1

$$\begin{aligned}
\Phi(p^m) &= \#\text{numbers relatively prime to } p^m \\
&= \#\text{numbers less than } p^m - \#\text{numbers not relatively prime to } p^m \\
&= |\{0,1,2,\dots,p^m-1\}| - \#\text{numbers with } p \text{ in factor decomposition} \\
&= p^m - |\{0,p,2p,3p,\dots,\dots\}| \\
&= p^m - p^m/p = p^m - p^{m-1}
\end{aligned}$$

2

We can prove that $\Phi(ab) = \Phi(a)\Phi(b)$ by showing, that there exists a mapping between Z_{ab} and $Z_a \times Z_b$. Such function $\alpha : Z_{ab} \to Z_a \times Z_b$ can be: $\alpha(x) = (x \mod a, x \mod b)$.
If $\alpha(x) = \alpha(y)$, then $x \equiv y \mod a$ and $x \equiv y \mod b$, thus $x \equiv y \mod ab$ and x, y are equal in Z_{ab}.
Vice versa, the conditions $x = x_1 \mod a$ and $x = x_2 \mod b$ specify a unique solution $x \in Z_{ab}$.
This proves, that we can construct a bijection between Z_{ab} and $Z_a \times Z_b$. Thus $|Z_{ab}| = |Z_a||Z_b|$ and $\Phi(ab) = \Phi(a)\Phi(b)$.