$$V = \{id, (12)(34), (13)(24), (14)(23)\}$$
$$|S_4| = 24, |V| = 4$$
$$|S_4/V| = 6$$

There are two groups of the order of 6. Namely C_6 and S_3. By showing that S_4/V is not isomorphic to C_6, we actually show that it is isomorphic to S_3.
For C_6 it is true, that elements are of order 6: $\forall x \in C_6 : x^6 = x^1$ and $x \neq x^i \forall i \in \{2, 3, 4, 5\}$. This is not true for S_4, because it contains elements of maximum order of 4. Since $V \subseteq S_4$, then there cannot be an element of order 6 in S_4/V.