7.1 Vilém Zouhar

cos

$$\begin{split} \cos(\alpha) + i\sin(\alpha) &= e^{i\alpha} \\ \cos(-\alpha) + i\sin(-\alpha) &= \cos(\alpha) - i\sin(\alpha) = e^{-i\alpha} \\ \cos(\alpha) &= \frac{e^{i\alpha} + e^{-i\alpha}}{2} \\ \beta &= \frac{2k}{n} \\ DFT(\cos(\beta k))_k &= \sum_{j=0}^{n-1} e^{i\beta kj} \frac{e^{i\beta k} + e^{-i\beta k}}{2} = 1/2 \cdot \sum_{j=0}^{n-1} \left[ e^{2i\beta k(j+1)} + e^{2i\beta k(j-1)} \right] = \\ &= 1/2 \cdot \sum_{j=0}^{n-1} \left[ \omega_n^{k(j+1)} + \omega_n^{k(j-1)} \right] \\ \operatorname{Pro} k(j+1) &\equiv n \vee k(j-1) \equiv n : DFT(\cos(\beta k))_k = n/2 \\ \operatorname{Jinak} DFT(\cos(\beta k))_k &= 0 \end{split}$$