THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH3280 Introductory Probability 2022-2023 Term 1
Suggested Solutions of Homework Assignment 1

Q1

(a). $S = \{(i,j) : i,j \in (1,\ldots,6)\}$.
(b). $A = \{(i,j) : i,j \in (1,\ldots,6) \; where \; i \geq j\}$.
(c). $B = \{(6,j) : j \in (1,\ldots,6)\}$.
(d). Since $B \subset A$, B implies A.
(e). $A \cap B^c$ is the event 'number of dots in first toss is not less than number of dots in second toss, while the first toss is not 6'.
(f). $A \cap C = \{(6,4),(5,3),(4,2),(3,1)\}$.

Q2

(a) The event that either A or B occurs is $A \cup B$, thus

$$Pr(A \cup B) = Pr(A) + Pr(B) = 0.35 + 0.54 = 0.89.$$

(b) The event that A occurs but B does not is $A \cap B^c$, thus

$$Pr(A \cap B^c) = Pr(A) = 0.35.$$

(c) The event that both A and B occur is $A \cap B$ which is the empty set. Hence the answer is 0.

Q3

Let C_1 be the event that Hong Kong males smoke cigarettes and C_2 be the event that Hong Kong males smoke cigars.
(a) Since

$$Pr(C_1^c \cap C_2^c) = Pr((C_1 \cup C_2)^c) = 1 - Pr(C_1 \cup C_2) = 1 - (Pr(C_1) + Pr(C_2) - Pr(C_1 \cap C_2))$$

the percentage of males who smoke neither cigars nor cigarettes is $[1 - (0.28 + 0.08 - 0.06)] \times 100\% = 70\%$.
(b) Since

$$Pr(C_2 C_1^c) = Pr(C_2) - Pr(C_2 C_1),$$

the percentage of males who smoke cigars but not cigarettes is $(0.08 - 0.06) \times 100\% = 2\%$.

Q4

(a) The total number of permutations of the five people is 5!. There are C_1^3 ways to choose a person X from Carl, Dan, and Eddy who are arranged between Adin and Bob as $AdinXBob$. We have 3! ways to permute the people when we regard $AdinXBob$ as an object. We also have 2 ways to arrange them within the object $AdinXBob$ because we can switch Adin and Bob. Hence the probability that there is exactly one person between Adin and Bob is

$$\frac{C_1^3 \times 3! \times 2!}{5!} = 0.3$$

(b) There are C_2^3 ways to choose two people X, Y from Carl, Dan, and Eddy who are arranged between Adin and Bob as $AdinXYBob$. We have 2! ways to permute the people when we regard $AdinXYBob$ as an object. We also have $2! \times 2!$ ways to arrange them within the object $AdinXYBob$ because we can switch Adin and Bob and switch X and Y. Hence the probability that there are exactly two people between Adin and Bob is

$$\frac{C_2^3 \times 2! \times 2! \times 2!}{5!} = 0.2$$

(c) There is only one way to choose three people X, Y, Z from Carl, Dan, and Eddy who are arranged between Adin and Bob as $AdinXYZBob$. We have $2! \times 3!$ ways to arrange them within the object $AdinXYZBob$ because we can switch Adin and Bob and permute X, Y and Z. Hence the probability that there are exactly three people between Adin and Bob is

$$\frac{2! \times 3!}{5!} = 0.1$$

Q5

(a) $A \cap B^c \cap C^c$
(b) $A \cup B \cup C$
(C) $A^c \cap B^c \cap C^c$
(d) $(A \cap B \cap C^c) \cup (A \cap B^c \cap C) \cup (A^c \cap B \cap C)$
(e) $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$
(f) $(A \cap B^c) \cup (B \cap A^c)$

Q6

Proof.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) \geq P(A) + P(B) - 1$$

□

Q7

Proof. Since
P(A,B occur while C does not occur) $= P(A \cap B \cap C^c) = P(A \cap B) - P(A \cap B \cap C)$,
P(A,C occur while B does not occur) $= P(A \cap C \cap B^c) = P(A \cap C) - P(A \cap B \cap C)$,
P(B,C occur while A does not occur) $= P(B \cap C \cap A^c) = P(B \cap C) - P(A \cap B \cap C)$,
then
P(exactly two of these events will occur) $= P(A \cap B \cap C^c) + P(A \cap C \cap B^c) + P(B \cap C \cap A^c) = P(A \cap B) + P(A \cap C) + P(B \cap C) - 3P(A \cap B \cap C)$. □