Digital Image Processing

Homework 1 Solutions

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1 Solution:

Prove that: $\nabla^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2}$

We know that:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Then, we find:

$x' = xcos(\theta) - ysin(\theta)$ $\qquad$ $y' = xsin(\theta) + ysin(\theta)$

Now, We find the x and y:

$$x = \frac{x' + ysin(\theta)}{cos(\theta)} \qquad x = \frac{y' - ysin(\theta)}{sin(\theta)}$$

$$y = \frac{xcos(\theta) - x'}{sin(\theta)} \qquad y = \frac{y' - xsin(\theta)}{cos(\theta)}$$

$$x = \frac{x' + ysin(\theta)}{cos(\theta)} = \frac{y' - ysin(\theta)}{sin(\theta)} \qquad y = \frac{xcos(\theta) - x'}{sin(\theta)} = \frac{y' - xsin(\theta)}{cos(\theta)}$$

We found $x' = xcos(\theta) - ysin(\theta)$ and we find the derivative of x and y.

$\dfrac{\partial x}{\partial x'} = cos(\theta)$ $\qquad$ $\dfrac{\partial y}{\partial x'} = -sin(\theta)$

And, we can write:

$\dfrac{\partial f}{\partial x'} = \dfrac{\partial f}{\partial x} cos(\theta) - \dfrac{\partial f}{\partial y} sin(\theta)$

Then, we take derivative this $\dfrac{\partial f}{\partial x'}$ function.

$$\frac{\partial^2 f}{\partial x'^2} = \frac{\partial^2 f}{\partial x^2} cos^2(\theta) - \frac{\partial}{\partial x}(\frac{\partial f}{\partial y}) \sin\theta \cos\theta - \frac{\partial}{\partial y}(\frac{\partial f}{\partial x}) \cos\theta \sin\theta + \frac{\partial^2 f}{\partial y^2} \sin^2\theta \qquad \text{(I)}$$

We know $y' = xsin(\theta) + ysin(\theta)$ and we can find $\dfrac{\partial x}{\partial y'} = \sin\theta$ and $\dfrac{\partial y}{\partial y'} = \cos\theta$.

And, we can write:

$\dfrac{\partial f}{\partial y'} = \dfrac{\partial f}{\partial x} \sin\theta + \dfrac{\partial f}{\partial y} \cos\theta$

Then, we take derivative this $\dfrac{\partial f}{\partial y'}$ function.

$$\frac{\partial^2 f}{\partial y'^2} = \frac{\partial^2 f}{\partial x^2} sin^2(\theta) + \frac{\partial}{\partial x}(\frac{\partial f}{\partial y}) \sin\theta \cos\theta + \frac{\partial}{\partial y}(\frac{\partial f}{\partial x}) \cos\theta \sin\theta + \frac{\partial^2 f}{\partial y^2} \cos^2\theta \qquad \text{(II)}$$

As a result of, We add (I) and (II) equations we can find,

$$\implies \frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

2 Solution :

A metric on a set X is a function $d : X$ x $X \to [0, \infty)$ satisfying:
$d(p,q) \leq d(p,r) + d(r,q) \to subaddivity$

The Minkowski distance is very common:
$\forall p = (p_1, p_2), q = (q_1, q_2) \in Z^2$
$d(p,q) = (|p_1 - q_1|^r + |p_2 - q_2|^r)^{\frac{1}{r}}$ For r = 1, it is city-block of L_1 distance.

Now, we apply *subaddivity* method to L_1, we find :

$d(p,q) \leq d(p,r) + d(r,q)$
$d(p,q) \leq (|p_1 - r_1| + |p_2 - r_2|) + (|r_1 - q_1| + |r_2 - q_2|)$
$(|p_1 - q_1| + |p_2 - q_2|) \leq (|p_1 - r_1| + |p_2 - r_2|) + (|r_1 - q_1| + |r_2 - q_2|)$

If everything in absolute value is considered positive.($p > r > q$)
$\Rightarrow (p_1 - q_1 + p_2 - q_2) \leq (p_1 - r_1 + p_2 - r_2) + (r_1 - q_1 + r_2 - q_2)$ Simply r_1 and r_2 gone. Then:
$\Rightarrow (p_1 - q_1 + p_2 - q_2) \leq p_1 + p_2 - q_1 - q_2$

For this result, equals on both sides. The right side can never be less than the left side.

3 Solution :

As we know T.B= A. Transformation matrix is unknown. We need to find T.

$$\begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

B= $\begin{bmatrix} 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 2 & 1 \end{bmatrix}$ $\begin{bmatrix} 3 & 1 \end{bmatrix}$

A= $\begin{bmatrix} 2 & 2 \end{bmatrix}$ $\begin{bmatrix} -1 & 4 \end{bmatrix}$ $\begin{bmatrix} -4 & 4 \end{bmatrix}$

$x.t_{11} + y.t_{12} + 1.t_{13} = x'$ (function 1)
$x.t_{21} + y.t_{22} + 1.t_{23} = y'$ (function 2)

$1.t_{11} + 2.t_{12} + 1.t_{13} = 2$ (1a)
$1.t_{21} + 2.t_{22} + 1.t_{23} = 2$ (1b)

$2.t_{11} + 1.t_{12} + 1.t_{13} = -1$ (2a)
$2.t_{21} + 1.t_{22} + 1.t_{23} = 4$ (2b)

$3.t_{11} + 1.t_{12} + 1.t_{13} = -4$ (3a)
$3.t_{21} + 1.t_{22} + 1.t_{23} = 4$ (3b)

Calculate:
$-(2a) + (3a) \Rightarrow t_{11} = -3$
$-(2b) + (3b) \Rightarrow t_{21} = 0$

Then 1a, 1b, 2a and 2b will be :
$2.t_{12} + 1.t_{13} = 5$ (1a)
$2.t_{22} + 1.t_{23} = 2$ (1b)
$1.t_{12} + 1.t_{13} = 5$ (2a)
$1.t_{22} + 1.t_{23} = 4$ (2b)

$(1a) + -(2a) \Rightarrow t_{12} = 0$
$(1b) + -(2b) \Rightarrow t_{22} = -2$

Then we find:
$t_{13} = 5$ $\qquad$ $t_{23} = 6$

Finally, Transaction Matrix: $\begin{bmatrix} -3 & 0 & 5 \\ 0 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

4 Solution :

C and D are two sets and B is a structuring element.

$$(C \cup D) \oplus B = (C \oplus B) \cup (D \oplus B) \tag{1}$$

This equation is not true. Because; dilation of $(D \in C)$ with a structuring element B is not same as the union of dilation of C with B and dilation of D with B.
But, distributivity over union. Dilation is distributive over the union of structuring elements.

If we want (1) equality to be true $\Rightarrow$ C and D should be structuring elements and B should be a set. Then this is true.