MATH3280A Tutorial 1

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A continuation of Example 5

Let $k \geq 2$ an integer and $t = (t_1, \ldots, t_k)$ be a vector of non-negative integers.

For each $0 \leq s \leq (t_1 + \cdots + t_k)$, how many vectors $(n_1, \ldots, n_k)$ are there such that

$$\begin{cases} n_1 + \cdots + n_k = s \\ 0 \leq n_i \leq t_i \quad , \forall\, 1 \leq i \leq k \end{cases} \qquad ?$$

We denote the total number of such vectors by $N_t(s)$ as a function of the target sum s depending on the treshold vector t.

A theoretical formula: For $0 \leq i \leq k$, define the multisets (i.e., allowing repeating elements)

$$T_i := \left\{ t \cdot b : b = (b_1, \ldots, b_k) \in \{0, 1\}^k \text{ and } b_1 + \cdots + b_k = i \right\},$$

where $\cdot$ denotes the standard inner product, then

$$N_t(s) = \sum_{i=0}^{k} (-1)^i \sum_{x \in T_i} \binom{s - (x+i) + k - 1}{k-1} \chi_{[x+i, +\infty)}(s) \quad .$$

Specific setup

```
t =  [12, 10, 25]
```

```
t = [12, 10, 25] .|> Int
```

$s =$ 18

Count by enumeration: $C_t(s) = 133$.

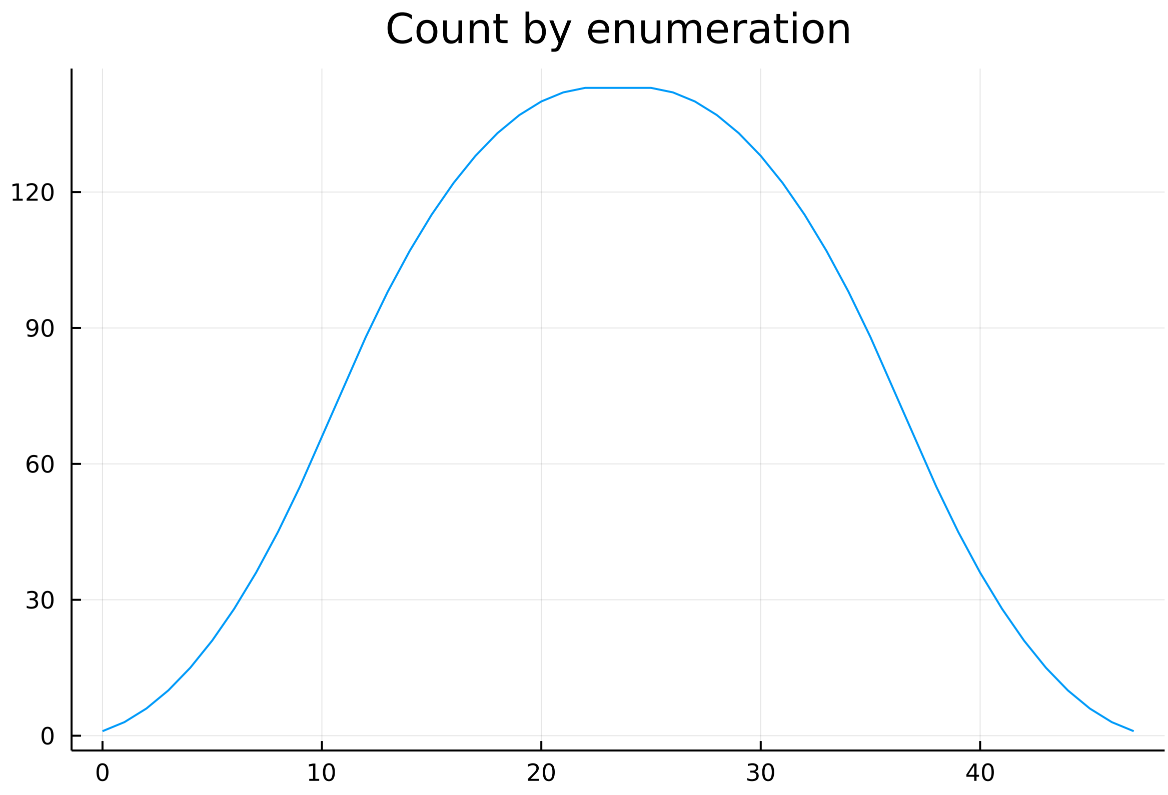
Output by formula: $N_t(s) = 133$.

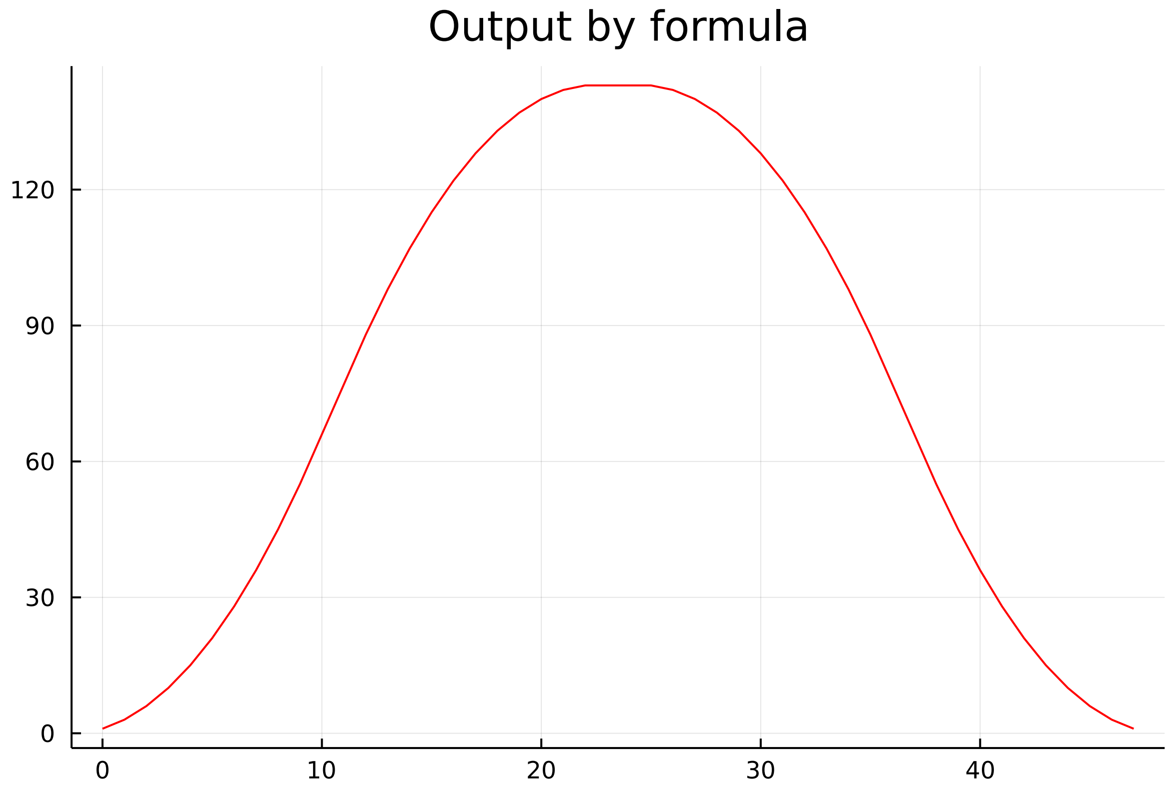
All outcomes:

```
countOutcomes =
  [1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 77, 88, 98, 107, 115, 122, 128, 133, 137,    mor
```

```
formulaOutcomes =
  [1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 77, 88, 98, 107, 115, 122, 128, 133, 137,    mor
```

Plot the outcomes





Appendix

`count_outcome (generic function with 1 method)`

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`formula_outcome (generic function with 1 method)`

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