1) $\bar{x} = A\bar{a} - \bar{b} = 0$

$\bar{y} = C\bar{x} + \bar{c} = 0$

$$\Gamma_x = A\Gamma_a A^T + \Gamma_b = AA^T + I$$

$$\Gamma_y = C\Gamma_x C^T + \Gamma_c = C(AA^T + I)C^T + I$$

2) $v = \begin{bmatrix} x \\ y \end{bmatrix}$ $\qquad E(v) = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\Gamma_v = \begin{bmatrix} \Gamma_x & \Gamma_{xy} \\ \Gamma_{yx} & \Gamma_y \end{bmatrix}$$

$$\begin{aligned}\Gamma_{xy} &= E[(x-\bar{x})(y-\bar{y})^T] \\ &= E[x(Cx+c)^T] \\ &= E[xx^TC^T + xc^T] \\ &= \Gamma_x C^T + \underbrace{\Gamma_{xc}}_{0} \\ &= \Gamma_x C^T\end{aligned}$$

$$\Gamma_v = \begin{bmatrix} \Gamma_x & \Gamma_x C^T \\ C\Gamma_x^T & \Gamma_y \end{bmatrix}$$

3) $z = y - x \qquad v = \begin{bmatrix} x \\ y \end{bmatrix}$

$$z = \begin{bmatrix} -I & I \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -I & I \end{bmatrix} v$$

$\bar{z} = 0 \qquad \Gamma_z = \begin{bmatrix} -I & I \end{bmatrix} \begin{bmatrix} \Gamma_x & \Gamma_x C^T \\ C\Gamma_x^T & \Gamma_y \end{bmatrix} \begin{bmatrix} -I \\ I \end{bmatrix}$

$$\Gamma_z = \Gamma_x - \Gamma_x C^T - C\Gamma_x^T + \Gamma_y$$

4) $\hat{x} = \bar{x} + K(y - \bar{y})$ avec $K = \Gamma_{xy} \cdot \Gamma_y^{-1}$

$$\hat{x} = \Gamma_x C^T \Gamma_y^{-1} y$$

$$\hat{x} = (AA^T + I) C^T (C(AA^T+I)C^T + I)^{-1} y$$