simply its weight

applying the Newton's second law, we have: $F_{\text{net}} = W \quad \Rightarrow \quad ma = mg$

so acceleration of the freely-falling object is: $\boxed{a = g}$[24]

➢ this shows **acceleration due to free fall** is simply equal to field strength g

so any object, regardless of its mass, has same acceleration due to free fall[25]

3.3.2 drag

when a body moves through air, water or any fluid, it experiences resistance called drag force

➢ factors that determine the value of fluid drag include

- relative speed of the object to the fluid ($v \nearrow \Rightarrow f \nearrow$)
- cross section of the object ($A \nearrow \Rightarrow f \nearrow$)
- shape of the object (streamlined shape has smaller drag)
- density of the fluid ($\rho \nearrow \Rightarrow f \nearrow$)

but what determines the drag force is a complicated issue[26]

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[24]In the derivation, the mass terms cancel out. Rigorously speaking, these are two different masses. One is the measure of inertia, and the other is a measure of gravitational force. It is experimentally found that the inertia mass and the gravitational mass are equal. The fact that the two masses are equal has profound reasons. We have shown here acceleration of free fall equals gravitational field strength, but Albert Einstein's suggests that it is actually impossible to distinguish between a uniform acceleration and a uniform gravitational field. This idea lies at the heart of his *general theory of relativity*. Those who are interested in this topic are recommended to start from here and do some online researches.

[25]In §2.4.2 and §2.5, the statement that acceleration of free fall is constant in absence of air resistance was asserted without further explanation. Now you know why.

[26]There are a few empirical formula for drag force, each of which is accurate under certain conditions.

For an object moving through a fluid at low speeds (*laminar flow*, no turbulence occurs), the resistance it experiences is proportional to its speed: $f = bv$, where b is some constant which depends on fluid viscosity and the effective cross-sectional area of the object.

If objects are moving at relative high speeds through the fluid such that *turbulence* is produced behind the

➢ drag force always acts in a direction to oppose relative motion of object through fluid

free fall through air

let's consider an object falling through air from a very high tower

forces acting are weight and air resistance (shown in the free-body diagram)

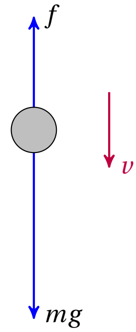
equation of motion for this falling object is:

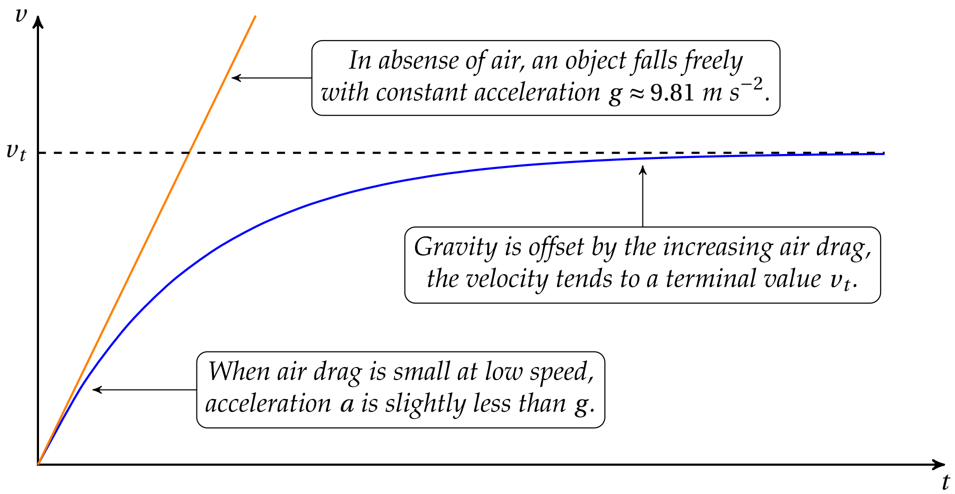
$$F_{\text{net}} = mg - f = ma$$

as v increases, air resistance f increases, so net force F_{net} decreases

this means acceleration a would decrease as object falls

i.e., speed will increase at a decreasing rate during the fall [27]





variation of velocity for a falling object through air

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object, drag force is proportional to the speed squared: $f = \frac{1}{2}\rho C_D A v^2$, where ρ is the fluid's density, A is the cross-sectional area, C_D is a dimensionless quantity called the drag coefficient.

[27]The velocity-time relation can be obtained for some simple models. Suppose air resistance is proportional to speed of the falling body, i.e., $f = bv$, then the equation of motion reads: $F_{\text{net}} = m\frac{\mathrm{d}v}{\mathrm{d}t} = mg - bv$, where acceleration is written explicitly as the rate of change in velocity. With the initial conditions $v = 0$ at $t = 0$, we can solve this differential equation to obtain the speed of this falling object at any given time t:

$$\mathrm{d}t = \frac{\mathrm{d}v}{g - \frac{b}{m}v} \quad \Rightarrow \quad \int_0^t \mathrm{d}t = \int_0^v \frac{\mathrm{d}v}{g - \frac{b}{m}v} \quad \Rightarrow \quad t = -\frac{m}{b}\ln\left(g - \frac{b}{m}v\right)\Big|_0^v = -\frac{m}{b}\ln\left(1 - \frac{bv}{mg}\right)$$

Rearrange the terms, we find: $v(t) = \frac{mg}{b}\left(1 - \mathrm{e}^{-\frac{bt}{m}}\right)$

- after sufficient long time, acceleration gradually decreases to zero

  velocity gradually increases and tends to a maximum value

  at this stage, equilibrium is restored: $f = mg$, object no longer accelerates

  this constant final velocity is known as the **terminal velocity**

- at low speeds, air resistance is negligible, so $F_{\text{net}} = ma \approx mg$

  acceleration of object at start of the fall is similar to g

  but as v increases, acceleration decreases so a becomes less than g

Example 3.6 An object of 5.0 kg falls through the atmosphere from a very high altitude. After some time, it falls at a constant speed of 70 m s^{-1}. Assume there is no significant change in gravitational field during the fall and the air resistance is proportional to speed: $f = bv$. (a) Find the value of the coefficient k. (b) Find the acceleration of the object when it is falling at 30 m s^{-1}.

✍ equilibrium between weight and air drag when falling at terminal speed, so

$$mg = bv_t \quad \Rightarrow \quad b = \frac{mg}{v_t} = \frac{5.0 \times 9.81}{70} \approx 0.70 \text{ kg s}^{-1}$$

at any instant, equation of motion is: $F_{\text{net}} = ma = mg - bv$

at 30 m s^{-1}, acceleration is: $a = \dfrac{mg - bv}{m} = \dfrac{5.0 \times 9.81 - 0.70 \times 30}{5.0} \approx 5.6 \text{ m s}^{-2}$ □

bubble rising in a liquid

let's now consider bubbles formed at the bottom of a soda water

forces acting on bubble are weight, water resistance and upthrust

equation of motion for the rising bubble is:

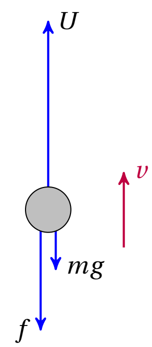
$$F_{\text{net}} = U - mg - f = ma$$

as bubble moves faster, f increases, then F_{net} decreases

so acceleration a would gradually decrease to zero as bubble rises

speed of bubble increases and reaches a maximum value

at terminal speed, $a \to 0$, one has: $U = f + mg$



3.3.3 normal contact

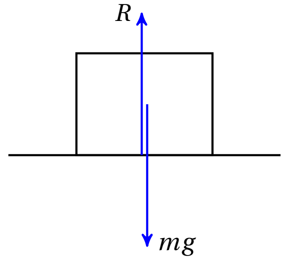
when two objects are in contact, the interaction between them is called the *contact force*

normal contact force is the component of contact force that is perpendicular to contact surface

- by definition, normal contact is always at right angle to surface of contact
- origin of normal contact is the *electrostatic interaction* between atoms

when two objects are pressed against each other, surface atoms get close

electrostatic repulsion between electron clouds of the atoms prevent them from penetrating through one another

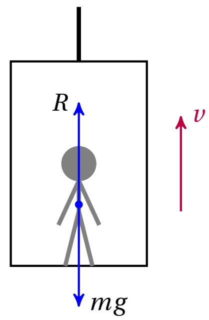


Example 3.7 A box of mass $m = 4.0$ kg is resting on a horizontal ground. What is the normal contact force acting?

✍ equilibrium between weight and normal contact, so

$$R - W = 0 \quad \Rightarrow \quad R = mg = 4.0 \times 9.81 \approx 39.2 \text{ N} \qquad \square$$

Example 3.8 A man of 80 kg stands in a lift. Find his apparent weight, i.e., the contact force, when the lift is (a) moving upwards at steady speed of 2.0 m s^{-1}, (b) accelerating upwards at 2.0 m s^{-2}, (c) moving upwards but slowing down at a deceleration of 1.5 m s^{-2}.



✍ forces acting on man are weight and normal contact

for either case, equation of motion for the man reads:

$$F_{\text{net}} = ma = R - mg$$

so normal contact force: $R = mg + ma$

when rising at steady speed, man is in equilibrium ($a = 0$), so: $R = mg = 80 \times 9.81 \approx 785$ N

when accelerating upwards ($a = +2.0 \text{ m s}^{-2}$): $R = 80 \times 9.81 + 80 \times 2.0 \approx 945$ N

when decelerating upwards ($a = -1.5 \text{ m s}^{-2}$): $R = 80 \times 9.81 + 80 \times (-1.5) \approx 665$ N $\square$

Example 3.9 A sleigh of mass 15 kg lies at rest on an icy ground. The surface is frictionless. A force P of 75 N is applied to the sleigh. Find the normal contact force and the acceleration of the sleigh if P is acting (a) horizontally, (b) at an angle α to the horizontal where $\tan\alpha = \frac{3}{4}$.

✍ free-body diagrams for both cases are shown

for both cases, no net force acts in vertical direction

net force in horizontal direction provides acceleration

when P acts horizontally:

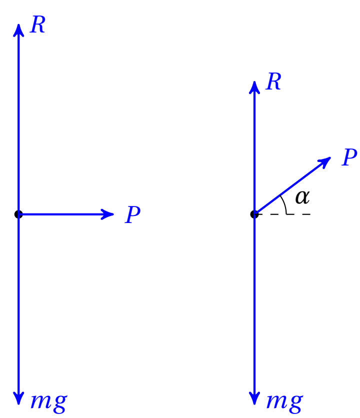
$$R = mg = 15 \times 9.81 \approx 147 \text{ N}$$

$$P = ma \quad \Rightarrow \quad a = \frac{P}{m} = \frac{75}{15} = 5.0 \text{ m s}^{-2}$$

when P acts at angle α:

$$R + P\sin\alpha = mg \quad \Rightarrow \quad R = 15 \times 9.81 - 75 \times \frac{4}{5} \approx 87 \text{ N}$$

$$P\cos\alpha = ma \quad \Rightarrow \quad a = \frac{P\cos\alpha}{m} = \frac{75 \times \frac{3}{5}}{15} = 3.0 \text{ m s}^{-2} \quad \square$$



(a) (b)

3.3.4 friction

friction is the component of contact force that is parallel to contact surfaces

when there is potential or actual sliding between surfaces, frictional force come into action

– for surfaces *tend* to move relative to each other, **static friction** acts to oppose this tendency

– if surfaces are already sliding over one another, then **kinetic friction** opposes this motion

➢ static friction f_S is self-adjusting

an object placed on a rough surface can stay at rest when acted by a small external force F

it can do so because f_S equalises external force to maintain equilibrium

if no external force acts, then $f_S = 0$

➢ there exists a maximum limiting friction $f_{\lim}$

when external force $F < f_{\lim}$, there is sufficient f_S to prevent object from sliding

when $F = f_{\lim}$, object is on the verge of sliding

when $F > f_{\lim}$, object start to move and static friction becomes kinetic friction f_K

➢ factors that determine frictional forces are

– nature of contacting surfaces (for both f_S and f_K)

– normal reaction R (for f_K)

these dependences are usually expressed by a mathematical equation $\boxed{f_K \approx f_{\lim} = \mu R}$

μ is the *coefficient of friction* whose value depends on the nature of the two surfaces[28]

➢ friction, on microscopic level, is an *electromagnetic force* in nature

when two surfaces are in contact, irregularities on the surface touch each other

surface atoms come very close and bonds are formed through electrostatic force

in some sense, surface atoms get *cold welded* to each other

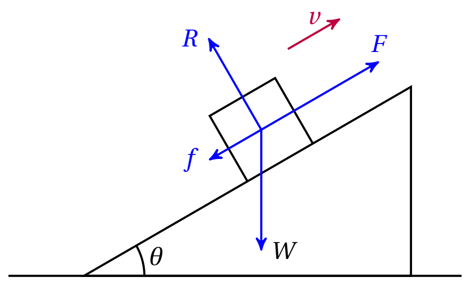
when surfaces try to move relative to each other, this electrostatic weld is origin of friction

3.4 inclined slope

inclined slope is probably the entry ticket into the business of mechanics

this notorious problem is found in any physics textbook and any exam paper on mechanics

[28]The idea of limiting friction is not required in the AS Physics syllabus, but it is required in *Mechanics 1* of the A-Level Mathematics course.



the problem is about a mass m placed on a plane inclined at angle θ to the horizontal

the mass could sit at rest on, slide down, or get pulled/pushed up the plane

motion of the mass could be affected by weight, friction, normal contact, or other forces

➢ forces can be resolved in directions parallel and perpendicular to the slope

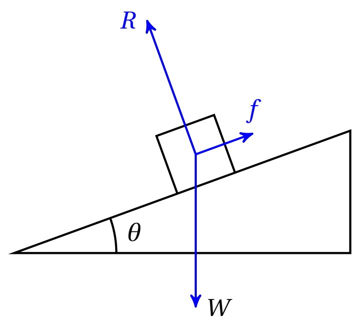
resolving along the slope leads to the equation of motion from which acceleration is found

➢ it is almost inevitable to break weight into two components[29]

– component of weight parallel down the slope is: $W_{/\!/} = mg\sin\theta$

– component of weight perpendicular to slope is: $W_{\perp} = mg\cos\theta$

Example 3.10 A block of mass m stays at rest on an inclined plane. The plane makes an angle θ with the horizontal. Find the normal contact force R and the frictional force f acting on the block.

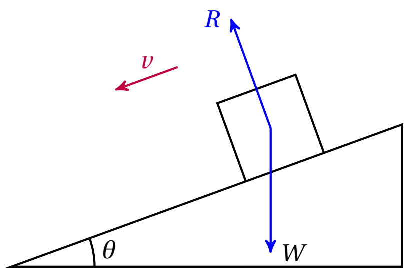


✍ block in equilibrium, so $F_{\text{net}} = 0$ in any direction

parallel to slope: $f = W_{/\!/} \quad \Rightarrow \quad f = mg\sin\theta$

normal to slope: $R = W_{\perp} \quad \Rightarrow \quad R = mg\cos\theta$ □

Example 3.11 A block of mass m slides down a *smooth* slope. The angle of the slope to the horizontal is θ. Find the acceleration of the block.



✍ only force acting along the slope is component of weight down the slope, so:

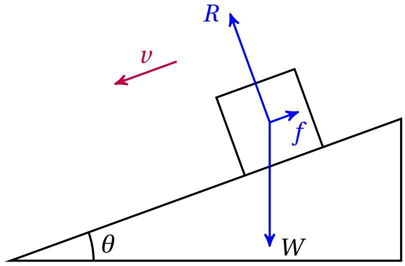
$$F_{\text{net}} = ma = mg\sin\theta \quad \Rightarrow \quad a = g\sin\theta$$

as $\theta \to 0$, $a \to 0$, this shows if plane becomes horizontal, the block simply stays put

as $\theta \to 90^\circ$, slope becomes vertical, block would undergo free fall, so naturally $a \to g$ □

[29]We have already done that in Example 1.7.

Example 3.12 A block of mass 2.0 kg slides down a rough slope from rest. The slope is inclined at angle $\theta = 20°$ to the horizontal, and the block experiences a constant friction of 5.0 N. (a) What is the block's acceleration? (b) What is the distance travelled in 2.5 seconds?



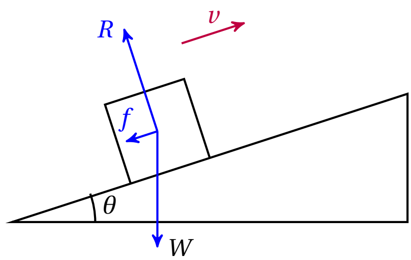
✍ resolving along slope:

$$F_{\text{net}} = mg\sin\theta - f = ma$$

$$a = \frac{2.0 \times 9.81 \times \sin 20° - 5.0}{2.0} \approx 0.855 \text{ m s}^{-2}$$

distance travelled: $s = \cancel{ut}^{\,0} + \frac{1}{2}at^2 = \frac{1}{2} \times 0.855 \times 2.5^2 \approx 2.67$ m □

Example 3.13 A block of mass 3.0 kg is travelling up an inclined slope at an initial speed of 2.8 m s^{-1}. The slope makes an angle of 18° with the horizontal. A constant friction of 7.5 N acts on the block. (a) What is the block's deceleration? (b) How far does the block travel along the slope before its speed decreases to zero? (c) Suggest whether the block could stay on the slope.



✍ resolving along slope (take direction of initial velocity as positive direction):

$$F_{\text{net}} = -mg\sin\theta - f = ma \quad \Rightarrow \quad a = \frac{-mg\sin\theta - f}{m} = \frac{-3.0 \times 9.81 \times \sin 18° - 7.5}{3.0} \approx -5.53 \text{ m s}^{-2}$$

$$v^2 - u^2 = 2as \quad \Rightarrow \quad s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 2.8^2}{2 \times (-5.5.3)} \approx 0.709 \text{ m}$$

note that component of weight down the slope is: $W_{/\!/} = mg\sin\theta = 3.0 \times 9.81 \times 18° \approx 9.1$ N

$W_{/\!/} > f$, so friction is not enough to prevent block from sliding back down the slope □

3.5 many-body problems

the problems we have been dealing with so far only involve one body

a mechanical system could consist of several objects that mutually interact

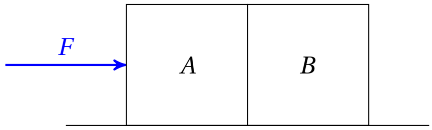
- ➢ one can take each individual and look into the *internal* forces between the objects of interest

  for any force acting between objects *within* system, there is an equal but opposite reaction force

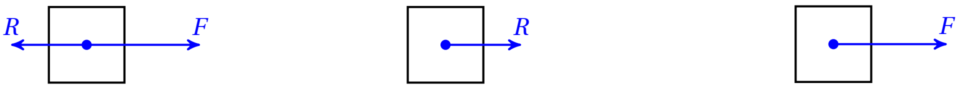
- ➢ the system can also be treated as a whole

  we can analyse *net external force* acting on entire system and work out combined acceleration

Example 3.14 Two boxes A and B are placed on a smooth surface. They are accelerated together by a horizontal force F as shown. Find the acceleration and the contact force between them.



✍ free-body diagrams for A, B, and entire system are given below



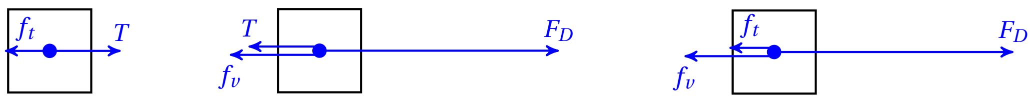
A B A and B combined

equations of motion can be written down for each free-body diagram and solved[30]

$$\begin{cases} \text{for } A: & F - R = M_A a \\ \text{for } B: & R = M_B a \\ \text{for system:} & F = (M_A + M_B) a \end{cases} \Rightarrow \begin{cases} a = \dfrac{F}{M_A + M_B} \\ R = \dfrac{M_B}{M_A + M_B} F \end{cases}$$ □

Example 3.15 A vehicle of mass 1500 kg is towing a trailer of mass 500 kg by a light inextensible tow-bar. The engine of the vehicle exerts a driving force of 9600 N, and the tractor and the trailer experience resistances of 3600 N and 1800 N respectively. Find the acceleration of the vehicle and the tension in the tow-bar.

✍ free-body diagrams for trailer, vehicle and entire system are given below



trailer vehicle system

equations of motion can be written down for each free-body diagram:

$$\begin{cases} \text{for trailer:} & T - f_t = M_t a \\ \text{for tractor:} & F_D - f_v - T = M_v a \\ \text{for system:} & F_D - f_v - f_t = (M_v + M_t) a \end{cases} \Rightarrow \begin{cases} T - 1800 = 500a \\ 9600 - 3600 - T = 1500a \\ 9600 - 3600 - 1800 = (1500 + 500) a \end{cases}$$

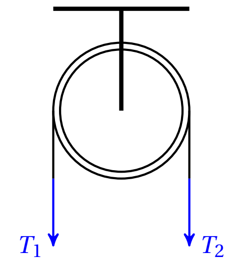
solving simultaneous equations[31], we find

$$a = 2.1 \text{ m s}^{-2}, \quad \text{and} \quad T = 2850 \text{ N}$$ □

[30] In fact, only two of the three equations are independent. You can easily check that adding the equation for A to that for B would produce the equation for the system. To solve the two unknowns for this problem, any two of the three equations shall do the job.

[31] Again, only two of the three equations are independent. You can freely choose your favourite two.

pulleys



a *pulley* is basically a wheel that carries a string/rope/cable

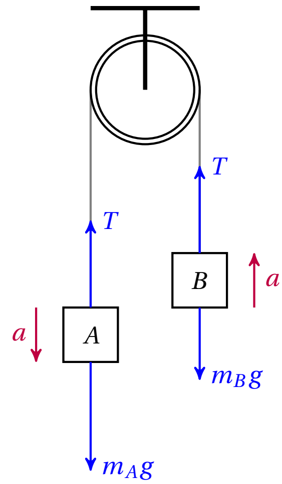
in this section, we only consider pulleys whose axis of rotation is fixed

such pulleys can be used to change direction of tension in a taut string

we also assume pulleys to be *ideal*: they have no mass and no friction

for an ideal pulley, tensions on both sides are equal: $T_1 = T_2$

Example 3.16 Two blocks of mass m_A and m_B ($m_A > m_B$) are joined together by a light inextensible string. The string passes over a smooth pulley as shown. The two blocks are suddenly released from rest. Find the acceleration of each block and the tension in the string.



✍ apply Newton's second law to each block:

$$\begin{cases} \text{for } A: & m_A g - T = m_A a \\ \text{for } B: & T - m_B g = m_B a \end{cases}$$

adding the two, one obtains equation of motion for whole system:

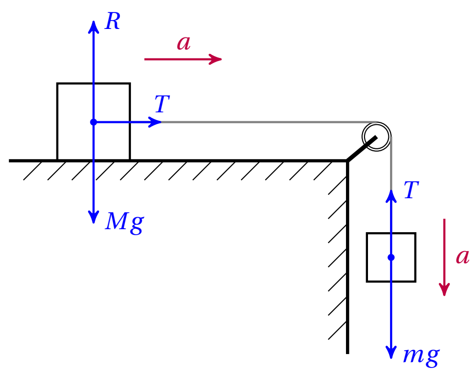
$$m_A g - m_B g = (m_A + m_B) a$$

solving these equations, we find

$$a = \frac{m_A - m_B}{m_A + m_B} g \qquad T = \frac{2 m_A m_B g}{m_A + m_B}$$

□

Example 3.17 A mass $M = 4.0$ kg is attached to a block of mass $m = 2.0$ kg through a light string which passes over a frictionless pulley as shown. When both masses are released, find the acceleration and the tension in the string.



✍ apply Newton's second law to each mass:

$$\begin{cases} \text{for } M: & T = Ma \\ \text{for } m: & mg - T = ma \end{cases}$$

adding the two equations, we have:

$$mg = (M + m) a$$

so acceleration is:

$$a = \frac{mg}{M+m} = \frac{2.0 \times 9.81}{4.0 + 2.0} = 3.27 \text{ m s}^{-2}$$

tension in string: $T = Ma = 4.0 \times 3.27 \approx 13.1$ N □

3.6 end-of-chapter questions

Newton's first law

Question 3.1 A little girl tries to lift a luggage bag of mass 25 kg. She pulls upwards with a force of 150 N. The bag does not move. What is the normal reaction from the floor?

Question 3.2 To push a trolley around in a supermarket with constant velocity, you need to exert a steady force. How does this fact agree with Newton's first law, which suggests that motion with constant velocity requires no force?

Question 3.3 A worker is pulling a wagon of mass of 40 kg across a lawn at a constant velocity. He applies a force of 200 N at an angle of 15° above the horizontal. (a) Draw a free-body diagram for the wagon. (b) Find the frictional force. (c) Find the normal contact force.

Newton's second law

Question 3.4 (a) Forces of 3.0 N and 4.0 N act at right angles upon a mass of 160 g. What is the acceleration produced? (b) If the angle between the two forces are allowed to vary, what is the maximum possible acceleration they produce on the same mass? (c) What about the minimum possible acceleration?

Question 3.5 Explain why it becomes increasingly easier for an rocket to accelerate as it travels through space. (Hint: consider the fuel carried by the rocket.)

Question 3.6 Many cars are equipped with airbags which can inflate quickly in case of a collision event. Using Newton's second law, suggest why airbags could protect the driver and the passenger in the car during a car crash.

Question 3.7 A rocket of mass 30,000 kg is launched vertically upwards at uniform acceleration of 1.6 m s^{-2}. What is the minimum thrust force required?

Question 3.8 A fire-fighter of mass 85 kg slides down a vertical pole. He descends through a distance of 6.0 m in 2.0 seconds. (a) Find the average acceleration. (b) Find the average frictional force acting on the fire-fighter.

Question 3.9 A trolley has mass m. A person needs to push the trolley with force F to produce an acceleration of a, and with force $2F$ to produce an acceleration of $3a$. Find, in terms of m and a, the constant resistive force opposing the trolley's motion.

Question 3.10 A girl stands onto a bathroom scale and finds the reading is 35.0 kg. She then takes the scale into a lift, what mass reading would she observe if the lift (a) is going down at a constant speed, (b) is accelerating downwards at 2.1 m s^{-2}?

Question 3.11 A pirate finds a box of gold coins at the bottom of a lake. The box and its contents have a total mass of 40 kg. The pirate pulls on the box by means of a cable, so that the box is made to rise vertically through the water. Meanwhile, the flow of water creates a constant horizontal force on the box, and the upthrust on the box is known to be 150 N. At one instant, the pirate applies a force of 380 N at an angle of 25° to the upward vertical, and the acceleration of the box is found to be 0.80 m s^{-2}. Assume all the forces acting are coplanar. (a) Draw a free-body diagram for the box. (b) Find the horizontal force due to water flow. (c) Find the drag force on the box.

Newton's third law

Question 3.12 A book placed on your desk experiences two forces: its weight and the support force. Identify the associated reaction forces.

Question 3.13 A student deduces that a rocket travelling in space can never accelerate because the propelling force acting on the rocket is cancelled by an equal and opposite force. Explain why this statement is incorrect.

Question 3.14 A U-shaped magnet lies on a top-pan balance and a mass reading of 180 g is registered. A current-carrying wire is then placed above the magnet. The wire experiences an additional force of 0.30 N that acts upwards. What is the mass reading on the balance?

terminal velocity

Question 3.15 A light ball and a heavy ball of the same size are released from a very high tower, state and explain whether they will reach the ground at the same time.

Question 3.16 A stone is dropped from rest from a high tower. Air resistance is not negligible as the stone reaches terminal speed. Sketch two separate graphs to show the variation of its displacement and acceleration with time.

Question 3.17 How does the terminal speed of a parachutist before opening the parachute compare to that after? Explain your reasons.

Question 3.18 A ball is thrown horizontally from the top of a cliff. Effects of air resistance cannot be neglected. What happens to the horizontal and vertical components of the ball's velocity?

Question 3.19 A small sphere of mass 20.0 g is dropped from rest in a viscous liquid. When the sphere is moving at a speed of v, the viscous drag has a magnitude of $f = \alpha v^2$, where α = 14 kg m^{-1}. (a) What is the sphere's acceleration at the instant when it is released? (b) What is the acceleration when it is moving at 5.0 cm s^{-1}? (c) What is the terminal velocity?

Question 3.20 A stone is thrown with some initial velocity at an angle to the horizontal. Sketch on the same graph the path of the stone if (a) air resistance is negligible, (b) air resistance is significant.

inclined slopes

Question 3.21 A 3.0 kg mass is placed on an inclined plane and it does not move. Given that the normal contact force acting on it is 28.0 N. (a) Find the angle of the plane to the horizontal. (b) Find the frictional force acting on the mass.

Question 3.22 A small mass slides down a frictionless slope with an acceleration of 2.8 m s^{-2}. Determine the angle that the slope makes with the horizontal.

Question 3.23 A car of mass 1400 kg is moving up a slope at a constant velocity of 13.5 m s^{-1}. The slope makes an angle of 6.0° to the horizontal. Total resistive force of 650 N acts on the car. What is the driving force required to push the car up the slope?

Question 3.24 A shopping trolley somehow loses control and runs down a straight slope from rest. The slope makes an angle of 3.0° to the horizontal. The resistive force acting on the trolley is a constant 15 N. The trolley and its contents have a total mass of 40 kg. (a) Find the acceleration of the trolley. (b) Determine the time for the trolley to travel a distance of 4.0 m along the the slope. (c) Suggest why the slope in shopping malls are not made any steeper.

Question 3.25 A heavy log of mass 240 kg is initially placed at a point P at the bottom of a slope. A motor drags the log up the slope through a cable. The slope is inclined at an angle of 16° to the horizontal. The motor provides a tension of 1200 N parallel to the slope. The friction that acts on the log is a constant 450 N. (a) Find the acceleration of the log. (b) Find the time taken to pull the log through a distance of 8.0 m to a point Q. (c) Find the velocity of the log at Q. (d) The cable breaks when the log reaches Q, find the distance moved beyond Q until the log's speed becomes zero. (e) The log will then slide back down the slope. Find the time for the log to return to its starting position. (f) Sketch a v-t graph for the log from the start at P until it returns to P.

many-body problems

Question 3.26 Block A of mass 5.0 kg is connected by means of a light string to block B of mass 3.0 kg. The two blocks are placed on a horizontal table. A force of 30 N is applied to pull on block A. Given that the friction on each block is 30% of its own weight. (a) Find the acceleration of the blocks. (b) Find the tension in the string.

Question 3.27 A box of mass $M = 3.6$ kg rests on a horizontal, rough surface. The box is connected to a block of mass $m = 2.0$ kg through a light cord that passes over a frictionless pulley as shown. The box is released from rest. Given that the box experiences a frictional force of 12 N and the block is initially at a height of $H = 0.80m$ above the floor. (a) Find the acceleration of the block. (b) Determine the time taken for the block to hit the floor.