3. domácí úkol | Vilém Zouhar

1

Využijeme vztahu dokázaného v ZS: $x^n - y^n = (x-y)(\sum_{i=1}^{n} x^{n-i}y^{i-1})$.

$$\int \frac{x^{17}-5}{x-1}dx = \int \frac{(x-1)(\sum_{i=0}^{16} x^i)}{x-1}dx - \int \frac{4}{x-1}dx = \left(\sum_{i=0}^{16} \int x^i dx\right) - 4\ln|x-1| =$$

$$\sum_{i=0}^{16} \frac{x^{i+1}}{i+1} - 4\ln|x-1| + c =$$

$$\frac{x^{17}}{17} + \frac{x^{16}}{16} + \frac{x^{15}}{15} + \frac{x^{14}}{14} + \frac{x^{13}}{13} + \frac{x^{12}}{12} + \frac{x^{11}}{11} + \frac{x^{10}}{10}\frac{x^9}{9} + \frac{x^8}{8} + \frac{x^7}{7} + \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x - 4\ln|x-1| + c$$

Původní je definovaná na $\mathbf{R}\backslash\{1\}$, výsledný integrál taky, tedy platí pro intervaly: $(-\infty, 1)$ a $(1, \infty)$.

2

$$\int \ln(x+\sqrt{1+x^2})dx = \textit{[per partes]} = x \cdot \ln(x+\sqrt{1+x^2}) - \int \frac{2x}{2\sqrt{1+x^2}} =$$

$$\left[f = 1, F = x, G = \ln(x+\sqrt{1+x^2}), g = \frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}}\right]$$

$$\left[\int \frac{2x}{2\sqrt{1+x^2}} = \textit{[sub. } y = 1+x^2, y' = 2x\textit{]} = \frac{1}{2}\int y^{-\frac{1}{2}}dy = 2y^{\frac{1}{2}} + c = \sqrt{1+x^2} + c\right]$$

$$= x \cdot \ln(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + c$$

Původní funkce je definovaná na $\mathbf{R}$ (neboť argument logaritmu je vždy kladný). Primitivní funkce je také definovaná na celém $\mathbf{R}$.

3

$$\int \frac{\cos^2 x}{\sin x \cdot (1-\cos x)}dx =$$

$$\left[sub.\ t = \tan\frac{x}{2}, 1 \cdot dx = \frac{2}{1+t^2}dt, \sin x = \frac{2t}{t^2+1}, \cos x = \frac{1-t^2}{1+t^2}\right]$$

$$= \int \frac{\left(\frac{1-t^2}{1+t^2}\right)^2}{\frac{2t}{t^2+1} \cdot \left(1 - \frac{1-t^2}{1+t^2}\right)} \cdot \frac{2}{1+t^2}dt = \int \frac{(1-t^2)^2 \cdot (1+t^2)}{(1+t^2)^2 \cdot t \cdot (1+t^2-1+t^2)}dt = \frac{1}{2}\int \frac{(1-t^2)^2}{(1+t^2) \cdot t^3}dt =$$

$$\left[\frac{1-2t^2+t^4}{(1+t^2) \cdot t^3} = \frac{At+B}{1+t^2} + \frac{C}{t^3} + \frac{D}{t^2} + \frac{E}{t} \Rightarrow (A,B,C,D,E) = (4,0,1,0,-3)\right]$$

$$= \frac{1}{2}\left[\int 2\frac{2t}{1+t^2}dt + \int \frac{1}{t^3} - \int \frac{3}{t}\right] =$$

$$\left[\int 2\frac{2t}{1+t^2}dt = [sub.\ y = 1+t^2, y' = 2t] = \int 2\frac{1}{y}dy = 2\ln|y| + c = 2\ln|1+t^2| + c\right]$$

$$= \frac{1}{2}\left[2\ln|1+t^2| + \frac{-1}{2t^2} - 3\ln|t|\right] + c = \frac{2\ln|1+\tan^2\frac{x}{2}| - \frac{1}{2\tan^2\frac{x}{2}} - 3\ln|\tan\frac{x}{2}|}{2} + c$$

Původní funkce je definovaná na $\mathbf{R}\backslash\bigcup_{k\in\mathbf{Z}}\{k \cdot \pi\}$, integrál taky na intervalech: $\mathbf{R}\backslash\bigcup_{k\in\mathbf{Z}}\{k \cdot \pi\}$ (kvůli tan ve jmenovateli)