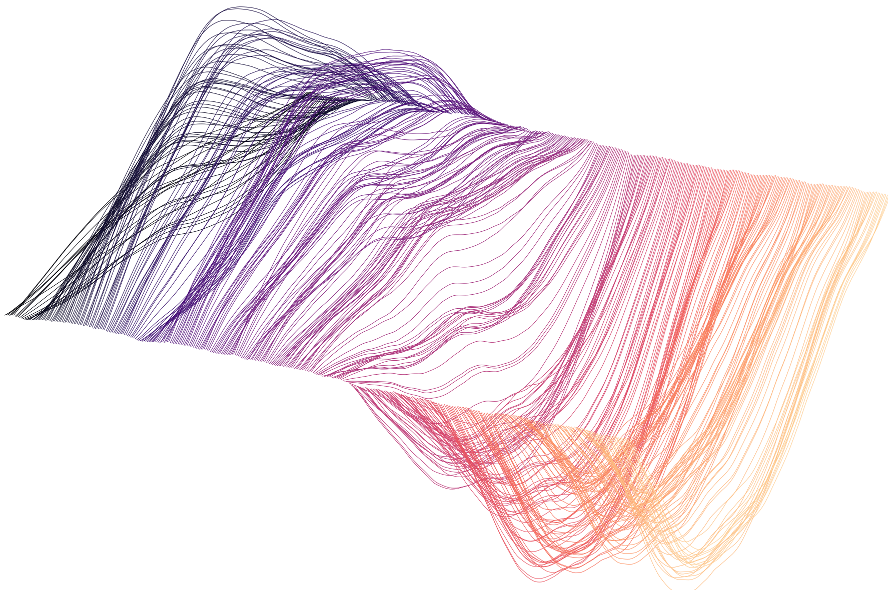
Department of Electrical Engineering and Automation

State-space deep Gaussian processes with applications

Zheng Zhao



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DOCTORAL DISSERTATIONS 163/2021

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Zheng Zhao

With the permission of the Aalto University School of Electrical Engineering, the defence of this doctoral thesis completed for the degree of Doctor of Science (Technology) will be held on 10 December 2021 at noon. The defence will take place simultaneously at a public examination in the lecture hall AS1 (Maarintie 8, Espoo) of the school and remotely via the link https://aalto.zoom.us/j/67529212279.

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Aalto University publication series
DOCTORAL DISSERTATIONS 163/2021

ISBN 978-952-64-0602-2 (printed)
ISBN 978-952-64-0603-9 (pdf)
ISSN 1799-4934 (printed)
ISSN 1799-4942 (pdf)
http://urn.fi/URN:ISBN:978-952-64-0603-9

Images: A realisation of a Wiener process taking value in a
Sobolev space with Dirichlet boundary condition

Unigrafia Oy
Helsinki 2021

Finland

NORDIC SWAN ECOLABEL
Printed matter
4041-0619

Aalto University

Abstract

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Name of the doctoral dissertation
State-space deep Gaussian processes with applications

Publisher School of Electrical Engineering

Unit Department of Electrical Engineering and Automation

Series Aalto University publication series DOCTORAL DISSERTATIONS 163/2021

Field of research

Manuscript submitted 10 August 2021 **Date of the defence** 10 December 2021

Permission for public defence granted (date) 28 October 2021 **Language** English

☐ **Monograph** ☒ **Article dissertation** ☐ **Essay dissertation**

Abstract
This thesis is mainly concerned with state-space approaches for solving deep (temporal) Gaussian process (DGP) regression problems. More specifically, we represent DGPs as hierarchically composed systems of stochastic differential equations (SDEs), and we consequently solve the DGP regression problem by using state-space filtering and smoothing methods. The resulting state-space DGP (SS-DGP) models generate a rich class of priors compatible with modelling a number of irregular signals/functions. Moreover, due to their Markovian structure, SS-DGPs regression problems can be solved efficiently by using Bayesian filtering and smoothing methods. The second contribution of this thesis is that we solve continuous-discrete Gaussian filtering and smoothing problems by using the Taylor moment expansion (TME) method. This induces a class of filters and smoothers that can be asymptotically exact in predicting the mean and covariance of stochastic differential equations (SDEs) solutions. Moreover, the TME method and TME filters and smoothers are compatible with simulating SS-DGPs and solving their regression problems. Lastly, this thesis features a number of applications of state-space (deep) GPs. These applications mainly include, (i) estimation of unknown drift functions of SDEs from partially observed trajectories and (ii) estimation of spectro-temporal features of signals.

Keywords Gaussian processes, stochastic differential equations, stochastic filtering and smoothing, state-space methods, signal processing, machine learning

ISBN (printed) 978-952-64-0602-2 **ISBN (pdf)** 978-952-64-0603-9

ISSN (printed) 1799-4934 **ISSN (pdf)** 1799-4942

Location of publisher Helsinki **Location of printing** Helsinki **Year** 2021

Pages 246 **urn** http://urn.fi/URN:ISBN:978-952-64-0603-9