A General Procedure to Build Mediaiton Models using BMASEM

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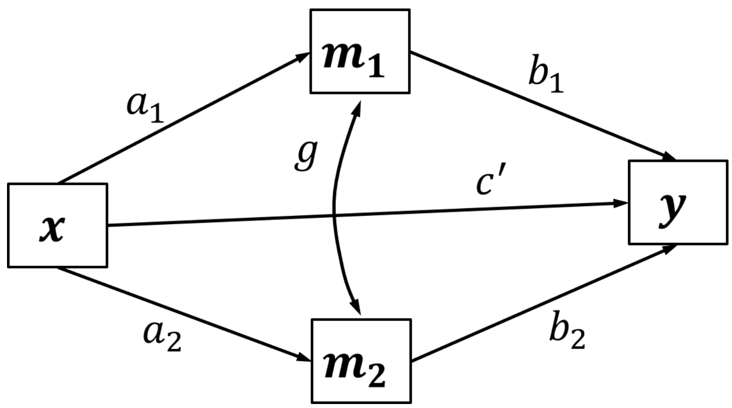
I was frequently asked about how to specify the correlation vector $\boldsymbol{\rho}_j$ or the correlation matrix $\boldsymbol{P}_j$ for mediation models while using BMASEM. Here I use the example depicted below to demonstrate a general procedure for building mediation models using BMASEM.



1. Write down the equations for the mediation model of interest. For example, for the mediation model displayed above, the equations are

$$\overset{\boldsymbol{\eta}_i}{\begin{pmatrix} x_i \\ m_{1,i} \\ m_{2,i} \\ y_i \end{pmatrix}} = \overset{\boldsymbol{\mu}}{\begin{pmatrix} \bar{x} \\ int_{m1} \\ int_{m2} \\ int_y \end{pmatrix}} + \overset{\boldsymbol{B}}{\begin{pmatrix} 0 & 0 & 0 & 0 \\ a_1 & 0 & 0 & 0 \\ a_2 & 0 & 0 & 0 \\ c' & b_1 & b_2 & 0 \end{pmatrix}} \overset{\boldsymbol{\eta}_i}{\begin{pmatrix} x_i \\ m_{1,i} \\ m_{2,i} \\ y_i \end{pmatrix}} + \overset{\boldsymbol{e}_i}{\begin{pmatrix} x_i - \bar{x} \\ e_{m1,i} \\ e_{m2,i} \\ e_{y,i} \end{pmatrix}} \tag{1}$$

where $\boldsymbol{e}_i \sim N(\boldsymbol{0}, \boldsymbol{V}_e)$ and

$$\boldsymbol{V}_e = \begin{pmatrix} \sigma_x^2 = 1 & 0 & 0 & 0 \\ 0 & \sigma_{em1}^2 & g & 0 \\ 0 & g & \sigma_{em2}^2 & 0 \\ 0 & 0 & 0 & \sigma_{ey}^2 \end{pmatrix}. \tag{2}$$

Note that for simplicity, the subscript j indexing primary studies is omitted.

2. Specify $\boldsymbol{B}$ and $\boldsymbol{V}_e$ as in Equations (1) and (2) using the R package Ryacas. (Example R code for this example is included in PSetup4Med.R)

3. Compute the model-implied correlation matrix $\boldsymbol{P}$ using symbolic matrix operations via the R package Ryacas (again, example R code is available in PSetup4Med.R for our example). Let $c = a_1b_1 + a_2b_2 + c'$. For our example, the resulting model-implied correlation matrix is as below (it can be obtained by running the example R code)

$$\begin{aligned} \boldsymbol{P} &= (\boldsymbol{I} - \boldsymbol{B})^{-1} \boldsymbol{V}_e \left[(\boldsymbol{I} - \boldsymbol{B})^{-1} \right]' \\ &= \begin{pmatrix} 1 & & & \\ a_1 & a_1^2 + \sigma_{em1}^2 & & \\ a_2 & a_1a_2 + g & a_2^2 + \sigma_{em2}^2 & \\ c & a_1c + b_1\sigma_{em1}^2 + b_2g & a_2c + b_1g + b_2\sigma_{em2}^2 & c^2 + b_1^2\sigma_{em1}^2 + 2b_1b_2g + b_2^2\sigma_{em2}^2 + \sigma_{ey}^2 \end{pmatrix}. \end{aligned}$$

4. Express $\boldsymbol{V_e}$ in terms of parameters in $\boldsymbol{B}$. Note that the diagonal elements of $\boldsymbol{P}$ equal 1. By contrasting the "model-implied" diagonal elements obtained in Step 3 and 1, we can solve for $\boldsymbol{V_e}$. For our example, because

$$\begin{pmatrix} a_1^2+\sigma_{em1}^2 \\ a_2^2+\sigma_{em2}^2 \\ c^2+b_1^2\sigma_{em1}^2+2b_1b_2g+b_2^2\sigma_{em2}^2+\sigma_{ey}^2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

we have that $\sigma_{em1}^2=1-a_1^2$, $\sigma_{em2}^2=1-a_2^2$, and $\sigma_{ey}^2=1-\left(c^2+b_1^2\left(1-a_1^2\right)+2b_1b_2g+b_2^2\left(1-a_2^2\right)\right)$. Therefore,

$$\boldsymbol{V}_e = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a_1^2 & g & 0 \\ 0 & g & 1-a_2^2 & 0 \\ 0 & 0 & 0 & 1-\left(c^2+b_1^2\left(1-a_1^2\right)+2b_1b_2g+b_2^2\left(1-a_2^2\right)\right) \end{pmatrix}.$$

5. Specify $\boldsymbol{\rho}$ by vectorizing $\boldsymbol{P}$. For our example,

$$\boldsymbol{\rho} = \begin{pmatrix} P[2,1] \\ P[3,1] \\ P[4,1] \\ P[3,2] \\ P[4,2] \\ P[4,3] \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ c \\ a_1a_2+g \\ a_1c+b_1\sigma_{em1}^2+b_2g \\ a_2c+b_1g+b_2\sigma_{em2}^2 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ c \\ a_1a_2+g \\ a_1c+b_1\left(1-a_1^2\right)+b_2g \\ a_2c+b_1g+b_2\left(1-a_2^2\right) \end{pmatrix}.$$

This is the model-implied correlation vector.