9. domácí úkol | Vilém Zouhar

1

Definiční obor: $D_f = \mathbf{R}^2$
Spojitost v bodě $(0,0)$:

$$(|x|-|y|)^2 \leq 0 \Rightarrow |x|^2+|y|^2 \leq 2|xy| \Rightarrow x^2+y^2 \leq 2|xy| \Rightarrow \sqrt{x^2+y^2} \leq \sqrt{2|xy|}$$

$$\text{Chceme: } \forall \epsilon > 0 \exists \delta > 0 : \forall x : |x-0| < \delta \Rightarrow |\frac{xy}{\sqrt{x^2+y^2}} - 0| < \epsilon$$

$$|\frac{xy}{\sqrt{x^2+y^2}}| \leq |\frac{xy}{\sqrt{2|xy|}}| = |\frac{\sqrt{|xy|}sgn(xy)}{\sqrt{2}}| < \epsilon$$

$$\Rightarrow \forall \epsilon > 0 \exists \delta > 0 : \forall x : |x-0| < \delta \Rightarrow |\frac{xy}{\sqrt{x^2+y^2}} - 0| < \epsilon \Rightarrow \text{funkce spojitá v (0,0)}$$

Parciální derivace:

$$\frac{\partial f}{\partial x}(x,y) = \frac{x^3}{(x^2+y^2)^{\frac{3}{2}}} \qquad (x,y) \neq (0,0)$$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{t\to 0} \frac{f(t,0)-f(0,0)}{t} = \lim_{t\to 0} \frac{\frac{t\cdot 0}{\sqrt{t^2+0^2}}}{t} = \lim_{t\to 0} \frac{1}{|t|} = \infty$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{y^3}{(x^2+y^2)^{\frac{3}{2}}} \qquad (x,y) \neq (0,0)$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{t\to 0} \frac{f(t,0)-f(0,0)}{t} = \lim_{t\to 0} \frac{\frac{t\cdot 0}{\sqrt{t^2+0^2}}}{t} = \lim_{t\to 0} \frac{1}{|t|} = \infty \qquad \text{(symetricky)}$$

2

Definiční obor: $D_f = \mathbf{R}^2 \backslash \{(a,-a), a \in \mathbf{R}\}$
Spojité dodefinování:

$$\lim_{x\to -y^-} \arctan(\frac{x-y}{x+y}) = \text{(ze spojitosti arctan)} = \arctan(\lim_{x\to -y^-} \frac{x+y}{x-y}) = \frac{\pi}{2}$$

$$\lim_{x\to -y^+} \arctan(\frac{x-y}{x+y}) = \text{(ze spojitosti arctan)} = \arctan(\lim_{x\to -y^+} \frac{x+y}{x-y}) = -\frac{\pi}{2} \quad \Rightarrow \text{nelze spojitě dodefinovat}$$

Parciální derivace:

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{1+\frac{(x-y)^2}{(x+y)^2}} \cdot \frac{\partial f}{\partial x}\frac{x-y}{x+y} = \frac{(x+y)^2}{(x+y)^2+(x-y)^2} \cdot \frac{(x+y)-(x-y)}{(x+y)^2} = \frac{y}{x^2+y^2} \qquad (x,y) \neq (0,0)$$

$$\frac{\partial f}{\partial x}(0,0) \quad \text{neexistuje}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{1}{1+\frac{(x-y)^2}{(x+y)^2}} \cdot \frac{\partial f}{\partial y}\frac{x-y}{x+y} = \frac{(x+y)^2}{(x+y)^2+(x-y)^2} \cdot \frac{-(x+y)-(x-y)}{(x+y)^2} = \frac{-x}{x^2+y^2} \qquad (x,y) \neq (0,0)$$

$$\frac{\partial f}{\partial y}(0,0) \quad \text{neexistuje}$$

$$\frac{\partial^2 f}{\partial x^2}(x,y) = \frac{-2xy}{(x^2+y^2)^2} \qquad (x,y) \neq (0,0)$$

$$\frac{\partial^2 f}{\partial x^2}(0,0) \quad \text{neexistuje}$$

$$\frac{\partial^2 f}{\partial y^2}(x,y) = \frac{2xy}{(x^2+y^2)^2} \qquad (x,y) \neq (0,0)$$

$$\frac{\partial^2 f}{\partial y^2}(0,0) \quad \text{neexistuje}$$

3

$$\nabla \sin(xyz) = (yz\cos(xyz), xz\cos(xyz), xy\cos(xyz)) = g(x,y,z)$$
$$D_{(2,1,1)}f(1,1,0) = g(1,1,0)\cdot(2,1,1) = (0,0,1\cos(0))\cdot(2,1,1) = 1$$