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Genetic Algorithms for Least-cost Design of Water Distribution Networks

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ABSTRACT

The paper describes the development of a computer model GANET, that involves the application of an area of Evolutionary Computing, better known as Genetic Algorithms, to the problem of least-cost design of water distribution networks. Genetic Algorithms represent an efficient search method for non-linear optimization problems, that is gaining in acceptance among water resources managers/planners. These algorithms share the favourable attributes of Monte Carlo techniques over local optimization methods in that they do not require linearizing assumptions nor the calculation of partial derivatives and avoid numerical instabilities associated with matrix inversion. In addition, their sampling is global, rather than local, thus reducing the tendency to become entrapped in local minima and avoiding dependency on a starting point. Genetic Algorithms are introduced in their original form followed by different improvements that were found to be necessary for their effective implementation in the optimization of water distribution networks. An example taken from the literature illustrates the approach used for the formulation of the problem. To illustrate the capability of GANET to efficiently identify good designs, three previously published problems have been solved. This led to the discovery of inconsistencies in predictions of network performance caused by different interpretations of the widely adopted Hazen-Williams pipe flow equation in the past studies. As well as being very efficient for network optimization, GANET is also easy to use, having almost the same input requirements as hydraulic simulation models. The only additional data requirements are a few Genetic Algorithm parameters that take values recommended in the literature. Two network examples, one of a new network design and one of parallel network expansion, illustrate the potential of GANET as a tool for water distribution network planning and management.

Key words: water distribution systems, optimization, genetic algorithms, modelling.

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INTRODUCTION

As a vital part of water supply systems, water distribution networks represent one of the largest infrastructure assets of industrial society. Simulation of hydraulic behaviour within a pressurised, looped pipe network is a complex task which effectively means solving a system of non-linear equations. The solution process involves simultaneous consideration of the energy and continuity equations and the head-loss function (Wood and Funk, 1993). A number of different methods for solving the steady-state network hydraulics have been developed over the years. Coupled with the availability of inexpensive powerful hardware, the development of models in the last two decades has improved considerably the ability to simulate hydraulic behaviour of large water distribution networks (Rossman, 1993; Wood, 1980). These models play an important role in layout, design and operation of water distribution systems.

Selection of pipe diameters (especially from a set of commercially available discrete-valued diameters) to constitute a water supply network of least capital cost has been shown to be an NP-hard problem (Yates et al., 1984). The cost of operating a water distribution system may be substantial (due to maintenance, repair, water treatment, energy costs, etc.), but still one of the main costs is that of the pipelines themselves. In recent years a number of optimization techniques have been developed primarily for the cost minimization aspect of network planning, although some reliability studies and stochastic modelling of demands have been attempted as reviewed by Walters and Cembrowicz (1993). The importance of obtaining the best network layout and the optimal pipe diameter for each pipe is emphasised by the fact that the decisions made during layout and design phases will determine the ultimate operations costs. Since joint consideration of network layout and design is extremely complex and since layout is largely restricted by the location of the roads, many different problem formulations and solution methods have been proposed and tested for least-cost design only (Alperovits and Shamir, 1977; Quindry et al., 1981; Ormsbee and Wood, 1986; Fujiwara and Khang, 1990; Murphy et al., 1993; and Eiger et al., 1994). Given a network layout and requirements (flow and pressure) the optimal design problem of water distribution systems has been viewed as the selection of pipe sizes which will minimize overall costs.

The paper describes the development of a computer model, known as GANET, that involves the application of a relatively new optimization technique to the problem of least-cost design of water distribution networks. The Genetic Algorithm (GA) is introduced in its original form followed by different possible improvements necessary for its effective implementation in the optimization of water distribution networks. An example taken from the literature illustrates the approach used for the formulation of the problem and the convergence characteristics of the GA. Two additional network examples, one for new design and one for rehabilitation of an existing system, illustrate the potential of GANET as a tool for water distribution network planning and management.

OPTIMAL DESIGN OF WATER DISTRIBUTION NETWORKS

Design of water distribution networks is often viewed as a least-cost optimization problem with pipe diameters being decision variables. Pipe layout, connectivity and imposed minimum head constraints at pipe junctions (nodes) are considered known. There are obviously other possible objectives, like reliability, redundancy and/or water quality, that can be included in the optimization process (Alperovits and Shamir, 1977). However, problems with quantifying these objectives for use within optimization design models kept researchers concentrating on the single, least-cost objective (Goulter, 1987). Even that, somewhat restricted formulation of optimal network design represents a difficult problem to solve. It is also worth noting that networks designed on a purely cost-effective basis and for a single loading condition tend to be driven to a branched layout (Quindry et al., 1981; Goulter and Morgan, 1985). This simply means that for optimal least-cost design, loops will, in normal circumstances, disappear, leaving an underlying tree-like structure. However, reliability/redundancy considerations usually dictate that loops are retained. Many researchers have used the minimum diameter constraint to ensure that loops are preserved in the final solution. The inclusion of some measure of reliability/redundancy is beyond the scope of this work.

Mathematical Formulation

The following mathematical statement of the optimal design problem is presented for a general water supply network (Shamir and Howard, 1968, Schaake and Lai, 1969; Quindry et al., 1981). The objective function is assumed to be a cost function of pipe diameters and lengths

$$f(D_1, \ldots, D_n) = \sum_{i=1}^{N} c(D_i, L_i) \qquad \textbf{(1)}$$

where $c(D_i,L_i)$ is the cost of the pipe i with the diameter D_i and the length L_i, and N is the total number of pipes in the system. The above function is to be minimized under the following constraints.

For each junction node (other than the source) a continuity constraint should be satisfied

$$\sum Q_{in} - \sum Q_{out} = Q_e \tag{2}$$

where Q_{in} is the flow into the junction, Q_{out} is the flow out of the junction and Q_e represents the external inflow or demand at the junction node. Under this convention demands Q_e which extract flow from the junction are positive.

For each of the basic loops in the network the energy conservation constraint can be written as

$$\sum h_f - \sum E_p = 0 \tag{3}$$

where E_p is the energy put into the liquid by a pump. The Hazen-Williams or Darcy-Weisbach formula may be used to express the head-loss term h_f. If more than one source node is available then additional energy conservation constraints are written for paths between any two of the nodes. For a total of P source nodes (reservoirs) there are P-1 independent equations needed.

The minimum head constraint for each node in the network is given in the form

$$H_j \geq H_j^{\min}; \; j = 1, \ldots, M \tag{4}$$

where H_j is the head at node j, H_j^{min} is the minimum required head at the same node and M is the total number of nodes in the system.

Hazen-Williams Equation

Many aspects of planning for water distribution networks require an understanding of the equations of closed conduit hydraulics. Practising engineers are primarily concerned with accurate predictions of the discharge capacity of pipe systems. To enable them to perform practical pipe flow calculations some function describing the relationship between pressure drop, flow rate, pipe length and pipe diameter is required. Several friction head loss/flow formulae have been developed for that purpose. The most well-known for pressurised pipe systems are the Darcy-Weisbach and the Hazen-Williams (H-W) formulae (Walski, 1984).

The empirical H-W equation is the most commonly used of the formulae. The following is the equation in its original form

$$\mathrm{v} = C R_h^{0.63} S_f^{0.54} \tag{5}$$

where v is the flow velocity in feet/second, R_h is the hydraulic radius in feet, S_f is the hydraulic gradient and C is a dimensional coefficient whose numerical value should change on conversion to different units. For a pipe of circular cross section R_h = D/4. In an attempt to keep the formula easy to use, the C-value is treated as a pipe constant. Conversion is achieved by introducing a numerical conversion constant α and thus the formula may be written in the form

$$\mathrm{v} = \alpha C D^{0.63} S_f^{0.54} \tag{6}$$

where D is the pipe diameter and α takes on a value of 0.55 for Imperial units and 0.355 for SI units. If the head loss h_f is desired with the flow Q known, the equation for a pipe can be written as

$$h_f = \omega \frac{L}{C^a D^b} Q^a \tag{7}$$

where ω is a new numerical conversion constant which depends on the units used, a is a coefficient equal to 1/0.54, b is a coefficient equal to 2.63/0.54, and L is the length of the pipe. However, different researchers have used different values for the numerical conversion constants α and ω in Eqs.(6) and (7). Table 1 gives a selection of the forms of the equation found in the optimization literature and the values of ω calculated for each of the forms to enable comparison.

To illustrate that there is no end to different interpretations of the H-W equation note that Jain et al. (1978) and Chiplunkar et al. (1985) suggested the following form

$$h_f = \frac{L\left(\frac{Q}{C}\right)^{1.8099}}{994.62 \cdot D^{4.8099}} \qquad \textbf{(8)}$$

where D and L are in meters and Q in m^3/s, while WATNET (WRc Engineering, 1989) uses

$$h_f = 1.2 \cdot 10^{10} L\left(\frac{Q}{C}\right)^{1.85} D^{-4.87} \qquad \textbf{(9)}$$

(D in millimeters, L in meters and Q in litres per second) which is equivalent to a value for the coefficient ω even lower than the lowest value found in published literature (Table 1), namely, ω = 10.4516, for D and L in metres and Q in m^3/s.

Solution Techniques Used for Least-Cost Pipe Network Optimization

The optimization problem formulated in the above manner is non-linear due to energy conservation constraints of Eq.(3). In addition, pipes for water supply are manufactured in a set of discrete-sized diameters thus introducing additional difficulties to the problem of searching for the optimal design. In order to solve this NP-hard problem exactly it is suggested that only explicit enumeration or an implicit enumeration technique such as Dynamic Programming, can guarantee the optimal solution (Yates et al., 1984). However, optimal pipe-network design problems of realistic size become intractable for enumeration techniques. For example, consider a network of 20 pipes and a set of 10 discrete diameters. The total solution space for this problem is equal to 10^{20} different designs. Even if one million design evaluations can be performed in a second, over 3 million years of CPU time would be needed for complete enumeration.

It is not surprising then that various simplified approaches have been suggested in the past to reduce the complexity of the original problem. The traditional approach of design engineers is to use a simulation model of the water distribution system to evaluate promising solutions obtained by the trial-and-error process. The costs for some of the better alternatives are then calculated to arrive at a recommended solution (Walski, 1985). The other approach which also uses a network simulation model is based on enumeration of a limited number of alternatives (Gessler, 1985). In this work Gessler devised tests to eliminate certain inferior solutions from being evaluated by a hydraulic simulation model (i.e., testing for pressure constraints). However, it was shown subsequently by Murphy and Simpson (1992) that the approach had failed to identify the optimal design for a moderately small network expansion problem.

One of the simplified approaches proposed by Alperovits and Shamir (1977) reduces the complexity of the original non-linear nature of the problem by solving a sequence of linear sub-problems. This approach has been adopted and subsequently improved by many researchers (Quindry et al., 1981; Goulter and Morgan, 1985; and Fujiwara and Khang, 1990). The optimum solution obtained by this method will consist of one or two pipe-segments of different discrete sizes between each pair of nodes. For a more realistic solution, the split-pipe design should be altered so that only one diameter is chosen for each pipe. The altered solution should then be checked to ensure that minimum head constraints are satisfied.

As the problem of design of a pipe network is non-linear, standard or application-tailored non-linear programming techniques could be used. Many of these models based on the steepest path ascent technique can be used coupled with a network simulation package. However, they still treat pipe diameters as continuous variables and have limitations to the size of problem they can handle (El-Bahrawy and Smith, 1985; Duan et al., 1990). The problem of converting a continuous-diameter solution to its discrete equivalent is less simple than it might appear since mere rounding to the nearest discrete value cannot guarantee an optimal or even feasible solution.

It should be noted here that all aforementioned approaches, i.e., heuristics, non-exhaustive enumeration, linear programming, and non-linear programming, require introduction of simplifications to ensure that a technical solution will be achieved. These simplifications depend on the solution methodology and the required accuracy of the output. Consequently, those methods do not guarantee the global optimum of the original discrete, non-linear optimization problem. Therefore, many repetitive applications of the algorithms are needed to ensure that the best solution found is of good quality, i.e., at least near optimal.

Over the course of the last two decades computer algorithms mimicking certain principles of nature have proved their usefulness in various domains of application. Among them, Genetic Algorithms (GAs), a subclass of general artificial-evolution search methods based on natural selection and the mechanisms of population genetics named Evolution Programs (EPs) by Michalewicz (1992), are the most popular. This form of search evolves throughout generations, improving the features of potential solutions by means of biologically-inspired operations. After reported successes in some problem domains, pipe network optimization has started to benefit from the use of GAs and other EPs. Goldberg and Kuo (1987), Walters and Lohbeck (1993), Murphy and Simpson (1992), and Walters and Cembrowicz (1993) developed and applied EPs to optimization of hydraulic networks.

STANDARD GENETIC ALGORITHMS

Genetic Algorithms, which are probably the best known type of EPs, are also referred to as *stochastic optimization* techniques. Stochastic optimization designates a family of optimization techniques in which the solution space is searched by generating candidate solutions with the aid of a pseudo-random number generator. As the run proceeds, the probability distribution by which new candidate solutions are generated may change, based on results of trials earlier in the run. Because of their stochastic nature there is no guarantee that the global optimum will be found using GAs although the number of applications suggests a good rate of success in identifying good solutions. The theory behind GAs was proposed by Holland (1975) and further developed by Goldberg (1989) and others in the 1980's. These algorithms rely on the collective learning process within a population of individuals, each of which represents a search point in the space of potential solutions. They draw their power from the theoretical principle of implicit parallelism (Holland, 19765). This principle enables highly fit solution structures (schemata) to receive increased numbers of offspring in successive generations and thus lead to better solutions.

A variety of applications has been presented since the early works of Holland [1975] and Goldberg [1983] and GAs have clearly demonstrated their capability to yield good approximate solutions even in cases of complicated multimodal, discontinuous, non-differentiable functions (e.g., see Savic and Walters, 1994 for a review). There are many variations of GAs but the following general description encompasses most of the important features. The analogy with nature is established by the creation within a computer of a set of solutions called a *population*. Each individual in a population is represented by a set of parameter values which completely describe a solution. These are encoded into *chromosomes*, which are, in essence, sets of character strings analogous to the chromosomes found in DNA. Standard GAs (SGAs) use a binary alphabet (characters may be 0's or 1's) to form chromosomes. For example a two-parameter solution $x = (x_1, x_2)$ may be represented as an 8-bit binary chromosome: 1001 0011 (i.e., 4 bits per parameter, x_1 = 1001, x_2 = 0011). At this point it should be noted that not all EPs restrict representation to the binary alphabet which makes them more flexible and applicable to a variety of decision-making problems.

The initial population of solutions, which is usually chosen at random, is allowed to evolve over a number of *generations*. At each generation, a measure (*fitness*) of how good each chromosome is with respect to an objective function is calculated. This is achieved by simply decoding binary strings into parameter values, substituting them into the objective function and computing the value of the objective function for each of the chromosomes. Next, based on their fitness values individuals are selected from the population and recombined, producing offspring which will comprise the next generation. This is the *recombination* operation, which is generally referred to as *crossover* because of the way that genetic material crosses over from one chromosome to another. For example, if two chromosomes are $x = (x_1, x_2)$ = 1111 1111 and $y = (y_1, y_2)$ = 0000 0000, the two offspring may be z = 1100 0000 and w = 0011 1111.

The probability that a chromosome from the original population will be selected to produce offspring for the new generation is dependent on its fitness value. Fit individuals will have higher probability of being selected than less fit ones resulting in the new generation having on average a higher fitness then the old population. Mutation also plays a role in the reproduction phase, though it is not the dominant role, as is popularly believed, in the process of evolution [Goldberg, 1989]. In SGAs mutation randomly alters each bit (also called *gene*) with a small probability. For example, if the original chromosome is $x = (x_1, x_2)$ = 1111 1111, the same chromosome after mutation may be x' = 1110 1111. If the probability of mutation is set too high, the search degenerates into a random process. This should not be allowed as a properly-tuned SGA is not a random search for a solution to a problem. As a simulation of a genetic process an SGA uses stochastic mechanisms, but the result is distinctly better than random.

GANET: A GA MODEL FOR LEAST-COST PIPE NETWORK DESIGN

The Genetic Algorithm for least-cost pipe network design (GANET) will be introduced using the two-loop network problem studied by Alperovits and Shamir (1977) and later by Quindry et al. (1981), Goulter et al. (1986), Fujiwara et al. (1987), Kessler and Shamir (1989, 1991), Bhave and Sonak (1992) and Sonak and Bhave(1993). The layout of the network with a single source at a 210 m fixed head and 8 pipes arranged in two loops is shown in Figure 1. The pipes are all 1000 m long with the assumed Hazen-Williams coefficient of 130. The node data for this network is summarised in Table 2. The minimum acceptable pressure requirements for nodes 2-7 are defined as 30 m above ground level. There are 14 commercially available diameters and Table 3 presents the total cost per one metre of pipe length for different pipe sizes.

Coding

Decision variables for this problem are eight pipe diameters and each pipe can take one of the fourteen sizes in Table 3. A binary string made of eight substrings is used for coding the problem into a suitable form for use within a GA. Since a three-bit substring cannot handle fourteen discrete pipe sizes (i.e., it can represent only $2^3 = 8$ discrete values) a four-bit substring is used. The eight substrings are joined together to form a 32-bit string which represents a solution to the problem. However, some of the binary codes are redundant since there are $2^4 = 16$ possible bit combinations and consequently two substring values will not correspond to any available pipe diameter. Fixed remapping, which maps a particular redundant substring value to a specific valid diameter, is used to handle this problem.

In addition, instead of a simple binary code a *Gray code* (Goldberg, 1989) interpretation of the bit string is used for decoding. A Gray code represents each number in the sequence of integers $[0...2^N-1]$ as a binary string of length N in an order such that adjacent integers have Gray code representations that differ in only one bit position. Marching through the integer sequence therefore requires flipping just one bit at a time. Goldberg [1989] calls this defining property of Gray codes the "adjacency property". The rationale for the Gray coding is found in the existence of the "neighbourhood structure" in the search space. It is assumed that groups of good solutions tend to lie close together, i.e., tend to be mutually accessible via mutation. For example, the binary coding of [0, 1, 2, 3, 4, 5, 6, 7] is [000, 001, 010, 011, 100, 101, 110, 111], while one Gray coding is [000, 001, 011, 010, 110, 111, 101, 100]. Notice that the step from three to four requires the flipping of all the bits in the binary representation while it requires only one flip in the Gray code representation.

Fitness

The evaluation function determines the cost of a solution by summing the cost of the pipes making up the network. Simulation of the network flows and pressure heads is then carried out to assess the feasibility of a solution. The network solver used in this work is based on the EPANET (Rossman, 1993) computer program. It employs the "gradient method" (Todini and Pilati, 1987) for solving the system of equations (2) and (3).

The minimum pressure constraint discriminates between feasible and infeasible solutions. Rather than ignoring infeasible solutions, and concentrating only on feasible ones, infeasible solutions are allowed to join the population and help guide the search, but for a certain price. A penalty term incorporated in the fitness function is activated for a pressure-infeasible solution thus reducing its strength relative to the other strings in the population. The penalty function used is graded, i.e., the penalty is a function of the distance from feasibility $f[d(H_j^{min}-H_j)]$. The form of the evaluation function used is

$$f(D_1,\ldots,D_n)=\sum_{i=1}^{N}c(D_i)\times L_i+p\times\left\{\max_j\left[\max\left(H_j^{\min}-H_j,0\right)\right]\right\} \quad \textbf{(6)}$$

where p is the penalty multiplier and the term in brackets {} is the maximum violation of the pressure constraint. The penalty multiplier is chosen to normalise nominal values of the penalties to the same scale as the basic cost of the network. The multiplier is a function of the generation number which allows a gradual increase in the penalty term.

$$p = \varphi \times \left(\frac{n_{gen}}{n_{gen}^{\max}} \right)^k \quad \textbf{(7)}$$

where φ is the constant penalty multiplier, n_{gen} is the generation number, $n_{gen}^{\max}$ is the maximum number of generations and k is a parameter (experimentally chosen to be 0.8). At the end of a GA run, the multiplier p should take a value which will not allow the best infeasible solution to be better than any feasible solution in the population.

Reproduction

To avoid problems with scaling, premature convergence, and selective pressure of traditional proportional fitness schemes (for example, roulette wheel selection), rank selection is used to choose parents for recombination (Grefenstette and Baker, 1989). The population is ordered according to the computed fitness values and parents are selected with a probability based on their rank in the population. A *linear ranking* selection is used in GANET (Whitley, 1989).

After experimenting with different crossover operators, the authors have adopted the *uniform crossover* (Syswerda, 1989) operator for the problem of optimal pipe sizing although one-point and two-point crossover operators are also available in GANET. The uniform crossover creates a single offspring from two parents, by choosing each gene in the offspring from one of the parents, selected randomly. The recommended range for probability of crossover is $p_c \in (0.6,1)$ [Goldberg, 1989].

Mutation

Random mutation is an operation by which the mutation operator simply replaces each gene by a random value with probability equal to the mutation rate p_m. The mutation rate is usually set very low, e.g., $p_m \in (0.01,0.10)$. For the two-loop example the mutation rate is set to be p_m= 0.03 (≈1/32), or on average, only one gene is changed in each offspring's 32-bit chromosome. In addition, instead of randomly determining the value of the gene selected for mutation, the gene value is inverted to its binary complement.

AN ILLUSTRATIVE EXAMPLE

The simple network of Figure 1 used by Alperovits and Shamir (1977) serves as an illustrative example for the least-cost network design analysis performed by GANET. In addition to the preceding problem description, the minimum diameter (1 inch = 2.54 cm) and the maximum allowable hydraulic gradient (0.05) constraints were imposed as in the original problem formulation. If only discrete pipe sizes are considered the total solution space is comprised of $14^8 = 1.48\times10^9$ different network designs, thus making, even this illustrative example, difficult to solve. Not surprisingly then, all studies considering this example dealt with split-pipe solutions. Table 4 lists least-cost solutions as reported by different researchers over the period since 1977. Optimal pipe diameters D (in inches) and pipe-segment lengths L (in metres) are given as found in the literature. It should be noted, however, that different authors did not use the same head-loss coefficients of the Hazen-Williams formula in the energy conservation equation Eq.(3). Alongside these split-pipe solutions, two discrete-diameter solutions obtained by using a GA are also presented. These two solutions are the optimal designs found for $\omega = 10.9031$ and $\omega = 10.5088$ respectively, hence covering the range of published values.

Figure 2 shows a typical plot of the cost of the best GANET solution in each generation of a single run. The GA employed an elitist strategy, the mutation rate was set to $p_m = 0.03$ and the crossover rate was set to $p_c = 1.0$. Since the run was performed with a population size of 50 individuals, the total number of objective function evaluations can be calculated as a product of the population size and the generation number. In the first 50 generations (2500 evaluations), selection and crossover dominate with the best cost driven down from close to one million units to approximately 430×10^3 units. For further generations, both crossover and mutation are driving the process until the best solution of $f = 419\times10^3$ units is found. For this small network GANET was allowed up to 500 generations (250,000 evaluations) per run, which represents only 0.0169 percent of the 1.48×10^9 possible pipe combinations. Ten runs were performed using different seed numbers for the pseudo-random number generator and one of the two solutions from Table 4 was always identified. Each run took up to 10 minutes CPU time on a PC 486/DX2 50 computer.

To assess the quality of the solutions obtained and compare them with the solutions from the literature, hydraulic simulations were performed for varying numerical values of the conversion constant ω in the Hazen-Williams formula. The pressures associated with the lower limit on ω and the designs from Table 4 are presented in Table 5. The pressures associated with the upper limit on ω and the designs from Table 4 are presented in Table 6. It can be observed that the GA solutions using ω = 10.9031 and ω = 10.5088 are similar in costs (419×10^3 and 420×10^3) to the solution by Kessler and Shamir (1989), $f_{opt}=417.5\times10^3$, and slightly more expensive than the solution by Eiger et al. (1994), $f_{opt}=402.352\times10^3$. However, if implemented these designs would perform differently in terms of delivered pressures (see Tables 5 and 6). Kessler and Shamir's (1989) and Eiger et al.'s (1994) solutions become infeasible at the upper bound of 10.9031.

The following two examples illustrate the potential of GANET as a tool for water distribution network planning and management. The first of the problems deals with the design of a completely new network for the city of Hanoi, Vietnam. The second example considers the problem of optimally expanding an existing network, namely, the New York city water supply tunnels (USA). The availability of GANET with its global sampling capability enables the engineer to concentrate on the quality of the solutions rather than on the mathematical characteristics of the model used. The quality of the network designs is assessed on the basis of: (a) the investment costs, and (b) how well the minimum head constraints are satisfied. Uncertainty associated with the interpretation of the widely accepted Hazen-Williams equation is used to illustrate the ease of use and efficiency of GANET in performing a meaningful network optimization exercise.

NEW PIPE NETWORK DESIGN

The configuration of the water distribution trunk network in Hanoi, Vietnam (Fujiwara and Khang, 1990) is shown in Figure 3. This is a network which consists of 32 nodes and 34 pipes organized in 3 loops. No pumping facilities are considered since only a single fixed-head source at elevation of 100 m is available. The minimum head requirement at all nodes is fixed at 30 m. The set of commercially available diameters (in inches) is S = [12, 16, 20, 24, 30, 40] with the price of each pipe i calculated as $C_i = 1.1\times L_i\times D_i^{1.5}$. The assumption of a continuous cost function is based on the paper by Fujiwara and Khang (1990), since their method could not handle discontinuous objective functions directly. If only discrete pipe diameters are considered, the total search space becomes 6^{34}=2.87×10^{26} possible network designs.

Table 7 lists solutions for the Hanoi network found in the literature. The best network designs obtained by different authors are given in terms of cost (millions of dollars), selected diameters D (inches), and pipe lengths L (meters). Alongside these continuous-diameter and split-pipe solutions, two discrete-diameter solutions obtained by GANET are presented for comparison. The first GA solution is found by using the lower value of the numerical conversion constant ω in the Hazen-Williams equation (Table 1, ω = 10.5088, for D in meters and Q in m^3/s). The H-W formula with the lower value of ω will be referred to as the 'relaxed' head-loss equation. Similarly, the second GA solution is found by using the upper value of ω (Table 1, ω = 10.9031). The H-W formula with this value of ω will be referred to as the restrictive head-loss equation.

Decision variables for this problem are 34 pipe diameters, where each pipe can take one of the six sizes. A Gray-coded string comprising 34 substrings is used for coding the problem into a suitable form for use within GANET. Since a three-bit substring can handle more than six discrete pipe sizes (i.e., it can represent 2^3 = 8 discrete values) some redundancy occurs within the chromosome. The 34 substrings are joined together to form a 102-bit string which can represent each possible solution to the problem. The following GA parameters were used: the solution-pool size (population) = 100, the probability of crossover = 1.0, the probability of mutation = 1/102, and the number of generations allowed = 10,000. Twenty runs were necessary using different seed number for the pseudorandom number generator to reach the solutions in Table 7. Each run took up to 3 hours of CPU time on a PC 486/DX2 50 computer.

The solution by Fujiwara and Khang (1990) is a continuous-diameter solution which is converted to a split-pipe network design using commercially available diameters. The latter solution identifies 27 pipes having two segments of different size. It can also be observed that pipes 15, 26, and 32 (or 33), each of which belongs to a different loop of the network, are of minimum possible diameter, D = 12 in. As previously mentioned this is a common feature of optimal least-cost solutions obtained for a single loading condition, namely, that loops will, in normal circumstances, disappear, leaving an underlying tree-like structure (Quindry et al., 1981). Only one of the aforementioned pipes of minimum diameter in the solution by Fujiwara and Khang (1990), i.e., pipe 15, coincides with the minimum-size pipe in the solution by Eiger et al. (1994). If only the cost is used to assess the quality of the designs (Table 7), the one by Fujiwara and Khang (1990) is superior to other solutions. However, Table 8 shows that this solution yields pressure heads far from feasibility even for the relaxed head-loss equation. This may be attributed to inaccuracy in the solution routine employed to analyze the hydraulic behaviour of the network.

There is an obvious similarity between the solution identified by the method of Eiger et al. (1994) and the GA solution obtained using the relaxed head-loss equation. Both are of similar cost (0.76 percent difference) and identify the same pipes of minimum diameter in the three loops (i.e., pipes 15, 28, and 31). However, because of its split-pipe nature the former is less realistic, e.g., pipe 11 with a 24-inch segment being 1198.96 m long and a 30-inch pipe segment being only 1.04 m long. The infeasibility of the solution by Eiger et al. (1994) is only marginal for the relaxed set of parameters assumed in the Hazen-Williams equation in Table 8.

The GA solution No. 2 is the most expensive (2.79 percent more expensive than the solution by Eiger et al., 1994), but it identifies the same pipes to be of minimum possible size. As expected, all solutions, except the GA No. 2 solution, were found infeasible for ω = 10.9031, as shown in Table 9. It may also be observed that the violations of the pressure constraint of the GA solution No. 1 are small (nodes 27, 30, and 31).

PARALLEL NETWORK EXPANSION

A number of studies in pipe network optimization have examined the expansion of the New York water supply system (Schaake and Lai, 1969; Quindry et al., 1981; Morgan and Goulter, 1985; Bhave, 1985; and Murphy et al., 1993). The common objective of the studies was to determine the most economically effective design for additions to the then existing system of tunnels that constituted the primary water distribution system of the City of New York (Figure 4). The same input data, e.g., existing pipe data, discrete set of available diameters, and as-sociated unit pipe costs, were used in this study. The Imperial system of units was used to enable easy comparison with previous studies. Because of age and increased demands the existing gravity flow tunnels have been found to be inadequate to meet the pressure requirements (nodes 16, 17, 18, 19 and 20) for the projected consumption level. The proposed method of expansion was the same as in previous studies, i.e., to reinforce the system by constructing tunnels parallel to the existing tunnels. For 15 available diameters, i.e., 16 possible decisions including the 'do nothing' option, and 21 pipes to be considered for duplication, the total solution space is $16^{21} = 1.93\times10^{25}$ possible network designs.

The best results found in previous optimization studies are summarised in Table 10. First of all, it is apparent from this table that solutions obtained from the literature differ in the number of pipes to be duplicated. The work by Schaake and Lai (1969) identifies 18 pipes to be duplicated, while 4 other studies recommend only 6 such pipes. In addition to that, even the works that found the same number of pipes to be laid in parallel, differ as to which pipes are identified, e.g., Morgan and Goulter (1985) identify pipes 7, 16, 17, 18, 19, and 21, while Murphy et al. (1993) identify pipes 15, 16, 17, 18, 19, and 21.

Since each of the 21 decision variables can take one of the sixteen discrete diameters, a Gray-coded string made of 21 substrings is used for coding the problem into a suitable form for use within GANET. Because a four-bit substring can handle exactly sixteen discrete pipe sizes there is no redundancy in the coded chromosome. The 21 substrings are joined together to form an 84-bit string, which can represent each possible solution to the problem. The following GA parameters were used: the solution-pool size (population) = 100, the probability of crossover = 1.0, the probability of mutation = 1/84, and the number of generations allowed = 10,000. As for the previous example, GA results were obtained for $\omega = 10.9031$ and $\omega = 10.5088$, the best solutions being given in Table 10. Since GAs are stochastic-search techniques the solution found was not always the same and therefore, several runs were necessary to ensure that the solutions identified were of good quality. When compared to the other techniques found in the literature, the GA performed at least as well as other techniques.

Similarities to solutions obtained using different optimization methods with different interpretations of the H-W equation, can be observed in the two GA solutions (the last two columns in Table 10). These two solutions identify different diameters for the same pipes that are to be duplicated and also the solution for the lower value of ω identifies pipe 7 while the solution for the upper value of ω identifies pipe 15 to be duplicated.

As with the Hanoi network problem, the sensitivity of the designs obtained for the New York City tunnels was examined for the upper and lower values of the coefficient ω of the Hazen-Williams equation. The pressures associated with the lower value of $\omega = 10.5088$ (Q in m^3/s and D in metres), the exponents $a = 1.85$ and $b = 4.87$ and the designs from Table 10 are presented in Table 11. The pressures associated with the upper value of $\omega = 10.9031$, the exponents $a = 1/0.54$, and $b = 2.63/0.54$ and the designs from Table 10 are presented in Table 12.

By analysing the last two tables it can be observed that the solution by Fujiwara and Khang (1990) is infeasible even for the relaxed head-loss equation. It is also interesting to note that the solution by Bhave (1985) satisfies the minimum head constraint almost exactly (i.e., as an equality) for the case of the restrictive head-loss equation.

SUMMARY AND CONCLUSIONS

The computational complexity of the problem of determining optimal designs for water distribution networks is extremely high. This is true even without taking into account annual costs (with consideration to the cost of money, depreciation, inflation, energy, manpower, and maintenance costs) or without considering other legitimate objectives such as reliability, water quality, etc. This paper describes the development of a Genetic Algorithm program, called GANET, for least-cost design of water distribution networks. This program is used to make a comparative study and provide an insight in the expected performance of solutions identified in the past studies.

The combinatorial optimization problem of least-cost design of water distribution networks is formulated and it is shown that Genetic Algorithms are particularly suited to this type of problem. The comparison of solutions obtained by GANET and other optimization techniques shows that the former produced good designs without unnecessary restrictions imposed by split-pipe or linearizing assumptions.

The research community involved in optimization of water distribution networks has started to become aware of the shortcomings of the methods which are able to find only local minima (Hansen et al., 1991; Eiger et al., 1994). Although Evolutionary Computing methods including Genetic Algorithms and Evolution Strategies, cannot guarantee that the global optimum is found, there have been successful applications of these techniques to the design of water distribution networks. In addition to the advantage of working directly with the actual discrete diameter sizes these artificial-evolution search methods are conceptually simple, and have a global sampling capability. Using the very efficient Evolutionary Computing techniques employed by GANET several problem from the literature were solved. On comparing the solutions to those previously published it was discovered that for a realistic range of parameter values some of the latter do not satisfy the minimum pressure constraints and thus may not be appropriate as recommended designs.

The extension of GANET to consider multiple loading conditions was not considered in this study, but it may be achieved without major changes in the program. Because the same decision variables are considered for single- and multiple-loading cases, the extension would not require an increase in the chromosome-string length, and consequently, the search convergence speed should not deteriorate. The change would only cause an increase in the run time since additional evaluations of the network hydraulic behaviour under different loading conditions are needed. Additions of pumps, reservoirs, etc., as decision variables can also be incorporated into GANET but they are beyond the scope of this paper.

Although GANET is only a research tool at present, it is not complicated to use and it does not require a large amount of mathematical sophistication to understand its mechanisms. In addition, it can be easily adapted to aid the design process of not only completely new water networks but also of parallel expansion network problems. It is hoped that the inherent simplicity will help GANET gain acceptance by practitioners familiar with basic network simulation skills. The authors feel strongly that a tool like GANET should not be considered as a decision-making tool, but as a tool able to provide alternative solutions from which designers/decision makers may choose.

Finally, the use of the Hazen-Williams equation and modifications necessary to adapt this head-loss formula to different units is a source of additional uncertainty associated with the results obtained. Also, the fact that the Hazen-Williams coefficient is assumed independent of pipe diameter, velocity of flow and viscosity, requires extreme caution to be exercised when applying this formula to optimization of water distribution systems. Special attention should be paid to optimizing a system of different loading conditions since the Hazen-Williams formula should not be used outside its region of applicability.

ACKNOWLEDGEMENT

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FIGURE CAPTIONS

Figure 1. A two-loop network

Figure 2. Course of the evolution for the best two-loop network

Figure 3. The Hanoi network

Figure 4. Layout for New York City water supply system

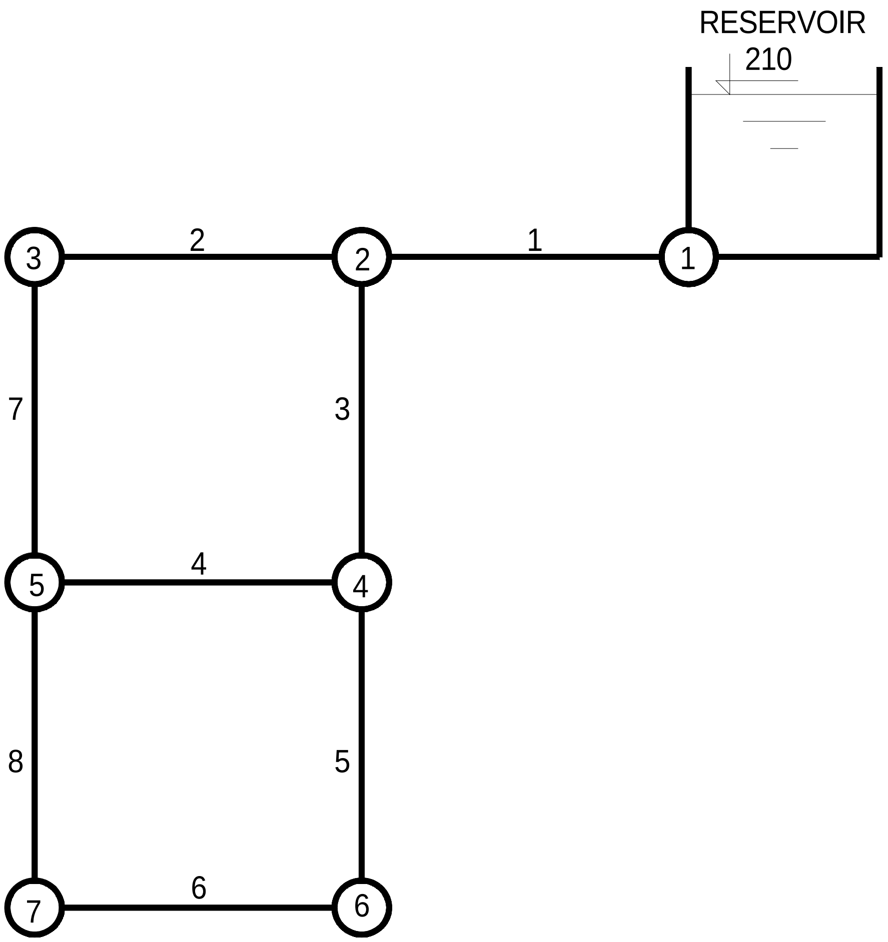

Figure 1. A two-loop network

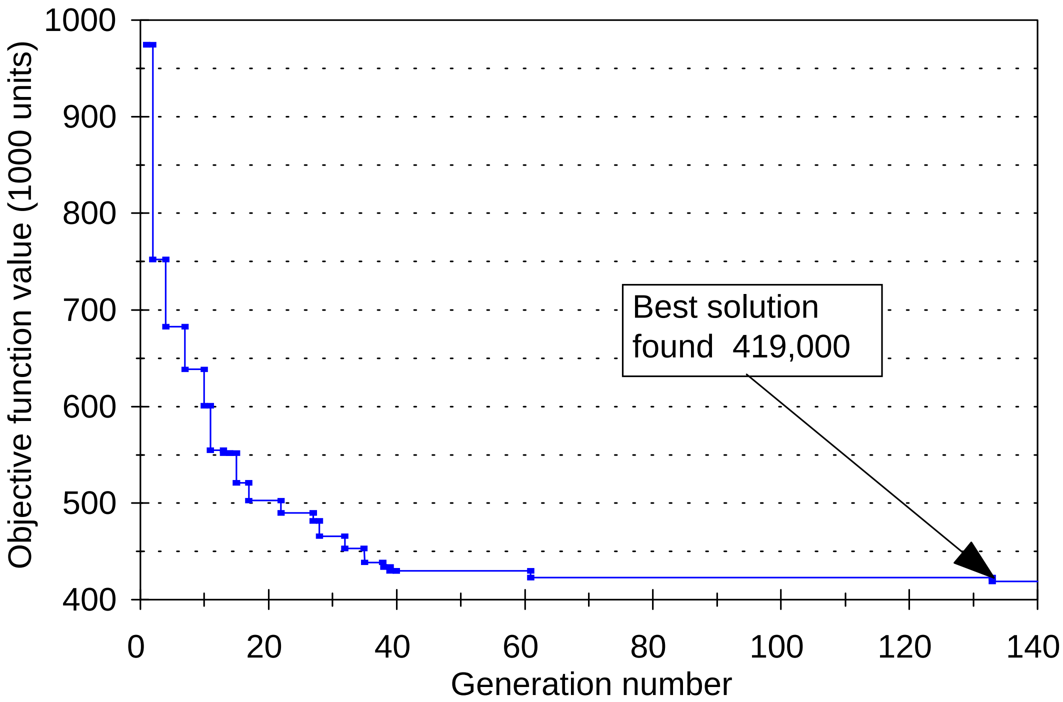

Figure 2. Course of the evolution for the best two-loop network

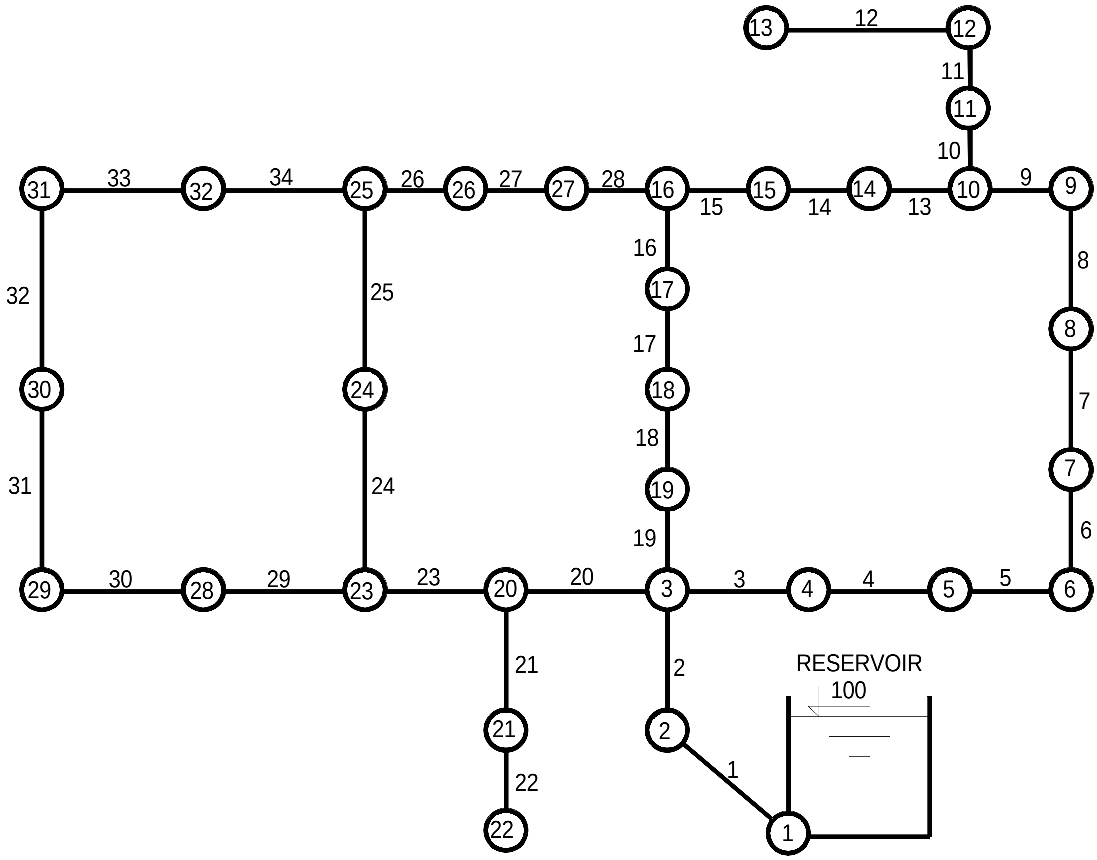

Figure 3. The Hanoi network

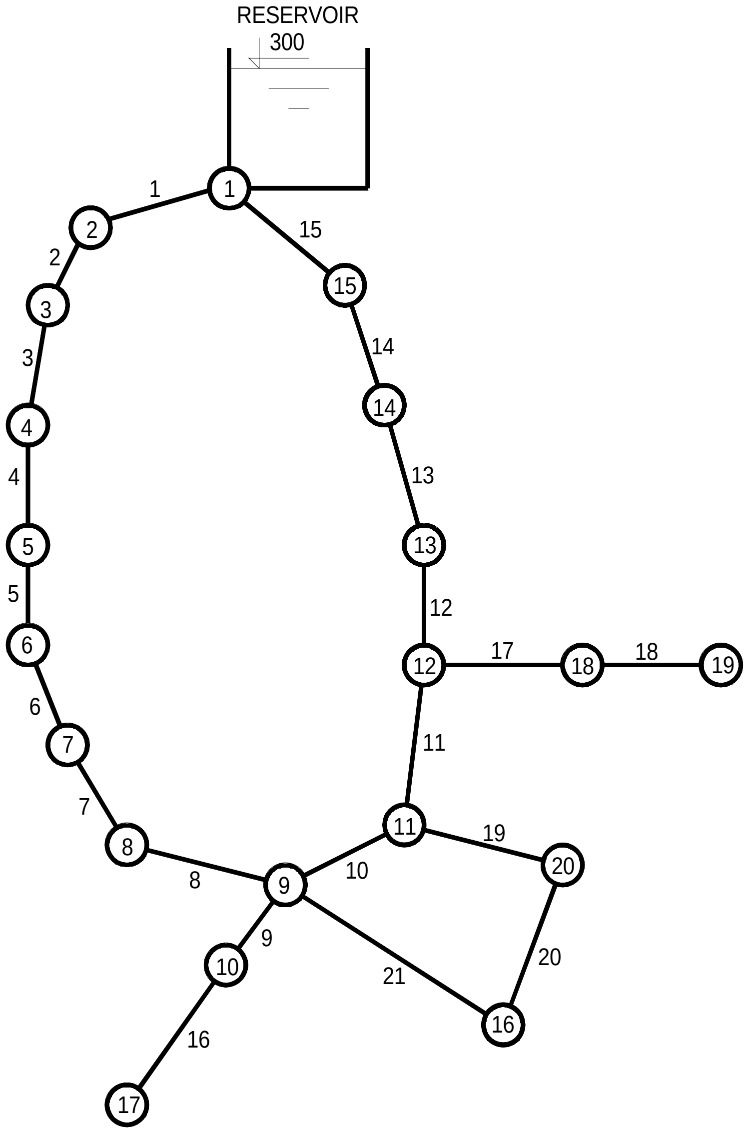

Figure 4. Layout for New York City water supply system

Table 1. Hazen-Williams equation used by different authors

No	Authors	Original equation used	Coefficient ω in equation $h_f = \omega \frac{L}{C^a D^b} Q^a$		
			D[m] Q[m^3/s]	D[ft] Q[cfs]	D[in] Q[cfs]
1	Alperovits and Shamir (1977)	$h_f = 8.515 \times 10^5 L \left(\frac{Q}{C}\right)^{1.852} D^{-4.87}$ for D[in] and Q[cfs]	10.6792	4.7269	8.515×10^5
2	Alperovits and Shamir (1977)	$h_f = 1.526 \times 10^4 L \left(\frac{Q}{C}\right)^{1.852} D^{-4.87}$ for D[cm] and Q[m^3/h]	10.7109	4.7409	8.540×10^5
3*	Quindry et al. (1981)	$h_f = \left(6.2 \times 10^{-4}\right)^{-\frac{1}{0.54}} L \left(\frac{Q}{C}\right)^{\frac{1}{0.54}} D^{-\frac{2.63}{0.54}}$ for D[in] and Q[cfs]	10.9031	4.8306	8.710×10^5
4	Ormsbee and Wood (1986)	$h_f = 3.204 L C^{-1.852} v^{1.852} D^{-1.166}$ for D[ft] and v[ft/sec]	10.6866	4.7302	8.521×10^5
5*	Fujiwara and Khang (1990)	$h_f = 162.5 L \left(\frac{Q}{C}\right)^{1.85} D^{-4.87}$ for D[in] and Q[m^3/h]	10.5088	4.6847	8.439×10^5
6	Murphy, and Simpson (1992)	$h_f = 4.727 L \left(\frac{Q}{C}\right)^{1.852} D^{-4.8704}$ for D[ft] and Q[cfs]	10.6744	4.7270	8.524×10^5
7	Murphy et al. (1993)	$h_f = 4.7291 L \left(\frac{Q}{C}\right)^{1.852} D^{-4.8704}$ for D[ft] and Q[cfs]	10.6792	4.7291	8.528×10^5
8	Simpson, et al. (1994)	$h_f = 10.675 L \left(\frac{Q}{C}\right)^{1.852} D^{-4.8704}$ for D[m] and Q[m^3/s]	10.6750	4.7272	8.524×10^5

*Chosen as an upper and lower bound on the coefficient ω

Table 2. Node data for the first network example

Node	Demand m^3/h	Ground level m
1 (source)	-1120.0	210.00
2	100.0	150.00
3	100.0	160.00
4	120.0	155.00
5	270.0	150.00
6	330.0	165.00
7	200.0	160.00

Table 3. Cost data for pipes

Diameter (inches)*	1	2	3	4	6	8	10	12	14	16	18	20	22	24
Cost (units)	2	5	8	11	16	23	32	50	60	90	130	170	300	550

*1 inch = 2.54 cm

Table 4. Solutions of the two-loop network

Pipe	Alperovits and Shamir (1977)		Goulter et al. (1986)		Kessler and Shamir (1989)		Eiger et al. (1994)		GA No. 1	GA No. 2
	L	*D*	*L*	*D*	*L*	*D*	*L*	*D*	*D*	*D*
[-]	[m]	[in]	[m]	[in]	[m]	[in]	[m]	[in]	[in]	[in]
1	256.00	20	383.00	20	1000.00	18	1000.00	18	18	20
	744.00	18	617.00	18						
2	996.38	8	1000.00	10	66.00	12	238.02	12	10	10
	3.62	6			934.00	10	761.98	10		
3	1000.00	18	1000.00	16	1000.00	16	1000.00	16	16	16
4	319.38	8	687.00	6	713.00	3	1000.00	1	4	1
	680.62	6	313.00	4	287.00	2				
5	1000.00	16	1000.00	16	836.00	16	628.86	16	16	14
					164.00	14	371.14	14		
6	784.94	12	98.00	12	109.00	12	989.05	10	10	10
	215.06	10	902.00	10	891.00	10	10.95	8		
7	1000.00	6	492.00	10	819.00	10	921.86	10	10	10
			508.00	8	181.00	8	78.14	8		
8	990.93	6	20.00	2	920.00	3	1000.00	1	1	1
	9.07	4	980.00	1	80.00	2				
Cost (units)	497,525		435,015		417,500		402,352		419,000	420,000

Table 5. The pressure heads for the two-loop network, H_i obtained for $\omega = 10.5088$, $a = 1.85$ and $b = 4.87$

Node i	Alperovits and Shamir (1977)	Goulter et al. (1986)	Kessler and Shamir (1989)	Eiger et al. (1994)	GA No. 1 $\omega = 10.5088$	GA No. 2 $\omega = 10.9031$
2	53.96	54.30	53.26	53.26	53.26	55.97
3	32.32	33.19	30.08	30.30	30.45	30.77
4	44.97	44.19	43.64	43.87	43.48	46.60
5	32.31	32.32	30.10	30.62	33.77	32.29
6	31.19	31.19	30.08	29.85*	30.49	30.86
7	31.57	31.57	30.09	29.85*	30.62	30.99

*Infeasible solution (H_i < 30 m)

Table 6. The pressure heads for the two-loop network, H_i obtained for $\omega = 10.9031$, $a = 1/0.54$ and $b = 2.63/0.54$

Node i	Alperovits and Shamir (1977)	Goulter et al. (1986)	Kessler and Shamir (1989)	Eiger et al. (1994)	GA No. 1 $\omega = 10.5088$	GA No. 2 $\omega = 10.9031$
2	53.80	54.15	53.09	53.09	53.09	55.86
3	31.89	32.78	29.59*	29.81*	29.97*	30.30
4	44.71	43.91	43.35	43.59	43.18	46.38
5	31.65	31.65	29.38*	29.90*	33.13	31.61
6	30.83	30.83	29.69*	29.46*	30.11	30.50
7	31.11	31.11	29.59*	29.34*	30.13	30.52

*Infeasible solution ($H_i < 30$ m)

Table 7. Solutions of the Hanoi network problem

Link l	Fujiwara and Khang (1990) continuous solution		Fujiwara and Khang (1990) split-pipe solution		Eiger et al. (1994) split-pipe solution		GA No. 1 discrete-diameter solution (ω = 10.5088)	GA No. 2 discrete-diameter solution (ω = 10.9031)
	D[in]	L[m]	D[in]	L[m]	D[in]	L[m]	D[in]	D[in]
1	40.0	100	40	100	40	100.00	40	40
2	40.0	1350	40	1350	40	1350.00	40	40
3	38.8	900	40	850	40	900.00	40	40
			30	50				
4	38.7	1150	40	1090	40	1150.00	40	40
			30	60				
5	37.8	1450	40	1300	40	1450.00	40	40
			30	150				
6	36.3	450	40	360	40	450.00	40	40
			30	90				
7	33.8	850	40	500	40	850.00	40	40
			30	350				
8	32.8	850	40	390	40	850.00	40	40
			30	460				
9	31.5	800	40	230	40	158.83	40	30
			30	570	30	641.17		
10	25.0	950	30	260	30	950.00	30	30
			24	690				
11	23.0	1200	24	1010	30	1.04	24	30
			20	190	24	1198.96		
12	20.2	3500	24	210	24	3500.00	24	24
			20	3290				
13	19.0	800	20	690	16	800.00	20	16
			16	110				
14	14.5	500	16	400	12	500.00	16	16
			12	100				
15	12.0	550	12	550	12	550.00	12	12
16	19.9	2730	20	2700	12	2730.00	12	16
			16	30				
17	23.1	1750	24	1490	20	1115.87	16	20
			20	260	16	634.13		
18	26.6	800	30	470	24	800.00	20	24
			24	330				
19	26.8	400	30	250	24	400.00	20	24
			24	150				
20	35.2	2200	40	1580	40	2200.00	40	40
			30	620				
21	16.4	1500	20	240	20	985.88	20	20
			16	1260	16	514.12		

Link *l*	Fujiwara and Khang (1990) continuous solution		Fujiwara and Khang (1990) split-pipe solution		Eiger et al. (1994) split-pipe solution		GA No. 1 discrete-diameter solution (ω = 10.5088)	GA No. 2 discrete-diameter solution (ω = 10.9031)
	D[in]	L[m]	D[in]	L[m]	D[in]	L[m]	D[in]	D[in]
22	12.0	500	12	500	12	500.00	12	12
23	29.5	2650	30	2540	40	2650.00	40	40
			24	110				
24	19.3	1230	20	1120	30	1230.00	30	30
			16	110				
25	16.4	1300	20	230	30	1300.00	30	30
			16	1070				
26	12.0	850	12	850	20	850.00	20	20
27	20.0	300	24	60	16	292.72	12	12
			20	240	12	7.28		
28	22.0	750	24	540	12	750.00	12	12
			20	210				
29	18.9	1500	20	1250	16	1500.00	16	16
			16	250				
30	17.1	2000	20	840	12	2000.00	16	16
			16	1160				
31	14.6	1600	16	1300	12	1600.00	12	12
			12	300				
32	12.0	150	12	150	16	150.00	12	12
33	12.0	860	12	860	20	227.08	16	16
					16	632.92		
34	19.5	950	20	890	24	950.00	20	20
			16	60				
Cost [$mill]	5.354		5.562		6.027		6.073	6.195

Table 7 (cont'd). Solutions of the Hanoi network problem

Table 8. The pressure heads for the Hanoi network problem, H_i obtained for $\omega = 10.5088$, $a = 1.85$ and $b = 4.87$

Node *i*	Fujiwara and Khang (1990) continuous solution	Fujiwara and Khang (1990) split-pipe solution	Eiger et al. (1994) split-pipe solution	GA No. 1 discrete-diameter solution ($\omega = 10.5088$)	GA No. 2 discrete-diameter solution ($\omega = 10.9031$)
1	100.00	100.00	100.00	100.00	100.00
2	97.16	97.16	97.16	97.16	97.16
3	61.95	61.95	61.95	61.95	61.95
4	56.48	56.43	57.52	57.21	57.20
5	49.62	49.65	52.02	51.33	51.31
6	41.50	41.52	46.27	45.13	45.09
7	39.18	39.19	44.94	43.68	43.63
8	35.26	35.29	43.38	41.93	41.88
9	31.65	31.63	42.16	40.54	40.49
10	28.40*	28.39*	39.09	40.34	40.28
11	24.62*	24.62*	37.54	38.79	38.73
12	20.42*	20.43*	34.12	38.78	38.73
13	10.69*	10.58*	29.92*	34.58	34.53
14	23.76*	23.77*	32.30	36.59	36.48
15	20.94*	20.92*	29.97*	34.71	34.53
16	18.67*	18.59*	29.93*	32.08	33.85
17	37.39	37.26	40.40	33.36	36.11
18	51.06	51.00	53.07	43.32	46.70
19	58.34	58.33	58.90	55.54	59.25
20	47.51	47.54	50.68	50.92	51.03
21	23.01*	22.83*	35.08	44.79	44.90
22	17.85*	17.67*	29.92*	39.63	39.75
23	32.34	32.44	44.39	44.83	45.04
24	20.49*	20.72*	38.50	39.64	40.20
25	12.87*	13.12*	34.66	36.38	37.24
26	14.41*	14.71*	30.40	32.67	33.77
27	15.56*	15.73*	29.92*	31.66	33.04
28	24.81*	24.90*	38.79	36.48	41.61
29	15.60*	15.79*	29.92*	32.04	35.54
30	10.36*	10.59*	30.20	31.29	33.07
31	10.31*	10.55*	30.48	31.81	33.33
32	10.36*	10.61*	32.51	32.17	33.55

*Infeasible solution ($H_i < 30$ m)

Table 9. The pressure heads for the Hanoi network problem, H_i obtained for $\omega = 10.9031$, $a = 1/0.54$ and $b = 2.63/0.54$

Node i	Fujiwara and Khang (1990) continuous solution	Fujiwara and Khang (1990) split-pipe solution	Eiger et al. (1994) split-pipe solution	GA No. 1 discrete-diameter solution ($\omega = 10.5088$)	GA No. 2 discrete-diameter solution ($\omega = 10.9031$)
1	100.00	100.00	100.00	100.00	100.00
2	97.07	97.07	97.07	97.07	97.07
3	60.76	60.76	60.76	60.76	60.76
4	55.13	55.07	56.19	55.88	55.87
5	48.06	48.09	50.54	49.82	49.80
6	39.70	39.72	44.61	43.44	43.40
7	37.31	37.32	43.24	41.94	41.90
8	33.29	33.31	41.64	40.15	40.10
9	29.57*	29.54*	40.38	38.72	38.66
10	26.23*	26.22*	37.23	38.51	38.45
11	22.35*	22.35*	35.64	36.91	36.86
12	18.03*	18.04*	32.13	36.91	36.85
13	8.05*	7.94*	27.82*	32.60	32.55
14	21.47*	21.48*	30.27	34.66	34.55
15	18.58*	18.56*	27.88*	32.74	32.56
16	16.26*	16.18*	27.84*	30.05	31.86
17	35.50	35.36	38.60	31.37	34.19
18	49.55	49.49	51.62	41.60	45.08
19	57.04	57.03	57.62	54.17	57.98
20	45.89	45.92	49.15	49.40	49.51
21	20.74*	20.55*	33.14	43.11	43.22
22	15.45*	15.27*	27.85*	37.82	37.94
23	30.30	30.40	42.68	43.14	43.35
24	18.12*	18.36*	36.63	37.80	38.38
25	10.31*	10.57*	32.68	34.45	35.33
26	11.89*	12.20*	28.31*	30.65	31.78
27	13.08*	13.25*	27.83*	29.61*	31.03
28	22.56*	22.65*	36.93	34.56	39.83
29	13.11*	13.30*	27.84*	30.01	33.60
30	7.74*	7.98*	28.11*	29.24*	31.06
31	7.69*	7.93*	28.40*	29.77*	31.33
32	7.74*	7.99*	30.48	30.13	31.56

*Infeasible solution ($H_i < 30$ m)

Table 10. Solutions of the New York City tunnels problem

Pipe	Schaake and Lai (1969)	Quindry et al. (1981)	Gessler (1982)	Bhave (1985)	Morgan and Go-ulter (1985)	Fujiwa-ra and Khang (1990)	Murphy et al. (1993)	GA No. 1 ω = 10.5088	GA No. 2 ω = 10.9031
[-]	*D* [in]	*D* [in]	*D* [in]	*D* [in]	*D* [in]	*D* [in]	*D* [in]	*D* [in]	*D* [in]
1	52.02	0.00	0	0.00	0	0.00	0	0	0
2	49.90	0.00	0	0.00	0	0.00	0	0	0
3	63.41	0.00	0	0.00	0	0.00	0	0	0
4	55.59	0.00	0	0.00	0	0.00	0	0	0
5	57.25	0.00	0	0.00	0	0.00	0	0	0
6	59.19	0.00	0	0.00	0	0.00	0	0	0
7	59.06	0.00	100	0.00	144	73.62	0	108	0
8	54.95	0.00	100	0.00	0	0.00	0	0	0
9	0.00	0.00	0	0.00	0	0.00	0	0	0
10	0.00	0.00	0	0.00	0	0.00	0	0	0
11	116.21	119.02	0	0.00	0	0.00	0	0	0
12	125.25	134.39	0	0.00	0	0.00	0	0	0
13	126.87	132.49	0	0.00	0	0.00	0	0	0
14	133.07	132.87	0	0.00	0	0.00	0	0	0
15	126.52	131.37	0	136.43	0	0.00	120	0	144
16	19.52	19.26	100	87.37	96	99.01	84	96	84
17	91.83	91.71	100	99.23	96	98.75	96	96	96
18	72.76	72.76	80	78.17	84	78.97	84	84	84
19	72.61	72.64	60	54.40	60	83.82	72	72	72
20	0.00	0.00	0	0.00	0	0.00	0	0	0
21	54.82	54.97	80	81.50	84	66.59	72	72	72
Cost $mil	78.09	63.58	41.80	40.18	39.20	36.10	38.80	37.13	40.42

Table 11. The pressure heads H_i for critical nodes (obtained for ω = 10.5088, a = 1.85 and b = 4.87)

Node i	Min. head [ft]	Schaake and Lai (1969)	Quindry et al. (1981)	Gessler (1982)	Bhave (1985)	Morgan and Goulter (1985)	Fujiwara and Khang (1990)	Murphy et al. (1993)	GA No. 1 ω = 10.5088	GA No. 2 ω = 10.9031
16	260.0	261.40	261.35	260.76	261.27	261.98	259.74*	260.95	260.16	261.54
17	272.8	274.08	273.94	273.43	273.69	273.12	272.60*	273.18	272.86	273.77
19	255.0	256.56	256.47	256.32	256.42	255.47	254.73*	256.17	255.21	256.80

*Infeasible solution

Table 12. The pressure heads H_i for critical nodes (obtained for ω = 10.9031, a = 1/0.54 and b = 2.63/0.54)

Node i	Min. head [ft]	Schaake and Lai (1969)	Quindry et al. (1981)	Gessler (1982)	Bhave (1985)	Morgan and Goulter (1985)	Fujiwa-ra and Khang (1990)	Murphy et al. (1993)	GA No. 1 ω = 10.5088	GA No. 2 ω = 10.9031
16	260.0	260.18	260.12	259.47*	260.01	260.73	258.43*	259.67*	258.86*	260.28
17	272.8	273.24	273.09	272.53*	272.81	272.21*	271.67*	272.28*	271.94*	272.88
19	255.0	255.19	255.09	254.91*	255.01	254.02*	253.26*	254.75*	253.75*	255.40

*Infeasible solution