1

1.1

$$\frac{\binom{95}{3}}{\binom{100}{3}}$$

1.2

$$\frac{\binom{95}{1}\cdot\binom{5}{2}}{\binom{100}{3}}+\frac{\binom{5}{3}}{\binom{100}{3}}$$

1.3

1.3.1

$$\frac{95^3}{100^3}$$

1.3.2

$$\frac{95^2\cdot 5\cdot 3}{100^3}$$

2

2.1

$$P(A_i)=P([i\text{-tý dopis správně}])=\frac{1}{n}$$

$$P(A_i\cap A_j)=\frac{1}{n\cdot(n-1)}$$

$$\dots$$

$$P(A_1\cup A_2\cup\dots A_n)=\sum_{I\subseteq[n]}(-1)^{|I|+1}P(\bigcup_{i\in I}A_i)=\sum_{I\subseteq[n]}(-1)^{|I|+1}\frac{(n-|I|)!}{n!}=\sum_{k=1}^{n}(-1)^{k+1}\cdot\binom{n}{k}\frac{(n-k)!}{n!}=\sum_{k=1}^{n}\frac{(-1)^{k+1}}{k!}$$

$$\sum_{k=0}^{n}\frac{(-1)^{k+1}}{k!}+1=1-\sum_{k=0}^{n}\frac{(-1)^k}{k!}$$

2.2

V limitě: $1-e^{-1}$

3

3.1

$$\binom{r}{k}(\frac{1}{n})^k \cdot (1-\frac{1}{n})^{r-k}$$

3.2

$B_j :=$ [Na j-tém cvičení nikdo]
$Z = P([\text{na každém cvičení alespoň jeden}]) = 1 - P([\exists \text{ cvičení bez studentů}])$
$P(B_j) = (1-\frac{1}{n})^r$
$P(B_j \cap B_k) = (1-\frac{2}{n})^r$
$\dots$
$Z = P(B_1 \cup B_2 \cup \dots \cup B_n) = \sum_{I \subseteq [n]} (-1)^{|I|+1} \cdot P(\bigcup_{i \in I} B_i) = \sum_{I \subseteq [n]} (-1)^{|I|+1}(1-\frac{|I|}{n})^r = -\sum_{k=1}^{n}(-1)^k \binom{n}{k}(1-\frac{k}{n})^r$
$\frac{-1}{n^r}\sum_{k=1}^{n}(-1)^k(n-k)^r \cdot \binom{n}{k}$

3.3

TODO

4

4.1

$$1-(\frac{364}{365})^n$$

4.2

$$1-\prod_{i=1}^{n}\frac{365-i}{365} = 1-\frac{365!}{365^n(365-n)!}$$

4.3

$$\frac{1}{2} \geq \frac{365!}{365^n(365-n)!} \to 23$$