1

1.1

$$f(x) = \begin{cases} 1 & x \in [0,1] \\ 0 & else \end{cases}$$

1.2

$$F(x) = \begin{cases} 0 & x \in (-\infty, 0) \\ x & else \\ 1 & x \in (1, \infty) \end{cases}$$

$$P(0.5) = 0$$
$$P(X \leq 0.5) = 0.5$$

1.3

$$Y = X^2, F_Y(y) = P(Y \leq y) = P(X^2 \leq y) = P(X \leq \sqrt{y}) = F(\sqrt(x)) \rightarrow \sqrt(x)$$
$$f_y(x) \rightarrow \frac{1}{2\sqrt(x)}$$

1.4

$$T = (b-a) \cdot X + a$$

2

2.1

$$\int_0^\infty ce^{-x/5}dx = 5c \cdot [e^{-y}]_0^\infty = 5c[0+1] = 1 \Rightarrow c = 1/5$$

2.2

$$F(t) = \int_0^t 0.2 \cdot e^{-x/5} = 0.2 \cdot [-e^{-x/5}]_0^t = 1 - e^{-t/5}$$

2.3

$$F(X \geq 5) = 1 - F(X < 5) = 1/e$$

2.4

$$F(X \in (2,5)) = F(X \leq 5) - F(X \leq 2) = -1/e + 1/e^{2/5}$$

2.5

$$P(X \geq 10 | X \geq 5) = \frac{P(X \geq 10 \wedge X \geq 5)}{P(X \geq 5)} = \frac{P(X \geq 10)}{1/e} = \frac{1/e^2}{1/e} = 1/e$$

2.6

$$Y = 5 + 3X$$
$$P(Y \leq y) = P(5 + 3X \leq y) = P(X \leq (y-5)/3) = 1 - e^{-\frac{y-5}{15}} = F_Y(y), y \in [5, infty)$$
$$f_Y(y) = 1/15 \cdot e^{\frac{y-5}{15}}, y \in [5, \infty)$$
$$F(Y \geq 35) = 1 - F(Y \leq 35) = e^{-2}$$

2.7

$$Z = \lceil X \rceil$$
$P(Z = z) = P(\lceil X \rceil = z) = P(X \in [z-1, z)) = \cdots$ diskrétní rozdělení

Druhý je vždy lepší

2.8

$$U = 1 - e^{-X/5} = F(X)$$
$$P(U \leq u) = P(F(X) \leq u) = P(X \leq F^{-1}(u)) = F(F^{-1}(u)) = u(u \in [0, 1))$$

2.9

Strčíme tam $F^{-1} \to P(F^{-1}(x) \leq u) = P(x \leq F(u)) = F_X(F_U(u)) = F_U(u)$

3

3.1

$$\begin{aligned}
&c\int_0^2 1 - |x-1| dx = 1 \\
&c\int_0^1 x \qquad dx + c\int_1^2 2 - x \qquad dx = 1 \\
&c[x^2/2]_0^1 + c[2x - x^2/2]_1^2 = 1 \\
&c/2 + c[4 - 2 - 2 + 1/2] = 1 \\
&c = 1
\end{aligned}$$

3.2

$$\begin{aligned}
F(t) &= \int_0^t 1 - |x-1| dx, t \in (0, 2) \\
&= \frac{(x-1)^2 sgn(1-x) + 2x - 1}{2} \\
&P(X \geq 1/2) = 1 - P(X \leq 1/2) = 7/8
\end{aligned}$$

4

4.1

$$\int_{-\infty}^{\infty} c \cdot e^{-|x|} = c[\int_{-\infty}^{0} e^x dx + \int_0^{\infty} e^{-x} dx] = c[[e^x]_{-\infty}^0 = [e^{-x}]_0^{\infty}] =$$
$$= c[1 + 1] = 1 \Rightarrow c = 1/2$$
$$|F(t)| = 1/2 \cdot \int_0^t e^x dx = e^{-t} - 1$$
$$P(X \geq 2) = 1 - P(X \leq 2) = 1 - 1/2 \cdot (e^2 - 1) = 1 - e^{-2}/2 \approx 93.23$$