

1.

My codes for calculating parameters:

```
[B,A] = butter(2, 5/(100/2));
disp("b1, b2, b3:")
disp(B)
disp("a2, a3:")
disp(A([2, 3]))
```

Values:

```
b1, b2, b3:
    0.0201    0.0402    0.0201

a2, a3:
   -1.5610    0.6414
```

2. My codes for applying the filter:

```
% Load data
noisy_data = load("./noisy.data");
y = noisy_data(:, 1);
x = noisy_data(:, 2);
u = noisy_data(:, 3);
% Get filter parameters
[B,A] = butter(2, 5/(100/2));
% Apply filter
filtered = filter(B, A, y);

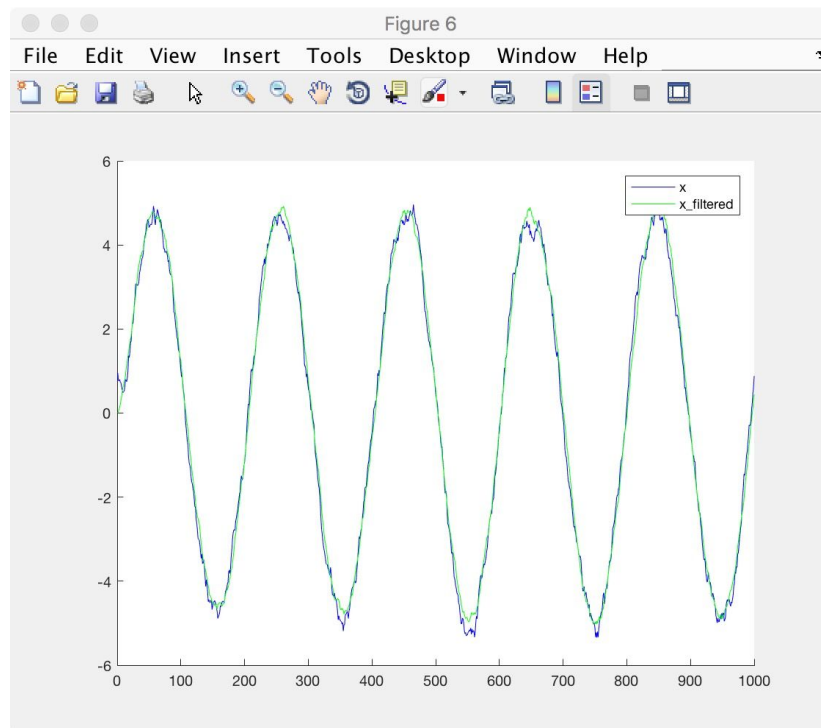
figure(1);
hold on;
h1 = plot(x, 'b');
h2 = plot(filtered, 'g');
legend([h1,h2], 'x', 'filtered');

figure(2);
subplot(3, 1, 1);
plot(y);
ylabel('y');
subplot(3,1,2);
plot(x);
ylabel('x');
subplot(3,1,3);
plot(filtered);
ylabel('filtered_x');

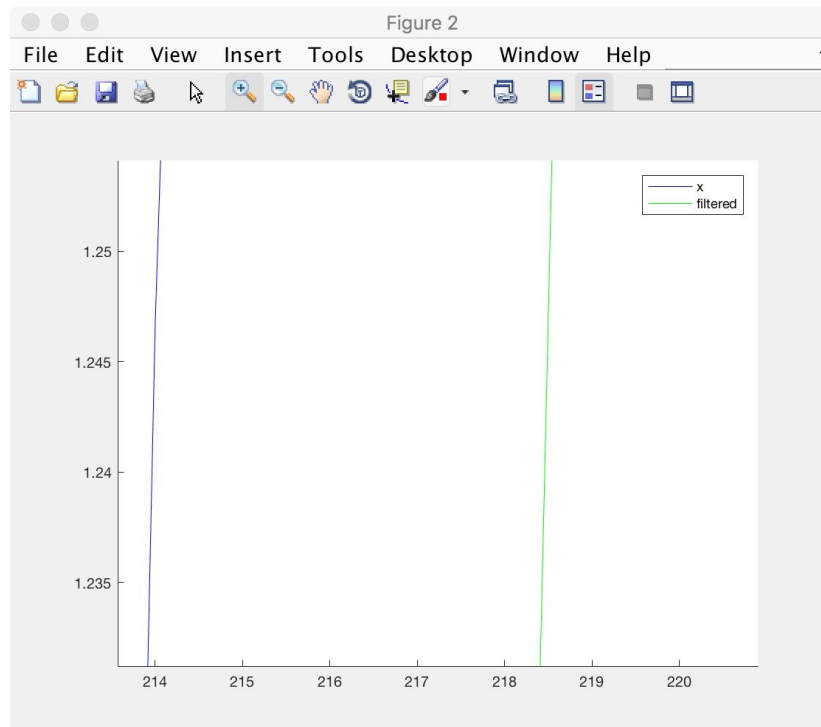
disp(finddelay(x,filtered));
|
```

Results:

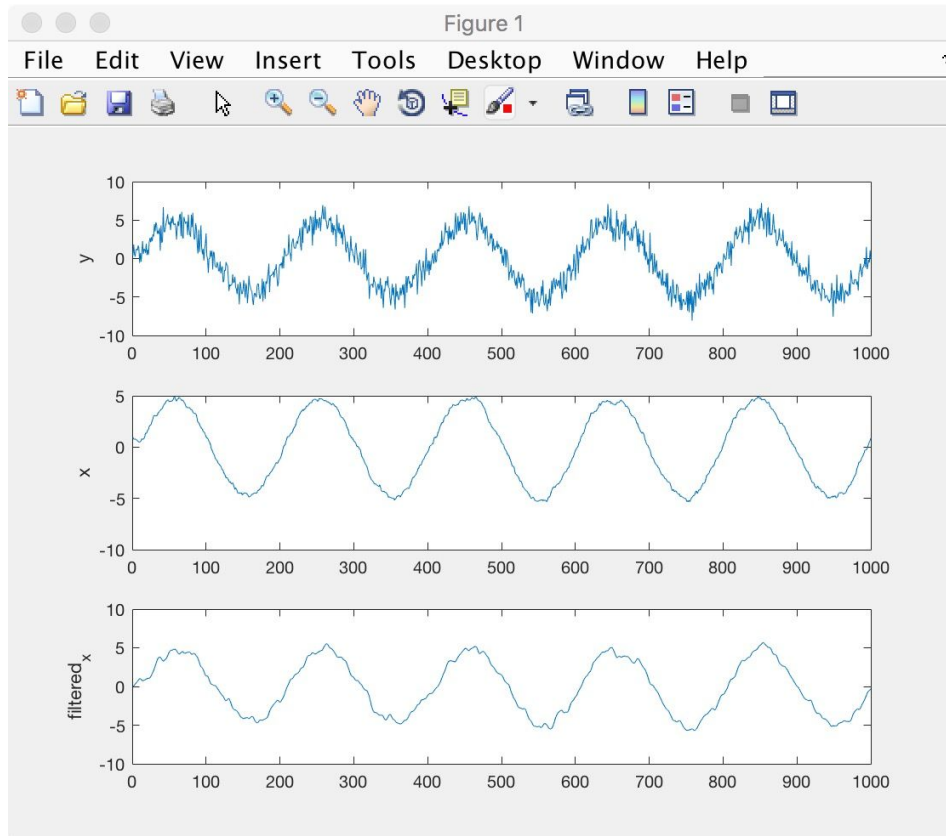
The plot of x and filtered x data:



To estimate the delay, I zoomed in the plot:



Here we can see that the delay is about 5 steps.



From the plots, the quality of the filter is ok and it filters out most of the noisy signal. However, it also brings some delay to the data. From the second plot, I zoom in the plot to estimate the delay. We can see that the delay is about 5 steps. I use the `finddelay` function to estimate the delay directly and result is 6 steps, which is $0.01 \times 6 = 0.06$ s.

3.

A snippet of my codes

```

data = load('noisy.data');
y = data(:,1);
x = data(:,2);
u = data(:,3);

A = 0.9;
B = 0.5;

C = 1;
r = 1;
Q = 0.01;

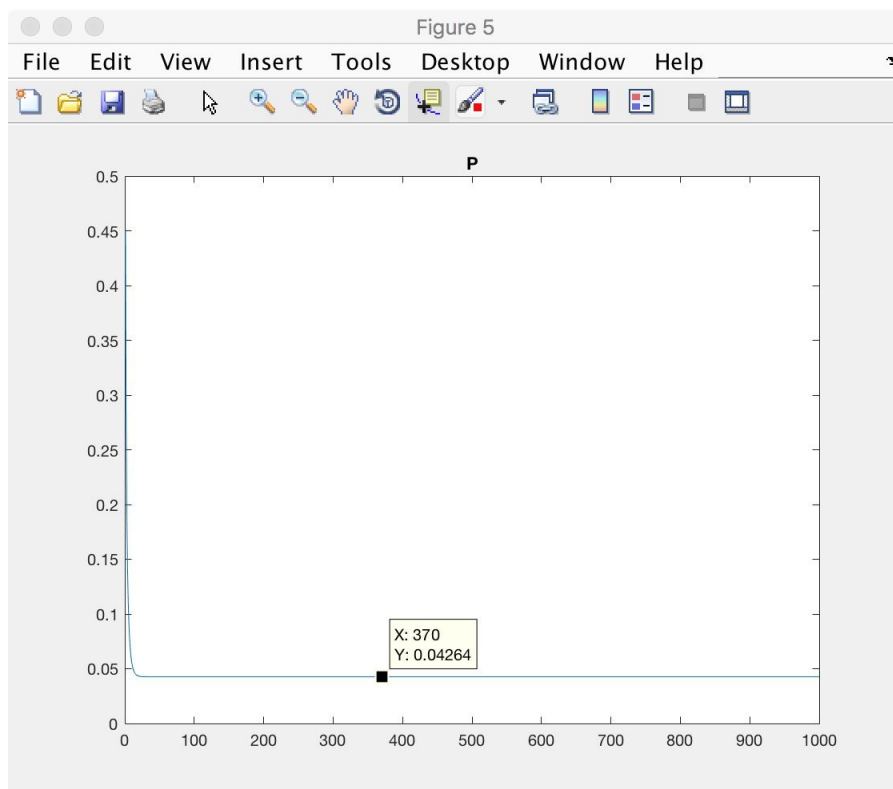
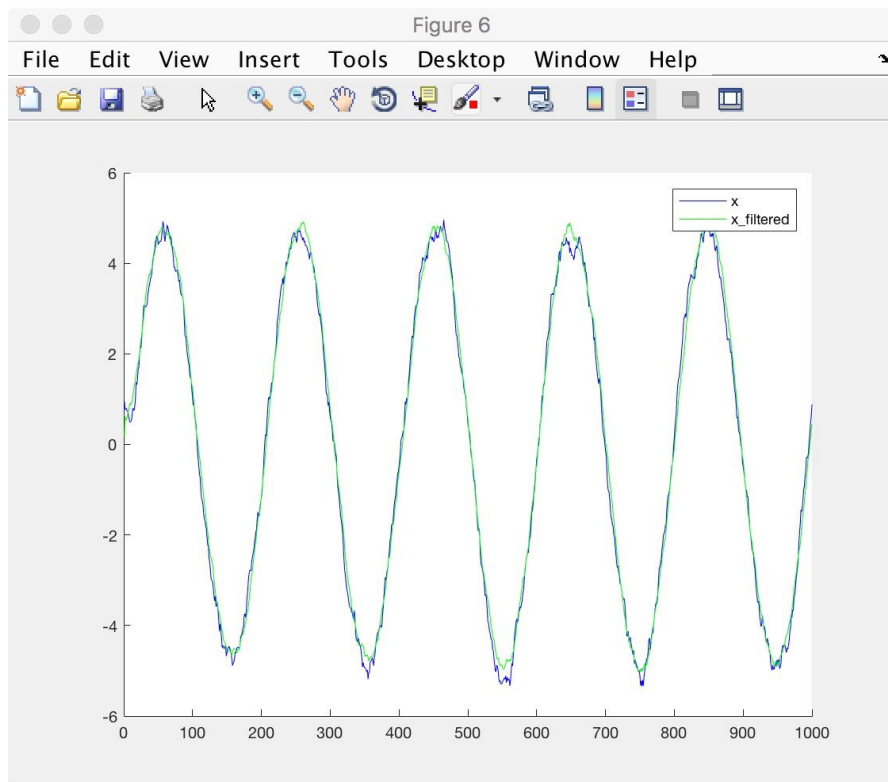
K = zeros(1000,1);
P_post = zeros(1000,1);
x_post = zeros(1000,1);

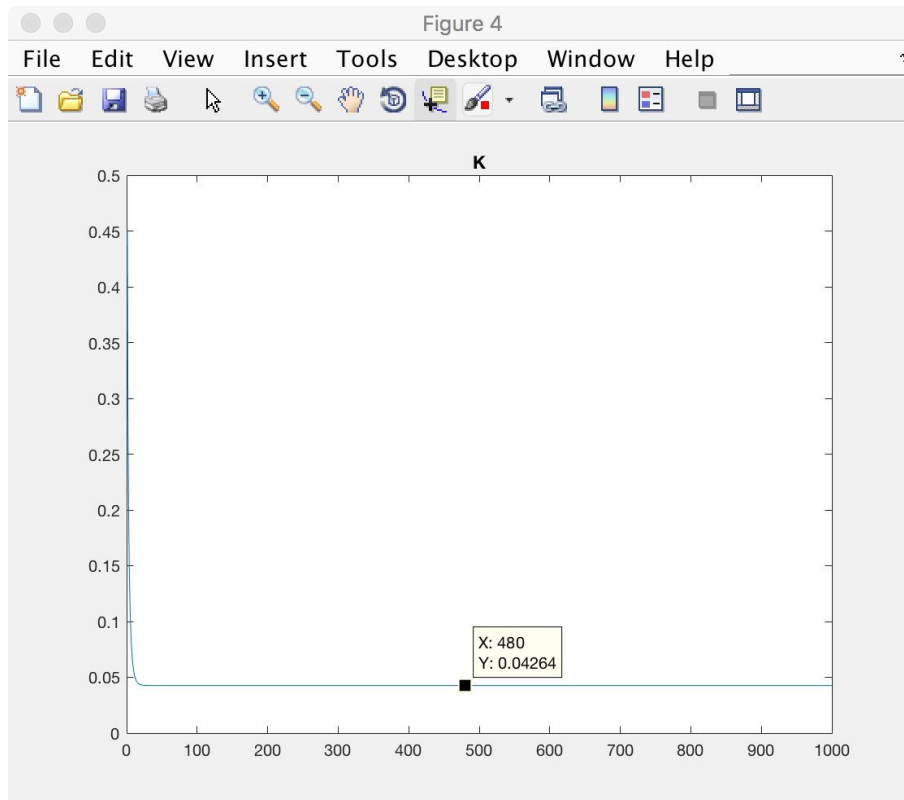
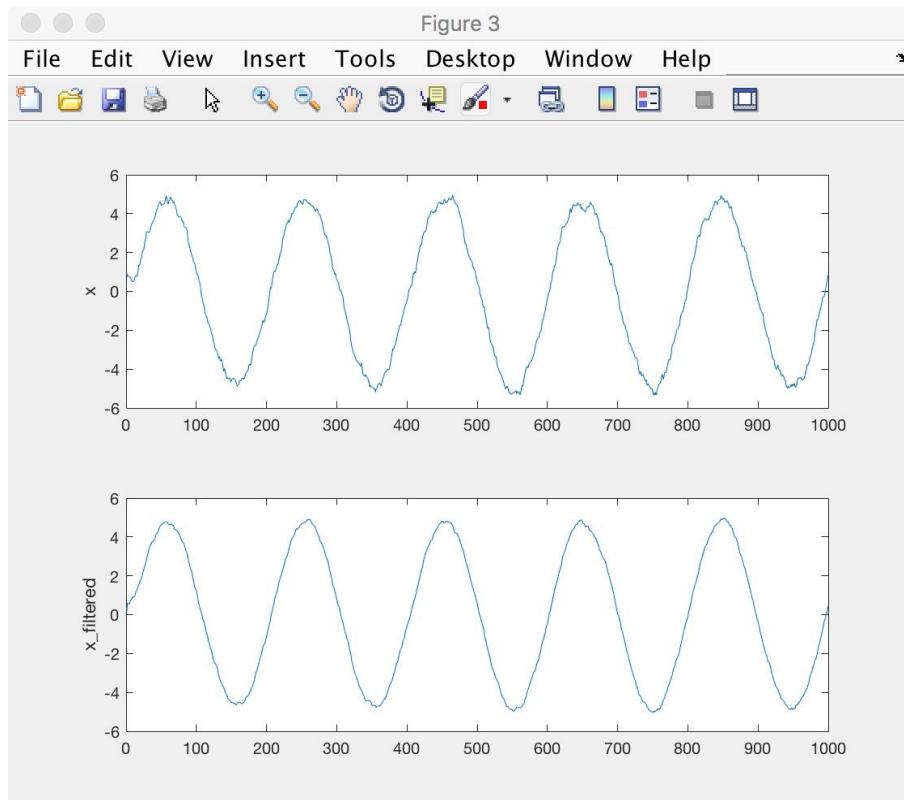
x_pri = 0;
% Set P(0) = 0 is equalvalent to set P_pri(1) = Q
P_pri = Q;

for t = 1:1000
    K(t) = P_pri * C / ( C * P_pri * C + r );
    x_post(t) = x_pri + K(t) * ( y(t) - C * x_pri );
    P_post(t) = ( 1 - K(t) * C ) * P_pri;
    x_pri = A * x_post(t) + B * u(t);
    P_pri = A * P_post(t) * A + Q;
end

```

Here are some of the plots from my codes:





I initialize P_{post} as A^*1*A+Q , which means that I initialize P as 1. Actually, I tried different initialization of P and they all converge very fast to the same value. I used `finddelay` function to

estimate the delay the it is only 1 step, which is 0.01s. Also compared to b), kalman filter actually filter out the noisy data better and has less delay.