We provide four examples of applying the proposed Bayesian MASEM to synthesizing mediation and CFA results across multiple studies. In two of these examples, the random mediation paths / factor loadings and factor correlations are not correlated to each other, and are included in the directory “Uncorrelated random SEM parameters”. The other two examples with correlated random parameters are in the directory “Correlated random SEM parameters”. Under each directory, we have one mediation example and one CFA example. For each example, we have a correct model and an incorrect model, which would be described in detail below. Researchers may choose to directly use OpenBUGS to run the analysis. All necessary files for each example such as the data, initial value, and model files are provided in corresponding folders. Researchers may also choose to call OpenBUGS via the R package R2OpenBUGS. To do so, researchers need to place the model script files, R code files, and data files (files with .dat extension) in appropriate directories and revise corresponding R script in the R code files accordingly. We also provide R code we used to generate the example data sets (files entitled GetData.R).

1 Mediation Example

Data are generated from a parallel mediation model with two correlated mediators m_1 and m_2 (see Panel A in Figure 1). During data analysis, we consider both this data generation model and an incorrect model, a sequential mediation model (see Panel B in Figure 1). In the condition with correlated SEM parameters, a_{1i} and b_{1i} in the parallel mediation model are correlated, and a_i, b_{1i}, b_{2i}, and c_i' in the sequential mediation model are correlated.

To prepare the model script file for OpenBUGS, we need to specify the model implied correlation vector $\boldsymbol{\rho}_i$ in terms of model parameters. Specifically, for the parallel mediation model, $\boldsymbol{\rho}_i$ is specified as

$$\boldsymbol{\rho}_i = \begin{pmatrix} \rho_{21,i} \\ \rho_{31,i} \\ \rho_{41,i} \\ \rho_{32,i} \\ \rho_{42,i} \\ \rho_{43,i} \end{pmatrix} = \begin{pmatrix} a_{1i} \\ a_2 \\ c_i \\ a_{1i}a_2 + g \\ a_{1i}c_i + b_{1i}\left(1 - a_{1i}^2\right) + b_2 g \\ a_2 c_i + b_{1i} g + b_2\left(1 - a_2^2\right) \end{pmatrix} \tag{1}$$

where $c_i = a_{1i}b_{1i} + a_2 b_2 + c'$is the total effect of X. For the sequential mediation model, $\boldsymbol{\rho}_i$ is specified as

$$\boldsymbol{\rho}_i = \begin{pmatrix} \rho_{21,i} \\ \rho_{31,i} \\ \rho_{41,i} \\ \rho_{32,i} \\ \rho_{42,i} \\ \rho_{43,i} \end{pmatrix} = \begin{pmatrix} a_i \\ a_i b_{1i} \\ c_i \\ b_{1i} \\ a_i c_i' + b_{1i} b_{2i} \\ a_i b_{1i} c_i' + b_{2i} \end{pmatrix} \tag{2}$$

where $c_i = a_i b_{1i} b_{2i} + c_i'$ is the total effect of X.

Further, to compute PP p value, we need to specify the first derivative of $\boldsymbol{\rho}_i$ with respect to model parameters. For the parallel mediation model, this derivative is

$$\left.\frac{\partial \boldsymbol{\rho}_i}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} = \left.\begin{array}{c c} / & \begin{array}{cccccc} \partial a_{1i} & \partial b_{1i} & \partial a_2 & \partial b_2 & \partial c' & \partial g \end{array} \\ \begin{array}{c} \partial\rho_{21,i} \\ \partial\rho_{31,i} \\ \partial\rho_{41,i} \\ \partial\rho_{32,i} \\ \partial\rho_{42,i} \\ \partial\rho_{43,i} \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ b_{1i} & a_{1i} & b_2 & a_2 & 1 & 0 \\ a_2 & 0 & a_{1i} & 0 & 0 & 1 \\ a_2 b_2 + c' & 1 & a_{1i} b_2 & a_{1i} a_2 + g & a_{1i} & b_2 \\ a_2 b_{1i} & a_{1i} a_2 + g & a_{1i} b_{1i} & 1 & a_2 & b_{1i} \end{pmatrix} \end{array}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} \tag{3}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ b_{10} & a_{10} & b_2 & a_2 & 1 & 0 \\ a_2 & 0 & a_{10} & 0 & 0 & 1 \\ a_2 b_2 + c' & 1 & a_{10} b_2 & a_{10} a_2 + g & a_{10} & b_2 \\ a_2 b_{10} & a_{10} a_2 + g & a_{10} b_{10} & 1 & a_2 & b_{10} \end{pmatrix}.$$

For the parallel mediation model, this derivative is

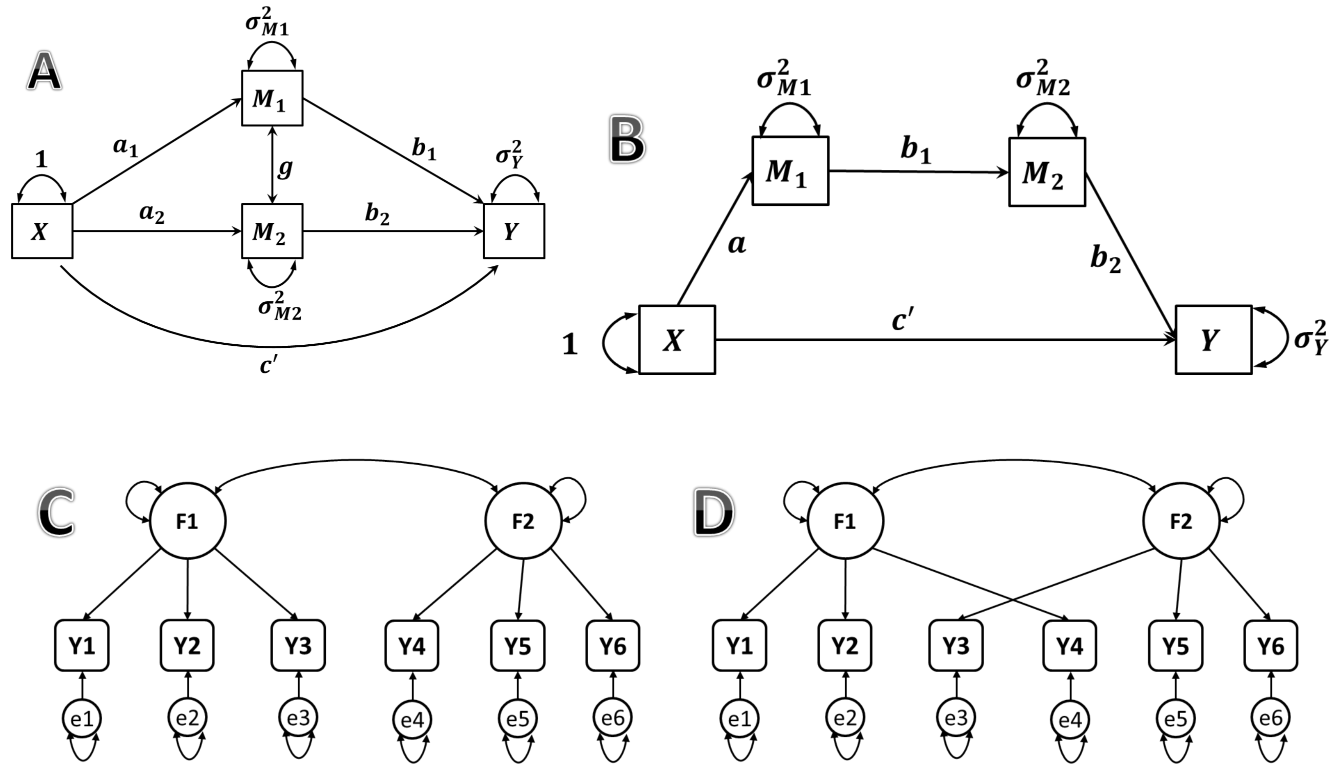

Figure 1: Illustration of the mediation and CFA examples.

$$
\left.\frac{\partial \boldsymbol{\rho}_i}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} = \left.\begin{array}{c c}
/ & \begin{array}{cccc} \partial a_i & \partial b_{1i} & \partial b_{2i} & \partial c'_i \end{array} \\
\begin{array}{c} \partial\rho_{21,i} \\ \partial\rho_{31,i} \\ \partial\rho_{41,i} \\ \partial\rho_{32,i} \\ \partial\rho_{42,i} \\ \partial\rho_{43,i} \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ b_{1i} & a_i & 0 & 0 \\ b_{1i}b_{2i} & a_i b_{2i} & a_i b_{1i} & 1 \\ 0 & 1 & 0 & 0 \\ c'_i & b_{2i} & b_{1i} & a_i \\ b_{1i}c'_i & a_i c'_i & 1 & a_i b_{1i} \end{pmatrix}
\end{array}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} \tag{4}
$$

$$
= \begin{pmatrix} 1 & 0 & 0 & 0 \\ b_{10} & a_0 & 0 & 0 \\ b_{10}b_{20} & a_0 b_{20} & a_0 b_{10} & 1 \\ 0 & 1 & 0 & 0 \\ c'_0 & b_{20} & b_{10} & a_0 \\ b_{10}c'_0 & a_0 c'_0 & 1 & a_0 b_{10} \end{pmatrix}.
$$

2 CFA Example

Data are generated from a two-factor CFA model (see Panel C in Figure 1). During data analysis, we consider both this data generation model and an incorrect model with misspecified factor loadings (see Panel D in Figure 1). In the condition with correlated SEM parameters, the loadings from F1 to Y3 (or to Y4 for the misspecified model) and F2 to Y6 are assumed to be correlated.

To prepare the model script file for OpenBUGS, we need to specify the model implied correlation vector $\boldsymbol{\rho}_i$ in terms of model parameters. Specifically, for the correct CFA model, $\boldsymbol{\rho}_i$ is specified as

$$
\boldsymbol{\rho}_i = \begin{pmatrix} \rho_{21,i} \\ \rho_{31,i} \\ \rho_{41,i} \\ \rho_{51,i} \\ \rho_{61,i} \\ \rho_{32,i} \\ \rho_{42,i} \\ \rho_{52,i} \\ \rho_{62,i} \\ \rho_{43,i} \\ \rho_{53,i} \\ \rho_{63,i} \\ \rho_{54,i} \\ \rho_{64,i} \\ \rho_{65,i} \end{pmatrix} = \begin{pmatrix} \lambda_{1i}\lambda_{2i} \\ \lambda_{1i}\lambda_{3i} \\ \lambda_{1i}\lambda_{4i}\phi_i \\ \lambda_{1i}\lambda_{5i}\phi_i \\ \lambda_{1i}\lambda_{6i}\phi_i \\ \lambda_{2i}\lambda_{3i} \\ \lambda_{2i}\lambda_{4i}\phi_i \\ \lambda_{2i}\lambda_{5i}\phi_i \\ \lambda_{2i}\lambda_{6i}\phi_i \\ \lambda_{3i}\lambda_{4i}\phi_i \\ \lambda_{3i}\lambda_{5i}\phi_i \\ \lambda_{3i}\lambda_{6i}\phi_i \\ \lambda_{4i}\lambda_{5i} \\ \lambda_{4i}\lambda_{6i} \\ \lambda_{5i}\lambda_{6i} \end{pmatrix} \tag{5}
$$

whereas for the misspecified CFA model, $\boldsymbol{\rho}_i$ is specified as

$$
\boldsymbol{\rho}_i = \begin{pmatrix} \rho_{21,i} \\ \rho_{31,i} \\ \rho_{41,i} \\ \rho_{51,i} \\ \rho_{61,i} \\ \rho_{32,i} \\ \rho_{42,i} \\ \rho_{52,i} \\ \rho_{62,i} \\ \rho_{43,i} \\ \rho_{53,i} \\ \rho_{63,i} \\ \rho_{54,i} \\ \rho_{64,i} \\ \rho_{65,i} \end{pmatrix} = \begin{pmatrix} \lambda_{1i}\lambda_{2i} \\ \lambda_{1i}\lambda_{3i}\phi_i \\ \lambda_{1i}\lambda_{4i} \\ \lambda_{1i}\lambda_{5i}\phi_i \\ \lambda_{1i}\lambda_{6i}\phi_i \\ \lambda_{2i}\lambda_{3i}\phi_i \\ \lambda_{2i}\lambda_{4i} \\ \lambda_{2i}\lambda_{5i}\phi_i \\ \lambda_{2i}\lambda_{6i}\phi_i \\ \lambda_{3i}\lambda_{4i}\phi_i \\ \lambda_{3i}\lambda_{5i} \\ \lambda_{3i}\lambda_{6i} \\ \lambda_{4i}\lambda_{5i}\phi_i \\ \lambda_{4i}\lambda_{6i}\phi_i \\ \lambda_{5i}\lambda_{6i} \end{pmatrix}. \tag{6}
$$

Further, to compute PP p value, we need to specify the first derivative of $\boldsymbol{\rho}_i$ with respect to model parameters. For the data generation model, this derivative is

$$
\left.\frac{\partial \boldsymbol{\rho}_i}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} = \left.\begin{array}{c c}
/ & \begin{array}{ccccccc} \partial\lambda_{1i} & \partial\lambda_{2i} & \partial\lambda_{3i} & \partial\lambda_{4i} & \partial\lambda_{5i} & \partial\lambda_{6i} & \partial\phi_i \end{array} \\
\begin{array}{c} \partial\rho_{21,i} \\ \partial\rho_{31,i} \\ \partial\rho_{41,i} \\ \partial\rho_{51,i} \\ \partial\rho_{61,i} \\ \partial\rho_{32,i} \\ \partial\rho_{42,i} \\ \partial\rho_{52,i} \\ \partial\rho_{62,i} \\ \partial\rho_{43,i} \\ \partial\rho_{53,i} \\ \partial\rho_{63,i} \\ \partial\rho_{54,i} \\ \partial\rho_{64,i} \\ \partial\rho_{65,i} \end{array} &
\begin{pmatrix}
\lambda_{2i} & \lambda_{1i} & 0 & 0 & 0 & 0 & 0 \\
\lambda_{3i} & 0 & \lambda_{1i} & 0 & 0 & 0 & 0 \\
\lambda_{4i}\phi_i & 0 & 0 & \lambda_{1i}\phi_i & 0 & 0 & \lambda_{1i}\lambda_{4i} \\
\lambda_{5i}\phi_i & 0 & 0 & 0 & \lambda_{1i}\phi_i & 0 & \lambda_{1i}\lambda_{5i} \\
\lambda_{6i}\phi_i & 0 & 0 & 0 & 0 & \lambda_{1i}\phi_i & \lambda_{1i}\lambda_{6i} \\
0 & \lambda_{3i} & \lambda_{2i} & 0 & 0 & 0 & 0 \\
0 & \lambda_{4i}\phi_i & 0 & \lambda_{2i}\phi_i & 0 & 0 & \lambda_{2i}\lambda_{4i} \\
0 & \lambda_{5i}\phi_i & 0 & 0 & \lambda_{2i}\phi_i & 0 & \lambda_{2i}\lambda_{5i} \\
0 & \lambda_{6i}\phi_i & 0 & 0 & 0 & \lambda_{2i}\phi_i & \lambda_{2i}\lambda_{6i} \\
0 & 0 & \lambda_{4i}\phi_i & \lambda_{3i}\phi_i & 0 & 0 & \lambda_{3i}\lambda_{4i} \\
0 & 0 & \lambda_{5i}\phi_i & 0 & \lambda_{3i}\phi_i & 0 & \lambda_{3i}\lambda_{5i} \\
0 & 0 & \lambda_{6i}\phi_i & 0 & 0 & \lambda_{3i}\phi_i & \lambda_{3i}\lambda_{6i} \\
0 & 0 & 0 & \lambda_{5i} & \lambda_{4i} & 0 & 0 \\
0 & 0 & 0 & \lambda_{6i} & 0 & \lambda_{4i} & 0 \\
0 & 0 & 0 & 0 & \lambda_{6i} & \lambda_{5i} & 0
\end{pmatrix}
\end{array}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} \tag{7}
$$

$$
= \begin{pmatrix}
\lambda_{20} & \lambda_{10} & 0 & 0 & 0 & 0 & 0 \\
\lambda_{30} & 0 & \lambda_{10} & 0 & 0 & 0 & 0 \\
\lambda_{40}\phi_0 & 0 & 0 & \lambda_{10}\phi_0 & 0 & 0 & \lambda_{10}\lambda_{40} \\
\lambda_{50}\phi_0 & 0 & 0 & 0 & \lambda_{10}\phi_0 & 0 & \lambda_{10}\lambda_{50} \\
\lambda_{60}\phi_0 & 0 & 0 & 0 & 0 & \lambda_{10}\phi_0 & \lambda_{10}\lambda_{60} \\
0 & \lambda_{30} & \lambda_{20} & 0 & 0 & 0 & 0 \\
0 & \lambda_{40}\phi_0 & 0 & \lambda_{20}\phi_0 & 0 & 0 & \lambda_{20}\lambda_{40} \\
0 & \lambda_{50}\phi_0 & 0 & 0 & \lambda_{20}\phi_0 & 0 & \lambda_{20}\lambda_{50} \\
0 & \lambda_{60}\phi_0 & 0 & 0 & 0 & \lambda_{20}\phi_0 & \lambda_{20}\lambda_{60} \\
0 & 0 & \lambda_{40}\phi_0 & \lambda_{30}\phi_0 & 0 & 0 & \lambda_{30}\lambda_{40} \\
0 & 0 & \lambda_{50}\phi_0 & 0 & \lambda_{30}\phi_0 & 0 & \lambda_{30}\lambda_{50} \\
0 & 0 & \lambda_{60}\phi_0 & 0 & 0 & \lambda_{30}\phi_0 & \lambda_{30}\lambda_{60} \\
0 & 0 & 0 & \lambda_{50} & \lambda_{40} & 0 & 0 \\
0 & 0 & 0 & \lambda_{60} & 0 & \lambda_{40} & 0 \\
0 & 0 & 0 & 0 & \lambda_{60} & \lambda_{50} & 0
\end{pmatrix}.
$$

For the CFA model with misspecified loadings, this derivative is

$$
\begin{aligned}
\left.\frac{\partial \boldsymbol{\rho}_i}{\partial \boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}
&= \left.\begin{array}{cc}
/ & \begin{array}{ccccccc} \partial\lambda_{1i} & \partial\lambda_{2i} & \partial\lambda_{3i} & \partial\lambda_{4i} & \partial\lambda_{5i} & \partial\lambda_{6i} & \partial\phi_i \end{array} \\
\begin{array}{c}
\partial\rho_{21,i}\\ \partial\rho_{31,i}\\ \partial\rho_{41,i}\\ \partial\rho_{51,i}\\ \partial\rho_{61,i}\\ \partial\rho_{32,i}\\ \partial\rho_{42,i}\\ \partial\rho_{52,i}\\ \partial\rho_{62,i}\\ \partial\rho_{43,i}\\ \partial\rho_{53,i}\\ \partial\rho_{63,i}\\ \partial\rho_{54,i}\\ \partial\rho_{64,i}\\ \partial\rho_{65,i}
\end{array} &
\begin{pmatrix}
\lambda_{2i} & \lambda_{1i} & 0 & 0 & 0 & 0 & 0 \\
\lambda_{3i}\phi_i & 0 & \lambda_{1i}\phi_i & 0 & 0 & 0 & \lambda_{1i}\lambda_{3i} \\
\lambda_{4i} & 0 & 0 & \lambda_{1i} & 0 & 0 & 0 \\
\lambda_{5i}\phi_i & 0 & 0 & 0 & \lambda_{1i}\phi_i & 0 & \lambda_{1i}\lambda_{5i} \\
\lambda_{6i}\phi_i & 0 & 0 & 0 & 0 & \lambda_{1i}\phi_i & \lambda_{1i}\lambda_{6i} \\
0 & \lambda_{3i}\phi_i & \lambda_{2i}\phi_i & 0 & 0 & 0 & \lambda_{2i}\lambda_{3i} \\
0 & \lambda_{4i} & 0 & \lambda_{2i} & 0 & 0 & 0 \\
0 & \lambda_{5i}\phi_i & 0 & 0 & \lambda_{2i}\phi_i & 0 & \lambda_{2i}\lambda_{5i} \\
0 & \lambda_{6i}\phi_i & 0 & 0 & 0 & \lambda_{2i}\phi_i & \lambda_{2i}\lambda_{6i} \\
0 & 0 & \lambda_{4i}\phi_i & \lambda_{3i}\phi_i & 0 & 0 & \lambda_{3i}\lambda_{4i} \\
0 & 0 & \lambda_{5i} & 0 & \lambda_{3i} & 0 & 0 \\
0 & 0 & \lambda_{6i} & 0 & 0 & \lambda_{3i} & 0 \\
0 & 0 & 0 & \lambda_{5i}\phi_i & \lambda_{4i}\phi_i & 0 & \lambda_{4i}\lambda_{5i} \\
0 & 0 & 0 & \lambda_{6i}\phi_i & 0 & \lambda_{4i}\phi_i & \lambda_{4i}\lambda_{6i} \\
0 & 0 & 0 & 0 & \lambda_{6i} & \lambda_{5i} & 0
\end{pmatrix}
\end{array}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} \\
&= \begin{pmatrix}
\lambda_{20} & \lambda_{10} & 0 & 0 & 0 & 0 & 0 \\
\lambda_{30}\phi_0 & 0 & \lambda_{10}\phi_0 & 0 & 0 & 0 & \lambda_{10}\lambda_{30} \\
\lambda_{40} & 0 & 0 & \lambda_{10} & 0 & 0 & 0 \\
\lambda_{50}\phi_0 & 0 & 0 & 0 & \lambda_{10}\phi_0 & 0 & \lambda_{10}\lambda_{50} \\
\lambda_{60}\phi_0 & 0 & 0 & 0 & 0 & \lambda_{10}\phi_0 & \lambda_{10}\lambda_{60} \\
0 & \lambda_{30}\phi_0 & \lambda_{20}\phi_0 & 0 & 0 & 0 & \lambda_{20}\lambda_{30} \\
0 & \lambda_{40} & 0 & \lambda_{20} & 0 & 0 & 0 \\
0 & \lambda_{50}\phi_0 & 0 & 0 & \lambda_{20}\phi_0 & 0 & \lambda_{20}\lambda_{50} \\
0 & \lambda_{60}\phi_0 & 0 & 0 & 0 & \lambda_{20}\phi_0 & \lambda_{20}\lambda_{60} \\
0 & 0 & \lambda_{40}\phi_0 & \lambda_{30}\phi_0 & 0 & 0 & \lambda_{30}\lambda_{40} \\
0 & 0 & \lambda_{50} & 0 & \lambda_{30} & 0 & 0 \\
0 & 0 & \lambda_{60} & 0 & 0 & \lambda_{30} & 0 \\
0 & 0 & 0 & \lambda_{50}\phi_0 & \lambda_{40}\phi_0 & 0 & \lambda_{40}\lambda_{50} \\
0 & 0 & 0 & \lambda_{60}\phi_0 & 0 & \lambda_{40}\phi_0 & \lambda_{40}\lambda_{60} \\
0 & 0 & 0 & 0 & \lambda_{60} & \lambda_{50} & 0
\end{pmatrix}.
\end{aligned}
\tag{8}
$$