Prove $nZ/mZ \equiv Z_{m/n}$ for $n|m$.

Proof:
LHS:

$$\begin{aligned} nZ =& \{0, \pm n, \pm 2n, \ldots\} \\ mZ =& \{0, \pm m, \pm 2m, \ldots\} \\ nZ/mZ =& \{\{0, \pm m, \pm 2m, \ldots\}, \{n, n \pm m, n \pm 2m, \ldots\}, \{2n, 2n \pm m, 2n \pm 2m, \ldots\}, \ldots\} \end{aligned}$$

For nZ/mZ we can use a handy transversal τ with the smallest positive elements from each coset. Such elements are $\{0, n, 2n, \ldots, (k-1)n\}$, where $k = \frac{m}{n}$.

RHS:

$$Z_{m/n} = \{0, 1, 2, 3, \ldots, \frac{m}{n} - 1\}$$

Mapping:
We propose a mapping $\psi : Z_{m/n} \to \tau(nZ/mZ)$ such that $\psi : x \mapsto nx$. Clearly this is a surjective function, because $\forall y \in \{0, n, 2n, \ldots, m-n\} \exists x \in \{0, 1, 2, \ldots, \frac{m}{n} - 1\} : nx = y$. Furthermore it is also injective, because $\forall a, b \in \{0, 1, 2, \ldots, \frac{m}{n} - 1\} : na = nb \Rightarrow a = b$. From this we conclude, that ψ is bijective.

No we only need to show, that ψ is an isomorphism. We denote $a + b \mod x$ as $a +_x b$.

$$\begin{aligned} &\forall a, b \in \{0, n, 2n, \ldots, m-n\} : \\ &\psi(a +_{m/n} b) = n(a +_{m/n} b) = na +_m nb = \psi(a) +_m \psi(b) \end{aligned}$$

Since we found an isomorphism between a transversal of nZ/mZ and $Z_{m/n}$ we conclude, that $nZ/mZ \equiv Z_{m/n}$.