Drone Stuff

Zhipeng Tang

February 2019

There are two things we need to make consensus on. First is the definition of axes, rotation directions. Mark said he needs to write that on a paper and post on a wall of his office. That is a great idea! Second is checking the correctness of the way I convert the Euler angle from the drone coordinate system to the camera coordinate system.

1 Basic Definition

According to our correspondence and your master thesis, I believe the **z-axis is pointing front, x-axis is pointing right, and y-axis is pointing down**. In addition, **the positive rotation direction is the counter-clock direction when we look at the arrow of axes (imaging the arrow is piercing our heads)**.

Figure 1 illustrates what I said:

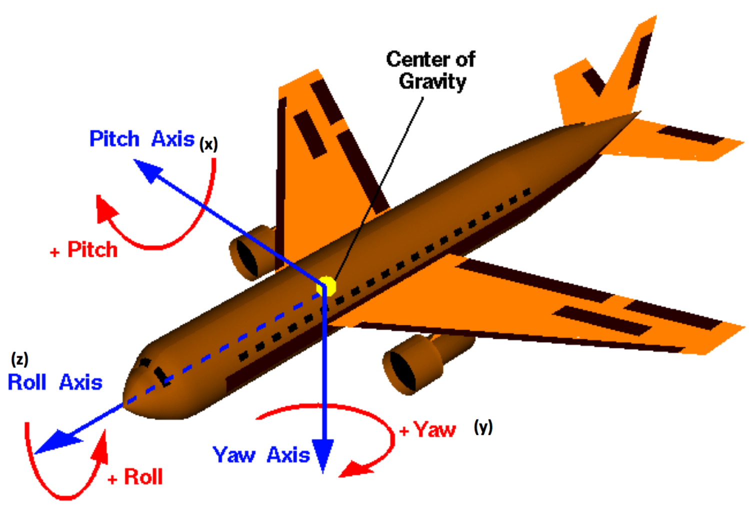

Figure 1: Definition of Aircraft Body Axes

2 Preliminary Knowledge

2.1 What is Rotation Matrix

2.1.1 Definition

The core of the conversion relies on Rotation Matrix, so we start from discussing this concept.

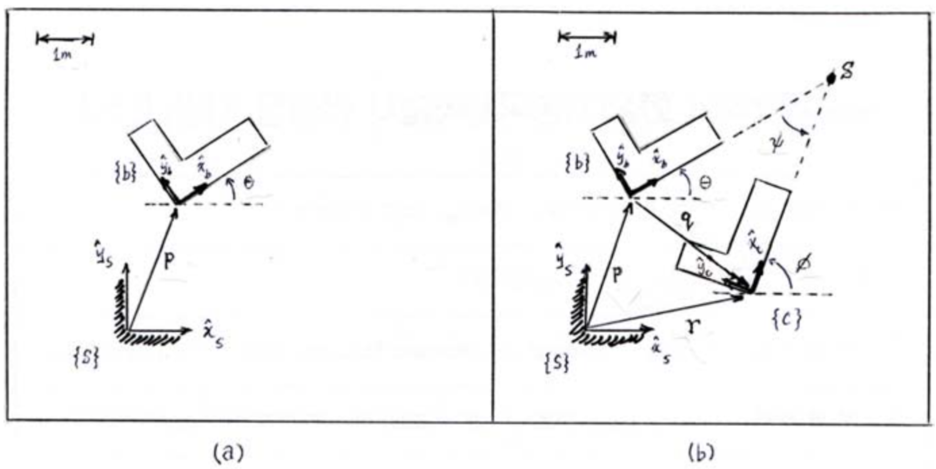

Figure 2: Rotation Matrix Example

First, we look at Fig 2 (a). $\{s\}$ is **the fixed frame** or **the world frame**. $\{b\}$ is **the body frame**, which is associated with a rigid body.

The origin and two axes of $\{b\}$ can be expressed in terms of the fixed frame as

$$\begin{aligned} p &= p_x\hat{x}_s + p_y\hat{y}_s \\ \hat{x}_b &= cos\theta\hat{x}_s + sin\theta\hat{y}_s \\ \hat{y}_b &= -sin\theta\hat{x}_s + cos\theta\hat{y}_s \end{aligned} \tag{1}$$

This can be represented more compactly as the vector and matrix form:

$$\begin{aligned} p &= \begin{bmatrix} p_x \\ p_y \end{bmatrix} \\ P &= \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \end{aligned} \tag{2}$$

P in the equation is **rotation matrix**. Notice that every column of the rotation matrix represents an axis of the frame after rotation expressed in terms of the linear combination of the original frame's axes. For example:

$$
\begin{aligned}
\hat{x}_b &= \begin{bmatrix} \hat{x}_s & \hat{y}_s \end{bmatrix} \begin{bmatrix} cos\theta \\ sin\theta \end{bmatrix} \\
\hat{y}_b &= \begin{bmatrix} \hat{x}_s & \hat{y}_s \end{bmatrix} \begin{bmatrix} -sin\theta \\ cos\theta \end{bmatrix}
\end{aligned} \tag{3}
$$

In other words, rotation matrix represents the orientation of the frame after rotation operation via two column vectors. When talking about a specific rotation matrix, we need to pay attention to the frame it is based on. For example, the rotation matrix P is indeed based on $\{s\}$. The two axes of $\{b\}$ is stored in the two columns of P. These two column vectors are expressed in terms of the axes of $\{s\}$.

In order to make things clear, for a rotation matrix R, we introduce the notation R_{ab}.

Definition 1 *Given two reference frames $\{a\}$ and $\{b\}$, the orientation of frame $\{b\}$ as seen from frame $\{a\}$ will be represented by the rotation matrix R_{ab}. That is, the three columns of R_{ab} are just vector representations of the $\hat{x}$, $\hat{y}$, and of frame $\{b\}$ expressed in terms of coordinates for frame $\{a\}$.*

2.1.2 Properties

We now introduce three important properties of the rotation matrix.

Property 1 *Consider a free vector v in physical space. Given two reference frames $\{a\}$ and $\{b\}$ in the physical space, let v_a and v_b denote representations of v with respect to these two frames, we have*

$$R_{ba}v_a = v_b$$

Property 1 tells us how to convert a vector from one reference frame to another.

Property 2 *Consider a vector v_b in reference frame $\{b\}$ and v_b rotates with the rotation of $\{b\}$. After the axes of $\{b\}$ rotate to the orientation of frame $\{a\}$, v_b rotates to v_b', where v_b' is still expressed in terms of frame $\{b\}$*

$$R_{ba}v_b = v_b'$$

Property 2 tells us how to compute the coordinates of the rotated vector.

Property 3 $R_{ab}R_{bc} = R_{ac}$

Property 3 can be illustrated by Figure 2 (b). In this figure, we have $R_{sc} = R_{sb}R_{bc}$.

2.2 ZYX Euler Angles

Every rotation matrix can be expressed by a ZYX Euler angle, and vice versa. What is ZYX Euler angle? Assume we have the ZYX Euler angle (α, β, γ) and a reference frame $\{0\}$ with axes (x, y, z), we perform the following steps:

1. Rotate about the z-axis by an angle α, we get a new frame $\{1\}$ with new axes (x', y', z')

2. Rotate about the y'-axis by an angle β, we get another new frame $\{2\}$ with new axes (x'', y'', z'')

3. Rotate about the x''-axis by an angle γ, we get the final frame $\{3\}$ with axes (x''', y''', z''')

Notice that we choose to rotate the axis of the new (body) frame each time in ZYX Euler Angle.

The ZYX Euler angle is corresponding to the following rotation matrix:

$$R_{03} = \begin{bmatrix} c_\alpha c_\beta & c_\alpha s_\beta s_\gamma - s_\alpha c_\gamma & c_\alpha s_\beta c_\gamma + s_\alpha s_\gamma \\ s_\alpha c_\beta & s_\alpha s_\beta s_\gamma + c_\alpha c_\gamma & s_\alpha s_\beta c_\gamma - c_\alpha s_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix} \tag{4}$$

2.3 Roll-Pitch-Yaw Angle

Roll-Pitch-Yaw Angle is another representation for rotation matrices. It is very similar to ZYX Euler angle.

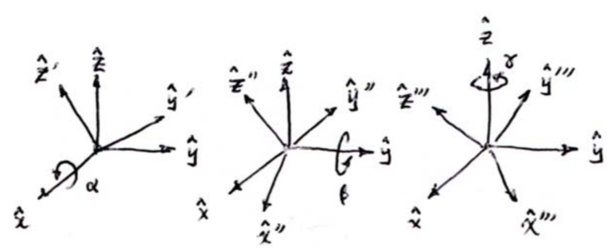

Figure 3: the Roll-pitch-yaw Angle

Assume we have the XYZ roll-pitch-yaw angle (α, β, γ) and a reference frame $\{0\}$ with axes (x, y, z), we perform the following steps:

1. Rotate about the x-axis by an angle γ, we get a new frame $\{1\}$ with new axes (x', y', z')

2. Rotate about the y-axis by an angle β, we get another new frame $\{2\}$ with new axes (x'', y'', z'')

3. Rotate about the z-axis by an angle α, we get the final frame $\{3\}$ with axes (x''', y''', z''')

Notice that we always rotate the axis of the original fixed frame in roll-pitch-yaw angle.

Interestingly, the rotation matrix corresponding to the XYZ roll-pitch-yaw angle is the same as the rotation matrix for ZYX Euler angle:

$$R_{(\alpha,\beta,\gamma)} = \begin{bmatrix} c_\alpha c_\beta & c_\alpha s_\beta s_\gamma - s_\alpha c_\gamma & c_\alpha s_\beta c_\gamma + s_\alpha s_\gamma \\ s_\alpha c_\beta & s_\alpha s_\beta s_\gamma + c_\alpha c_\gamma & s_\alpha s_\beta c_\gamma - c_\alpha s_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix} \tag{5}$$

3 Euler Angle Conversion

3.1 Problem Definition

Consider a drone equipped with a camera. The angle between the direction the drone points at and the direction the camera points at is θ (see Figure 4).

At beginning, the drone's frame (i.e. pose) is T_{drone1}. Based on this frame, the drone rotates according to a ZYX Euler angle (α, β, γ). The new frame after rotation is T_{drone2}.

The camera will rotate with the drone. Assume the camera's frame at beginning is $T_{camera1}$. After rotation, the camera's frame turns to $T_{camera2}$.

We want to compute the ZYX Euler angle $(\alpha', \beta', \gamma')$ which turns $T_{camera1}$ to $T_{camera2}$

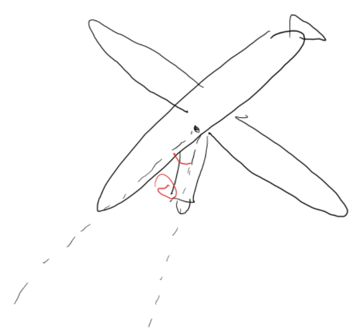

Figure 4: Illustration of the Drone and its Camera

3.2 Solution

Assume there is a fixed frame $\{s\}$ whose three axes are $\hat{x}_s$, $\hat{y}_s$, and $\hat{z}_s$. All of the aforementioned frames can be seen as rotation matrices of which columns are their axes expressed in terms of $\{s\}$. For example, just as the definition of rotation matrices, $\begin{bmatrix} \hat{x}_s & \hat{y}_s & \hat{z}_s \end{bmatrix} T_{drone1}$ is the actual axes of T_{drone1}. More specifically, T_{drone1} **is in fact equivalent to** $T_{s,drone1}$. I bother to talk about this because I want to explain the definition of the aforementioned frames exactly.

Apparently, the ZYX Euler angle (α, β, γ) for drone rotation can be expressed as the rotation matrix $R_{T_{drone1},T_{drone2}}$:

$$R_{T_{drone1},T_{drone2}} = \begin{bmatrix} c_\alpha c_\beta & c_\alpha s_\beta s_\gamma - s_\alpha c_\gamma & c_\alpha s_\beta c_\gamma + s_\alpha s_\gamma \\ s_\alpha c_\beta & s_\alpha s_\beta s_\gamma + c_\alpha c_\gamma & s_\alpha s_\beta c_\gamma - c_\alpha s_\gamma \\ -s_\beta & c_\beta s_\gamma & c_\beta c_\gamma \end{bmatrix} \tag{6}$$

According to the definition of rotation matrices, we have:

$$\begin{aligned} T_{s,drone2} &= T_{s,drone1} R_{T_{drone1},T_{drone2}} \\ T_{drone2} &= T_{drone1} R_{T_{drone1},T_{drone2}} \end{aligned} \tag{7}$$

Let's try to find the rotation matrix $R_{drone1,camera1}$ to rotate T_{drone1} to $T_{camera1}$. Because the angle between the drone and the camera is θ, we know that the ZYX Euler angle from the drone frame to the camera frame is $(0, 0, -\theta)$ (the minus symbol means the rotation is clockwise). Therefore, we can substitute the Euler angle into the formula and get the corresponding rotation matrix $R_{drone1,camera1}$. Therefore,

$$\begin{aligned} T_{s,camera1} &= T_{s,drone1} R_{drone1,camera1} \\ T_{camera1} &= T_{drone1} R_{drone1,camera1} \end{aligned} \tag{8}$$

Similarly, the rotation matrix $R_{drone2,camera2}$ which transform T_{drone2} to $T_{camera2}$ has

$$\begin{aligned} T_{s,camera2} &= T_{s,drone2} R_{drone2,camera2} \\ T_{camera2} &= T_{drone2} R_{drone2,camera2} \end{aligned} \tag{9}$$

Notice that the drone and the camera constitute a rigid body. This implies $R_{drone2,camera2}$ which is equivalent to $R_{drone1,camera1}$. We set

$$R_{drone,camera} = R_{drone1,camera1} = R_{drone2,camera2} \tag{10}$$

What we want to compute is $R_{T_{camera1},T_{camera2}}$, where $T_{camera2} = T_{camera1} R_{T_{camera1},T_{camera2}}$. Combine all equations together, we have:

$$\begin{cases} T_{drone2} = T_{drone1} R_{T_{drone1},T_{drone2}} \\ T_{camera1} = T_{drone1} R_{drone,camera} \\ T_{camera2} = T_{drone2} R_{drone,camera} \\ T_{camera2} = T_{camera1} R_{T_{camera1},T_{camera2}} \end{cases} \tag{11}$$

Solve this system of equations, we have

$$R_{T_{camera1},T_{camera2}} = (T_{drone1}R_{drone,camera})^{-1}T_{drone2}R_{drone,camera} \tag{12}$$

Then, we can convert $R_{T_{camera1},T_{camera2}}$ to the ZYX Euler angle, and that is what we want.