Limitations and Proper Use of the Hazen-Williams Equation

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Abstract: The Hazen-Williams equation is used widely in water supply and sanitary engineering. This equation uses a constant, the Hazen-Williams C, to indicate the roughness of a pipe interior. Because of the empirical nature of the equation, its range of applicability is limited. Many textbooks and software manuals give C values based on pipe type, condition, and age but do not give the range of applicability. Historic experimental data is used to demonstrate that C is a strong function of Reynolds number and pipe size and that the Hazen-Williams equation has narrow applicable ranges for Reynolds numbers and pipe sizes. The level of error when the Hazen-Williams equation is used outside its data ranges is significant. However, a valid C for a given pipe at a specific Reynolds number can be used to estimate a pipe's relative roughness, which then can be used by the rational Darcy-Weisbach equation without the range limitations. A method for doing so is given.

INTRODUCTION

The Hazen-Williams equation empirically relates the slope of an energy grade line to the hydraulic radius and the discharge velocity of water flowing full in a pipe. This equation uses a constant to characterize the roughness of the pipe's inner surface. Originally introduced in 1902, it still is used widely in water supply and sanitary engineering. However, being empirical, the Hazen-Williams equation is not dimensionally homogeneous and its range of applicability is limited.

The well-known Colebrook-White transition formula relates the friction factor in the Darcy-Weisbach equation to the Reynolds number and the relative roughness of the pipe inner wall. The Moody diagram (Moody 1944) facilitates the determination of this friction factor, which is implicit in the Colebrook-White formula. The Darcy-Weisbach equation is rational, dimensionally homogeneous, and applicable to water as well as to other fluids.

Frequently, the Hazen-Williams equation is presented in hydraulics, water supply, and sanitary engineering texts together with the Darcy-Weisbach equation. Vennard (1961), Streeter and Wylie (1985), Street et al. (1996), and Potter and Wiggert (1997) discuss the limitations of the former but most of the texts do not. Consequently, the Hazen-Williams equation is misapplied outside its data range. The first objective of this paper is to show quantitatively the limitations of the Hazen-Williams equation.

Despite its limitations, the Hazen-Williams equation has been used for a long time and there exists a valuable database for the inner surface roughness of older pipes (Hudson 1966). For these pipes, validated C values can be used to establish their relative roughnesses. After doing so, the knowledge of pipe roughness accumulated for the Hazen-Williams equation can be transformed and used by the Darcy-Weisbach equation for broader applications. This transformation also allows the error of a misapplied Hazen-Williams equation to be quantified. The second objective of this paper is to show such a transformation.

RELATING HAZEN-WILLIAMS *C* TO DARCY-WEISBACH *f*

The Hazen-Williams equation, in SI units, is

$$V = 0.849 C R_h^{0.63} S^{0.54} \tag{1}$$

where V = velocity; R_h = hydraulic radius; and S = slope of the energy grade line. The C is the Hazen-Williams coefficient. When U.S. customary units are used, the constant 0.849 in (1) should be changed to 1.318. Daugherty and Franzini (1965) and Hwang and Hita (1987) suggest that the equation is applicable for the flow of water in pipes larger than 5 cm and velocities less than 3 m/s.

The Darcy-Weisbach equation is

$$H = \frac{fL}{D}\frac{V^2}{2g} \tag{2}$$

where f = friction factor; L = pipe length; D = pipe diameter; V = discharge velocity; g = gravitational acceleration; and H = head loss over the length L. The friction factor is related to the Reynolds number R and the equivalent roughness ε, and the pipe diameter D by the Colebrook-White formula

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\mathsf{R}\sqrt{f}}\right) \tag{3}$$

Eq. (1) can be rearranged so that S is expressed in terms of a velocity head $V^2/2g$. In so doing, a $V^{0.1841}$ appears in the denominator of the right side of the equation. By introducing the kinematic viscosity ν, $V^{0.1841}$ is combined with $(D/\nu)^{0.1841}$ to form $\mathsf{R}^{0.1841}$. Meanwhile, R_h is replaced by $D/4$. This, combined with $(D/\nu)^{0.1841}$ introduced earlier, results in a $D^{1.0185}$ in the denominator. Let ε be the equivalent roughness of the pipe inner wall. Split $D^{1.0815}$ into D and $D^{0.0185}$ and rewrite the latter as $(D/\varepsilon)^{0.0185}\,\varepsilon^{0.0185}$. Finally, replace S by H/L. With these manipulations, (1) is recast in the form of (2) as

$$H = \left[\frac{133.84}{C^{1.8519}\mathsf{R}^{0.1481}}\left(\frac{\varepsilon}{D}\right)^{0.0185}\frac{1}{\varepsilon^{0.0185}\,\nu^{0.1481}}\right]\frac{L}{D}\frac{V^2}{2g} \tag{4}$$

Vennard (1961) introduced a similar equation in U.S. customary units with $D^{0.0185}$ dropped because of the small exponent. Consequently, the relative roughness ε/D is absent. Similar equations can be found in Streeter and Wylie (1985), Street et al. (1996), and Potter and Wiggert (1997).

By equating f in (2) with the quantity in the bracket on the right side of (4), C can be expressed as

$$C = 14.07 f^{-0.54}\mathsf{R}^{0.08}\left(\frac{\varepsilon}{D}\right)^{0.01}\varepsilon^{-0.01}\nu^{-0.08} \tag{5}$$

When U.S. customary units are used, the constant 14.07 in (5) should be changed to 17.22.

It is seen that, in the light of the Darcy-Weisbach equation, C is a function of the Reynolds number R, the relative roughness ε/D, the absolute roughness ε, and the kinematic viscosity ν. Alternatively, by canceling the ε's in (5), C can be viewed as a function of R, ε/D (because f depends on ε/D), D, and ν [see (7) in the following sections].

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Note. Discussion open until February 1, 1999. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on July 11, 1997. This paper is part of the ***Journal of Hydraulic Engineering***, Vol. 124, No. 9, September, 1998. ©ASCE, ISSN 0733-9429/98/0009-0951–0954/$8.00 + $.50 per page. Paper No. 16176.

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TABLE 1. Experimental Data (Columns 1–7) Used to Establish C

Data set (1)	D (in.) (2)	No. of observation (3)	Range of V (ft/s) (4)	Range of C (5)	Mean C (6)	Remarks (7)	ε/D (8)
1	3.22	8	0.36–5.15	119.5–120	120	Uncoated, new cast iron	0.0034
2	5.39	8	0.5–7.48	132.1–125.8	129	Uncoated, new cast iron	0.0012
3	7.40	6	1.6–8.22	125.0–116.0	121	Uncoated, new cast iron	0.0016
4	12	30	1.0–5.00	139.3–148.5	144	Coated, very straight, no specials	0.00019
5	12	4	1.6–3.1	107.0–121.5	114	Coated, Bonn service main, new	0.0028
6	12	4	1.6–3.1	106.0–117.0	111	Coated, Bonn service main, new	0.0034
7	16.02	20	1.0–5.0	146.0–145.8	146	Coated, well laid, new cast iron	0.00012
8	16.02	30	1.0–5.0	145.0–145.6	145	Coated, well laid, new cast iron	0.00014
9	16.48	4	1.6–3.1	129.0–133.0	131	Coated, Danzig main, new cast iron	0.00068
10	19.68	9	1.4–3.7	112.0–117.8	115	Uncoated, new cast iron	0.0019
11	29.96	30	1.25–2.90	138–142	140	Coated, straight, no specials, new	0.00021
12	36	2	4.2	129	129	Coated, Rochester main, new cast	0.00034
13	48	3	2.6–6.2	142.0–141.0	142	Coated, Rosemary siphon, new cast	0.00008
14	48	1	3.5	112.3	112.3	Coated, Edinburgh main, new cast	0.0011
15	3.25	7	0.58–6.14	124–143	133	Coated sheet iron, riveted	0.0011
16	48	35	1.0–5.0	119–105	112	Tuberculated Rosemary siphon, cast	0.0012
17	48	21	2.0–5.0	144–141	142	Cleaned Rosemary siphon, cast iron	0.00009

Note: The mean C (column 6) for data sets 12 and 14 were not stated by Williams and Hazen (1933), but they are obviously the same as those in column 5. Selected from Table 1 of Williams and Hazen (1933, pp. 4–5).

Williams and Hazen (1933) indicated that the C value might not be a constant because the exponents in the equation may vary with pipe size and with the slope of the energy grade line. However, the exponents were selected representing "approximately average conditions" so that C is practically constant and is viewed as an index of the smoothness of the interior of the pipe surface. This intent of C and the fact that hydraulic radius appears separately in the Hazen-Williams equation might have motivated many texts, references, and software manuals [e.g., Linsley and Franzini (1972), Metcalf and Eddy (1972), Viessman and Hammer (1985), Simon (1986), Prasuhn (1987), Rossman (1994), and Hwang and Houghtalen (1996)] to tabulate values of C for different pipe types without giving any indication of applicable pipe sizes. Some cursory information on the variations of C with pipe size can be found in Babbitt et al. (1962), Bauer et al. (1969), Stephenson (1981), Roberson et al. (1988), and Delleur (1995).

VARIATIONS OF HAZEN-WILLIAMS C WITH REYNOLDS NUMBER AND PIPE SIZE

Contained in Table 1 of Williams and Hazen (1933) is a set of experimental data used to establish C values. A portion of that table is reproduced in Table 1. The first 14 sets pertaining to new cast iron pipes are used here to demonstrate the variation of C with R and D. It is seen that the variations in C among the coated cast-iron pipes mask the difference in C between the coated and the uncoated cast iron pipes. For the purpose of showing C varying with R and D, these 14 pipes are assumed to have a common ε. Variations of C with ε are addressed later. This common ε is 0.0003 m, established by minimizing the root-sum square of the difference between the computed Cs from (5) with f from (3) and the Cs in column 6 of Table 1. Using this ε and a ν of 1.133×10^{-6} m²/s (for water at 15.56°C), Fig. 1 is generated from (5). Each curve in this figure corresponds to an ε/D. Because ε is fixed, the ε/D values indicate the pipe diameters. The R and ε/D ranges cover the transition zone and the complete turbulence-rough pipe zone of the Moody diagram. The boundary between these two zones is indicated by the dashed line computed from (Moody 1944)

$$\frac{1}{\sqrt{f}} = \frac{\mathsf{R}}{200}\frac{\varepsilon}{D} \tag{6}$$

The computed C values, two per data set and connected by a line segment, are superimposed in Fig. 1. The data used by Williams and Hazen (1933) for new cast-iron pipes lie entirely within the transition zone. For a given pipe inner surface type, the computed C value varies significantly with R and D. The experimental data used to establish the C values in Williams and Hazen (1933) has limited ranges in R and D.

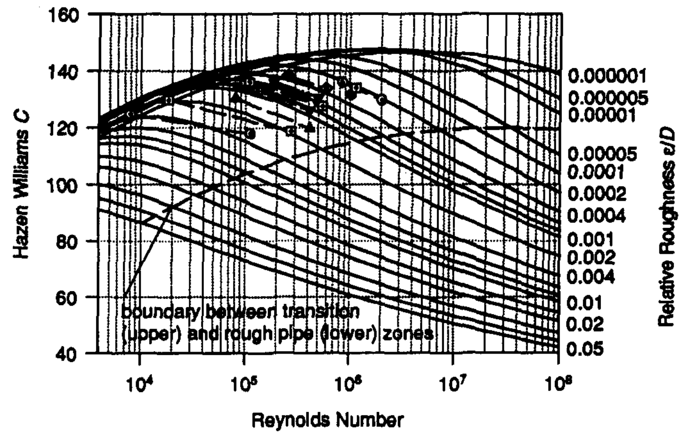

FIG. 1. Hazen-Williams C as Function of Reynolds Number and Relative Roughness for 15.56°C (60°F) Water Flowing in Pipes with Absolute Roughness of 0.0003 m (0.00099 ft)

FINDING RELATIVE ROUGHNESS FROM HAZEN-WILLIAMS C, REYNOLDS NUMBER, AND PIPE SIZE

By eliminating the explicit dependency of C on ε as shown in (5), the Hazen-Williams C is related to R, ε/D [in f via (3)], D, and ν as

$$C = 14.07f^{-0.54}\mathsf{R}^{-0.08}D^{-0.01}\nu^{-0.08} \tag{7}$$

When U.S. customary units are used, the constant 14.07 in (7) is 17.22.

For specified D and ν for water, C can be plotted as a function of R and ε/D. If C and R are known, which is the case in the database of the Hazen-Williams equation, then ε/D can be found from the plot. To illustrate this process and to demonstrate that C varies with D in ε/D as well as in D alone, plots with D of 0.08255, 0.3048, and 1.2192 m (3.25, 12, and 48 in.) are shown in Figs. 2, 3, and 4, respectively. Superimposed on each are the applicable experimental data from Table 1. For each data set, the maximum and the minimum C values

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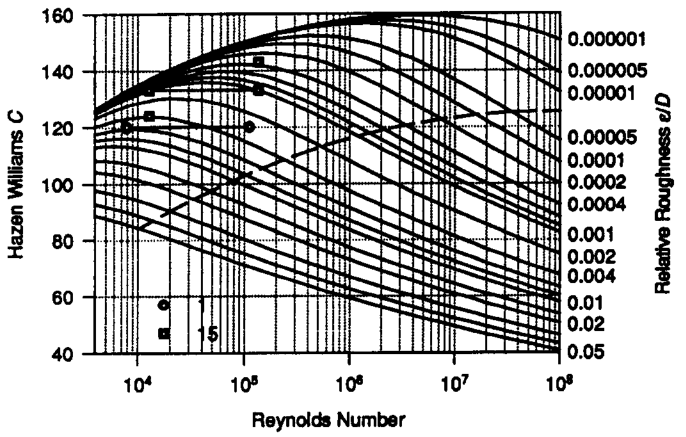

FIG. 2. Hazen-Williams *C* as Function of Reynolds Number and Relative Roughness for 15.56°C (60°F) Water Flowing in 0.0826-m (3.25 in.) Pipe

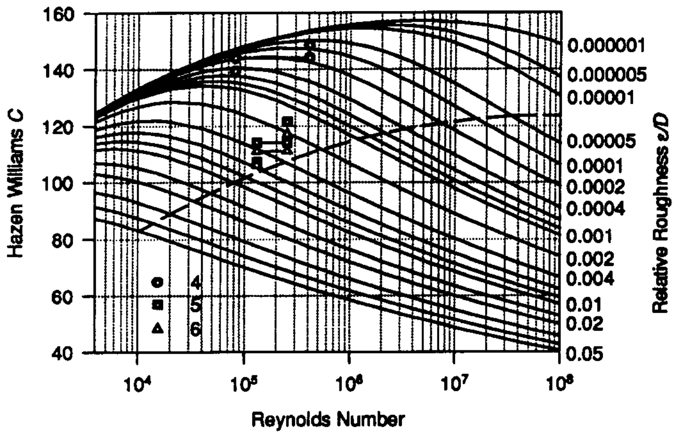

FIG. 3. Hazen-Williams *C* as Function of Reynolds Number and Relative Roughness for 15.56°C (60°F) Water Flowing in 0.305-m (12 in.) Pipe

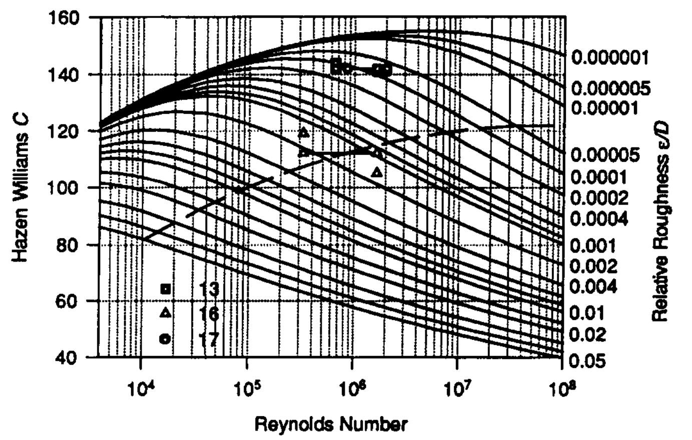

FIG. 4. Hazen-Williams *C* as Function of Reynolds Number and Relative Roughness for 15.56°C (60°F) Water Flowing in 1.219-m (48 in.) Pipe

are shown by the isolated symbols while the mean C value is indicated by a horizontal line segment over the Reynolds number range. It is seen that except for the 12-in. Bonn service main (data sets 5 and 6), the individual data points exhibit the trend of C variation with R as indicated by the curves of constant ε/D. This is especially so for the new, cleaned, and tuberculated Rosemary siphon (data sets 13, 16, and 17). The relative roughness for each pipe can be taken as the ε/D associated with the curve that passes through the point of mean C (column 6 of Table 1) and mean R.

The foregoing procedure illustrates what is involved in finding ε/D from C. However, it is awkward to use and an alternative is provided here. Several formulas that approximate (3) and are explicit for f exist (Barr 1975; Swamee and Jain 1976; Round 1980). They can be used with (7) to express ε/D explicitly. For example, by substituting the f in the explicit approximation formula of Swamee and Jain (1976)

$$f = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathsf{R}^{0.9}}\right)\right]^2} \tag{8}$$

into (7), it can be shown that

$$\varepsilon/D = 3.7\left(10^{-0.0432\,C^{0.926}D^{0.0093}(\mathsf{R}\nu)^{0.074}} - \frac{5.74}{\mathsf{R}^{0.9}}\right) \tag{9}$$

When U.S. customary units are used, the constant -0.0432 should be changed to -0.0359. The f computed from (8) approximates that of (3) within $\pm 1\%$ for $10^{-6} < \varepsilon/D < 10^{-2}$ and $5 \times 10^3 < \mathsf{R} < 10^8$ (Swamee and Jain 1976). Hence (9) yields an acceptable estimation for ε/D from known C, D, R, and ν. The relative roughnesses so computed are shown in column 8 in Table 1.

APPLICATION—ERROR OF USING HAZEN-WILLIAMS EQUATION OUTSIDE ITS RANGE

Let S_{hw} be the slope of the energy grade line in the Hazen-Williams equation. In computing S_{hw}, the mean C values are used in the conventional manner. Let S_{dw} be the slope of the energy grade line in the Darcy-Weisbach equation, $S_{dw} = H/L$. The ε/D needed to determine the friction factor f is computed from (9) using the same C values employed in obtaining S_{hw}. Because the relative roughness does not vary with the Reynolds number and because the Reynolds number effect on f is accounted for in the Colebrook-White formula, the S_{dw} is correct over a wide Reynolds number range.

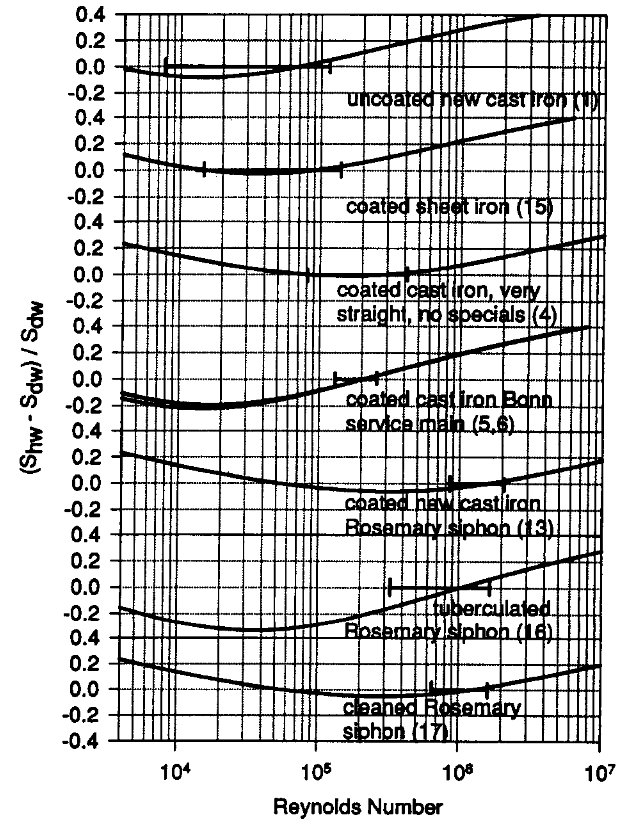

FIG. 5. Fractional Error of Hazen-Williams Equation

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The fractional error of the frictional slope from the Hazen-Williams equation can be expressed as $(S_{hw} - S_{dw})/S_{dw}$. This error varies with the Reynolds number and is shown for the eight data sets in Fig. 5. For each case, a horizontal line segment indicates the Reynolds number range over which the Hazen-Williams C is established. It is seen that the error is small within the experimental data range, but becomes considerable outside.

SUMMARY AND CONCLUSIONS

The empirical Hazen-Williams C indicates the roughness of the interior surface of a pipe. Significant variations of C with Reynolds number, relative roughness, and, separately, diameter or absolute roughness of the pipe are demonstrated. The original Williams and Hazen (1933) data for cast-iron pipes cover only a portion of the transition zone in the Moody diagram. Many text books, references, and software manuals tabulate C values for different pipe types without indicating the applicable Reynolds number and pipe size ranges. Thus one must be cautious when using the Hazen-Williams equation. The error resulting from applying the Hazen-Williams equation outside its database is significant.

However, the accumulated knowledge of Hazen-Williams C for older pipes is extensive and valuable. The relative roughness for such pipes can be estimated from a Hazen-Williams C that is known to be realistic for specific conditions. A method for such estimations is suggested. The relative roughness so established then can be used to determine the friction factor in the Darcy-Weisbach equation. Friction head losses then can be calculated correctly for Reynolds number and pipe size ranges wider than those used in establishing the C values.

Because the Darcy-Weisbach equation with the Colebrook-White formula for the friction factor is theoretically sound and has an extensive database, the usage of the Hazen-Williams equation is strongly discouraged.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

C = Hazen-Williams coefficient;
D = pipe diameter;
f = Darcy-Weisbach friction factor;
g = gravitational acceleration;
H = frictional head loss;
L = pipe length;
R = Reynolds number;
R_h = hydraulic radius;
S = slope of energy grade line;
V = discharge velocity;
ε = absolute equivalent roughness; and
ν = kinematic viscosity of water.

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