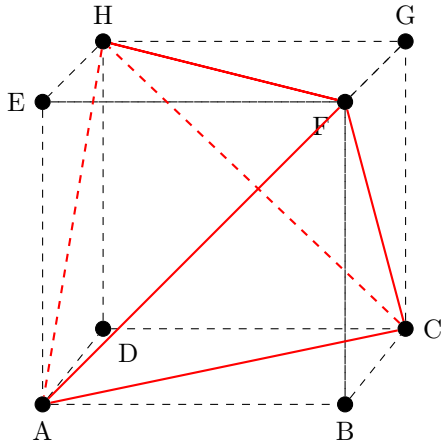


1 Tetrahedron



type	description	example	#
(4, 0, 0, 0)	id	(A)(C)(F)(H)	1
(1, 0, 1, 0)	rotation $2\pi/3$ around the axis between a point and the altitude to the opposing triangle	(A)(CFH)	4
(1, 0, 1, 0)	rotation $-2\pi/3$ around the axis between a point and the altitude to the opposing triangle	(A)(CHF)	4
(0, 2, 0, 0)	rotation around the axis between the centre of a line and the centre of the opposing segment	(AC)(HF)	3

Total: 12.

2 Even permutations

Note

Composition is performed from the right.

Lemma 1

Any 3-cycle can be composed from 3-cycles of the form $(1, i, j)$.

Proof

For 3-cycles without 1: (a, b, c) , this is trivially $(1, a, b)(1, b, c)$. For 3-cycles with 1: $(a, b, 1)$ we already have a solution.

Lemma 2

Any 3-cycle can be composed from 3-cycles of the form $(1, 2, j)$.

Proof

If the 3-cycle contains consecutively 1 and 2, then we are done. If it contains consecutively 2 and 1: $(a, 2, 1)$, then we apply the 3-cycle twice: $(a, 2, 1) = (1, 2, a)(1, 2, a)$.

If the 3-cycle contains 2, but not 1, then we simply apply Lemma 1. If the resulting 3-cycles contain 1, 2 consecutively, then ok. If not, we can break them down again using applying the 3-cycle twice as previously.

If the 3-cycle contains 1 and not 2, then $(1, a, b) = (1, 2, b)(1, 2, b)(1, 2, a)(1, 2, b)$ and from Lemma 1 we can construct any 3-cycle of this form.

Lemma 3

A_n is generated by 3-cycles of the form $(i, i+1, i+2)$.

Proof

From Lemma 2 we know, that any 3-cycle can be composed of more elementary cycles of the form $(1, 2, k)$. For $A_3 = \{(1), (1, 2, 3), (1, 3, 2)\}$ we need only the generator $(1, 2, 3)$. For A_4 this is also true (checked programmatically by exhaustive state search). For $n \geq 5$: $(1, 2, i)$ it is obvious for $i = 3$ and for $i = 4$ it is also true, because $(1, 2, 4) = (1, 2, 3)(1, 2, 3)(2, 3, 4)(1, 2, 3)$.

For $i \geq 5$: $(1, 2, i) = (1, 2, i-2)(1, 2, i-1)(i-2, i-1, i)(1, 2, i-2)(1, 2, i-1)$. From Lemma 2 we know, that any 3-cycle can be created by the product of 3-cycles of the form $(1, 2, j)$ and from the lecture we know, that any A_k is generated by 3-cycles.

Theorem

Since we can use $\sigma = (1, 2, 3, \dots, n)$, we can also use $\sigma^{-1} = \sigma^{n-1}$. Since we have $\alpha = (1, n-1, n)$, we can also use $(1, 2, 3) = \sigma^2 \alpha$. This way we can create arbitrary cycle of the form $(i, i+1, i+2) = \sigma^{1+i} \alpha$. From Lemma 3 we know, that 3-cycles of the form $(i, i+1, i+2)$ generate A_n , so it can be also generated from α, σ .