Relating the Hazen-Williams and Darcy-Weisbach Friction Loss Equations for Pressurized Irrigation

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Abstract. *The Hazen-Williams C factors change with Reynolds number and mean flow velocity. Figures are presented for determining pipe roughness heights or relative roughness ratios that are equivalent to values of the Hazen-Williams C factor. The results facilitate the ability to utilize friction data derived with the Hazen-Williams equation with the Darcy-Weisbach equation. C factors were also determined that are equivalent to known or measured pipe roughnesses. The Churchill equation for directly calculating the Darcy-Weisbach friction factor is compared with other established friction factor equations. The Churchill equation accurately predicts friction factors over all ranges in Reynolds number, pipe roughness, and temperature encountered in irrigation systems, including the laminar and transition flow regimes.*
Keywords. *Head loss, Pipe flow, Sprinkle, Trickle, Roughness.*

Prediction of friction head loss in pipe systems is an integral part of irrigation system design, operation and evaluation. Many engineers use the Hazen-Williams equation to predict friction losses in pipes, primarily due to its simplicity. However, the equation has significant limitations, especially in trickle irrigation systems, where it cannot be assumed that water temperature is the constant 20°C (70°F) assumed in the standard Hazen-Williams C factors, that flow is fully turbulent, and that C factors remain constant with increasing Reynolds number.

Churchill (1977) combined explicitly solved equations for the Darcy-Weisbach friction factor that are valid over all ranges of Reynolds number, thereby simplifying the application of the more accurate Darcy-Weisbach equation to sprinkle, trickle, and other closed pipe systems. This is especially beneficial in situations where water temperatures deviate from 20°C, as in trickle irrigation, or where flow velocities are relatively low, as near the terminals of sprinkle and trickle laterals.

The Churchill friction equation estimates friction factors across all flow regimes in closed irrigation systems including the transition zone between laminar and turbulent flow. Examples in this article demonstrate its application. Relationships between Darcy-Weisbach friction factors and Hazen-Williams C values are presented to convert Hazen-Williams C factors into roughness terms for the Darcy-Weisbach equation.

The Darcy-Weisbach Equation

The earliest known standardized head loss equation was developed by Chezy in approximately 1775 (Herschel, 1897) and is applicable to any type of channel cross-section:

$$h_f = C_f \frac{L}{R_h} \frac{V^2}{2g} \tag{1}$$

where h_f is the friction loss occurring along length L, (L), R_h is the hydraulic radius (L), V is mean flow velocity ($L\ T^{-1}$), and g is gravitational acceleration ($L\ T^{-2}$). C_f is a factor generally termed the coefficient of friction. The quotient h_f/L is the slope of the energy grade line.

The Chezy equation is a physically based and theoretically correct equation; i.e., friction head loss is linearly proportional to velocity head ($V^2/2g$). For circular cross-sections (pipes flowing full), R_h is equal to D/4, where D is the inside pipe diameter (L), thus reducing the Chezy equation to the Darcy-Weisbach form for full pipe flow (Darcy, 1854; Weisbach, 1845):

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{2}$$

where f is the Darcy-Weisbach friction factor for full pipe flow. The friction factor f is equal to 4 C_f. Units of L, D, V^2 and g in equation 2 cancel to length (L).

Most computations for f are related to the dimensionless Reynolds Number (N_R):

$$N_R = \frac{\rho V D}{\mu} \tag{3}$$

where ρ = fluid density, mass L^{-3}, μ = dynamic viscosity, mass $L^{-1}T^{-1}$, V = mean pipe velocity ($L\ T^{-1}$), and D is inside pipe diameter (L). Dynamic viscosity of water can be expressed as a function of water temperature over the range of 0 to 100°C (32 to 212°F) as:

$$\mu = 1.679\text{E-}3 \exp(-0.024T) \tag{4}$$

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where T is mean fluid temperature in °C and μ is in kg $m^{-1}s^{-1}$. Equation 4 was developed from a graphical viscosity relationship presented in Binder (1973). Density of water is relatively constant over the range of temperatures experienced in irrigation and can be taken as 1000 kg m^{-3}.

Variation of f with Relative Roughness and Turbulence

The Darcy-Weisbach friction factor varies with fluid turbulence (characterized by N_R) and with relative pipe roughness. Relative roughness is the ratio of e/D, where e is the mean size of roughness projections along the pipe wall, and D is the inside pipe diameter. Figure 1 represents typical variation in f with N_R for various ratios of e/D. In the log-log form, figure 1 is referred to as the Moody diagram (Moody, 1944). The symbols in figure 1 are measurements of f from Nikuradse (1933) for small pipes with artificial sand roughness representing six values of e/D. The solid lines in figure 1 are values of f for clean, commercial pipe presented by Moody (1944) and computed using the Churchill (1977) equation.

The sand roughness data of Nikuradse agree well with the Moody diagram and Churchill equation for commercial pipe in the laminar flow region ($N_R < 2000$) and in the region termed "complete turbulence, rough." In the latter region, f has a nearly constant value independent of N_R and depends only upon e/D. The left hand boundary of the completely turbulent, rough zone can be characterized as $f = 2.8\ N_R^{-0.37}$ shown as the dotted line in figure 1.

In between the laminar and completely turbulent zones is the so-called "transition" zone ($4000 < N_R < 16.2\ f^{-2.7}$), where f first increases and then generally decreases with increasing N_R. The sand roughness data of Nikuradse are lower than f values predicted by the Moody diagram in this zone due to the high uniformity of sand used in Nikuradse's tests. Roughness in commercial pipe generally varies widely so that the degree of extension of elements through the laminar film along the pipe surface at lower values for N_R is more variable as compared to uniform sand. As N_R increases, the thickness of the laminar film along the pipe surface decreases and roughness elements are more exposed to the turbulent stream (Streeter and Wylie, 1979).

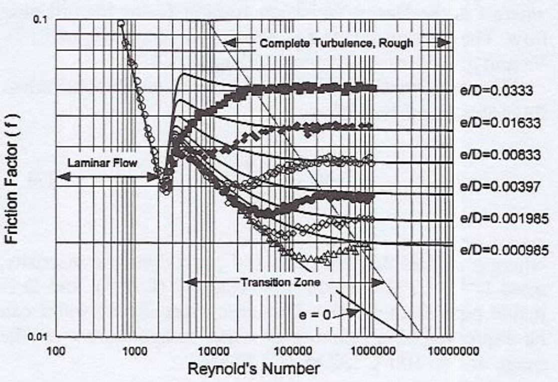

Figure 1–Moody diagram showing the Darcy-Weisbach friction factors (f) reported by Nikuradse (1933) for artificial sand roughness and friction factors computed by the Churchill equation for the same relative roughnesses (e/D).

General Values for Roughness Height

General values of roughness heights for common pipe materials reported in various literature sources are presented in table 1. Pipe roughness will generally increase with age due to pitting or etching by sand and by corrosion, and will increase with precipitation of salts on pipe walls. The roughness of pipes will also increase with deposition of sediment or attachment of algal, bacterial, or other slime to pipe walls. Roughness of iron and steel pipe increases with formation of rust.

Equations for Calculating Friction Factor, f

Many equations for predicting the Darcy-Weisbach friction factor have been presented over the last 60 years. The equations presented in the 1930s required implicit solution techniques. Equations introduced in the 1960s and 1970s provided explicit solutions. Many of these equations are still used and are described in this section to provide background to the Churchill equation. They are compared in figures 2 and 3 with the Churchill equation.

Laminar Flow. *Poiseuille's equation.* For full pipe flow under laminar conditions ($N_R < 2000$), f is approximated by the Poiseuille equation (Morris and Wiggert, 1973) as

$$f = \frac{64}{N_R} \tag{5}$$

On the Moody diagram, this equation is a straight line for logarithmic scales (figs. 2 and 3).

Table 1. General values of roughness height reported in the literature

Pipe Material	Pipe Roughness (mm) New	Ave.	Old
Smooth drawn tubing (glass, brass)	0.0015†‡		
PVC pipe	0.002‖	0.013#	
Aluminum (with couplers)		0.13#	
Concrete	0.3‡§		
Smooth with sm. joints	0.015-0.2*	0.3†	
Wood-floated or brushed	0.2-0.4*		
Unusually rough/rough joints	0.6-1.0*	0.3†	3‡§
Butt-welded steel			
New	0.04†‡§		
Light rust	0.15*	0.2#	0.37*
Hot-asphalt-dipped	0.06*	0.08#	0.15*
Heavy brush-coated enamels/tars	0.37*		0.95*
Incrustation/tuberculation		0.95-2.5*	2.5-6*
Epoxy enameled steel		0.005-0.05* 0.03#	
Galvanized iron	0.15†‡§		
Wrought iron	0.045*	0.13*	0.9-2.5*

* Brater and King (1976).
† Morris and Wiggert (1972).
‡ Streeter and Wylie (1979).
§ Binder (1973).
‖ Flammer et al. (1982).
Keller and Bliesner (1990).

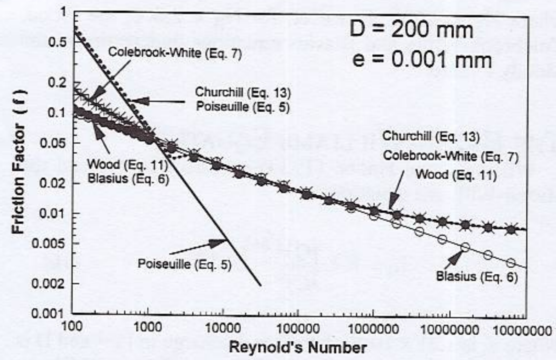

Figure 2–Comparison of the Darcy-Weisbach friction factor computed using various equations for 200 mm diameter pipe and 0.001 mm roughness height (Churchill f is the dashed line).

Turbulent Flow. *Blasius equation.* The Blasius equation (Blasius, 1913) was formulated for small diameter smooth pipes under turbulent flow. It has the form:

$$f = 0.32 N_R^{-0.25} \tag{6}$$

The Blasius equation plots as a straight line for all N_R on a logarithmic scale and follows the "smooth pipe" curve of the Moody diagram at $N_R > 4{,}000$ (figs. 2 and 3).

The Blasius equation was used by Keller and Bliesner (1990) to describe flow in smooth trickle pipes.

Colebrook and White equation. Colebrook and White (1937) and Colebrook (1938) presented equations for predicting the friction factor in the "transition range" between smooth pipe flow and fully turbulent or "rough pipe" flow:

$$\frac{1}{\sqrt{f}} = 1.74 - 0.87 \mathrm{Ln}\left(\frac{2e}{D} + \frac{18.7}{N_R\sqrt{f}}\right) \tag{7}$$

where D is inside pipe diameter and e is mean equivalent height of pipe roughness elements. Parameters D and e have the same measurement units. Equation 7 can be rearranged to produce a form equivalent to that presented by Colebrook (1938):

$$\frac{1}{\sqrt{f}} = 0.87 \mathrm{Ln}\left[0.27\left(\frac{e}{D}\right) + \frac{2.51}{N_R\sqrt{f}}\right] \tag{8}$$

An iterative solution is necessary to solve both equations 7 and 8.

Nikuradse equations. For turbulent flow in smooth pipe where $e/D = 0$ [such as polyvinyl chloride (PVC) or polyethylene (PE) pipe], the Colebrook equation (eq. 8) reduces to:

$$\frac{1}{\sqrt{f}} = 0.87 \mathrm{Ln}\left(N_R\sqrt{f}\right) - 0.8 \tag{9}$$

which is an equation commonly attributed to Nikuradse (1933) (Streeter and Wylie, 1976, p. 66; Binder, 1973, p. 111). This equation plots as a lower boundary of f values on the Moody diagram.

For large N_R ($> 10^6$), equation 7 reduces to:

$$\frac{1}{\sqrt{f}} = 1.74 - 0.87 \mathrm{Ln}\left(\frac{2e}{D}\right) \tag{10}$$

which is commonly referred to as the Nikuradse rough pipe equation (Morris and Wiggert, 1972). Because the N_R term in the Colebrook-White equation is absent in the Nikuradse rough pipe equation, this equation predicts a horizontal f curve on the Moody diagram and is therefore valid only for high N_R in the fully turbulent region.

Wood equation. Wood(1966) presented an equation for turbulent flow in rough pipes:

$$f = a + b\, N_R^{-c} \tag{11}$$

where

$$a = 0.094\, k^{0.225} + 0.53\, k \tag{11a}$$

$$b = 88\, k^{0.44} \tag{11b}$$

$$c = 1.62\, k^{0.134} \tag{11c}$$

$$k = e/D \tag{11d}$$

The Wood equation is valid over the range $N_R > 10^4$ and $10^{-5} < e/D < 0.04$ and produces estimates similar to the Colebrook-White equation (figs. 2 and 3) in that range. The advantage of the Wood equation as compared to the Colebrook-White/Colebrook equation is the explicit (direct) solution with generally equivalent accuracies across similar ranges of Reynolds numbers. The disadvantage of the Wood equation is its invalidity in the laminar range.

Swamee-Jain equation. Swamee and Jain (1976) introduced an explicit equation for turbulent flow:

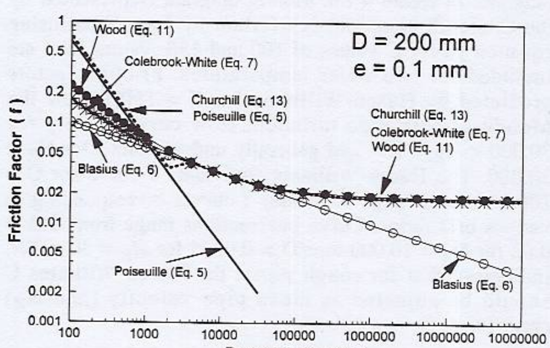

Figure 3–Comparison of the Darcy-Weisbach friction factor computed using various equations for 200 mm diameter pipe and 0.1 mm roughness height (Churchill f is the dashed line).

$$f = \frac{1.328}{\left\{ \mathrm{Ln}\left[\left(\frac{6.97}{N_R} \right)^{0.9} + 0.27\left(\frac{e}{D} \right) \right] \right\}^2} \qquad (12)$$

Equation 12 is similar to equation 14 for the Churchill "A" factor.

The Churchill Equation. Churchill (1977) introduced a friction factor equation that is valid for both rough and smooth pipes and for the full range of laminar, transition and fully turbulent flow regimes of the Moody diagram. The Churchill equation was developed by combining the Poiseuille equation for laminar flow with empirical friction factor relationships proposed by Colebrook (1938) and Nikuradse (1933) for turbulent regimes, and by employing the effects of using large exponents to weight the effects of the various flow regimes, as proposed by Wilson and Azad (1975). The resulting equation is:

$$f = 8\left[\left(\frac{8}{N_R} \right)^{12} + \frac{1}{(A+B)^{1.5}} \right]^{1/12} \qquad (13)$$

where A and B are computed as

$$A = \left\{ -2.457\ \mathrm{Ln}\left[\left(\frac{7}{N_R} \right)^{0.9} + 0.27\left(\frac{e}{D} \right) \right] \right\}^{16} \qquad (14)$$

and

$$B = \left(\frac{37530}{N_R} \right)^{16} \qquad (15)$$

The Churchill equation offers an explicit solution and is valid over all Reynolds numbers and e/D, thus eliminating the need for conditional statements and tests in computer code concerning flow regime and flow equation. However, the Churchill equation requires greater precision (i.e., double precision) when computing f, A, and B, due to the calculation of extremely large or extremely small numbers with large exponents (12 and 16 powers). The A parameter in the Churchill equation is primarily significant for large Reynolds numbers in the fully turbulent region; whereas, the B parameter is primarily significant at lower Reynolds numbers in the transition region.

Comparison of Darcy-Weisbach Friction Factor Equations

Some of the more commonly used equations for f are compared with the Churchill equation in figures 2 and 3 for roughness heights of 0.001 mm (smooth) and 0.1 mm (rough) for 200 mm diameter pipe. In both applications, the Wood, Colebrook-White, and Churchill equations agree over the range of turbulent flow conditions where $N_R >$ 5,000. The Churchill equation predicts the "dip" in f in the early transition range ($2{,}000 < N_R < 4000$) that was measured in the tests by Nikuradse (1933) for artificially roughened pipes (Streeter and Wylie 1979, p. 236; Binder, 1973, p. 110). The Churchill equation agrees with the Poiseuille ($64/N_R$) relationship in the laminar flow range where $N_R < 2{,}000$. However, for $N_R < 2{,}000$, the Wood, Colebrook-White and Blasius equations underestimate the Moody f value.

The Hazen-Williams Equation

Williams and Hazen (1933) empirically derived the Hazen-Williams equation:

$$h_f = K\,L\left(\frac{Q}{C} \right)^{1.852} D^{-4.87} \qquad (16)$$

where K is 1.21×10^{10} when Q is discharge in l s^{-1} and D is inside pipe diameter (mm). C is the Hazen-Williams "capacity" factor. The value of C ranges from 80 for extremely rough pipe to approximately 150 for smooth pipe. The factor K is 10.69 when Q is in m^3 s^{-1} and D (m).

When expressed in terms of the Darcy-Weisbach equation, the Hazen-Williams equation predicts a Darcy-Weisbach friction factor of the form:

$$f = 12.10\ K\ C^{-1.852} Q^{-0.148} D^{0.13} \qquad (17)$$

indicating that the Hazen-Williams "f", if used with the Darcy-Weisbach equation varies with Q and D. When expressed in terms of the Reynold's number and viscosity, the friction factor predicted by the Hazen-Williams equation becomes:

$$f = \frac{373}{C^{1.852} \mu^{0.148} D^{0.018} N_R^{0.148}} \qquad (18)$$

which is similar to forms presented by Hughes and Jeppson (1978) and Lamont (1981) with the exception of the viscosity term, which is included in equation 18 to allow plotting f vs. N_R for various water temperatures. Since μ is the denominator in the calculation for N_R, the effects of viscosity will cancel in equation 18. This occurs because the Hazen-Williams equation was developed assuming constant water temperature.

To illustrate the relative comparison between friction estimated by the Hazen-Williams equation and the Moody diagram, in figure 4 the Moody diagram (represented by the Churchill equation) is overlain by f predicted using equation 18 for C values of 100 and 150. Values for f are included for two water temperatures. Friction factors predicted by Hazen-Williams for C = 150 follow the Moody smooth pipe turbulent flow curve closely for $30{,}000 < N_R < 10^8$, and generally underpredict f for $N_R <$ 30,000. The Hazen-Williams equivalent f curve for C = 100 crosses a range of Moody f-curves corresponding to various e/D ratios. Curve intersections range from e/D = 0.02 for N_R = 10,000 to e/D = 0.0005 for $N_R = 30 \times 10^6$, indicating that for rough pipes, the Hazen-Williams C should be adjusted as mean pipe velocity (and N_R) changes.

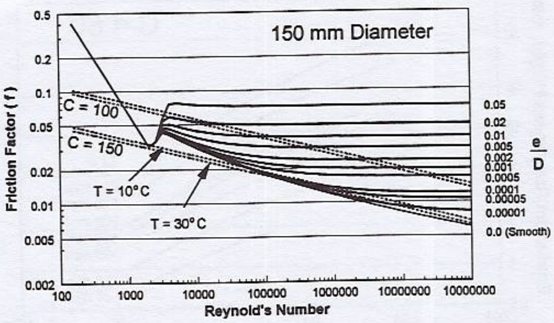

Figure 4–Moody diagram showing the Darcy-Weisbach friction factor (f) computed by the Churchill equation and equivalent f values computed by the Hazen-Williams equation for 150 mm diameter pipe.

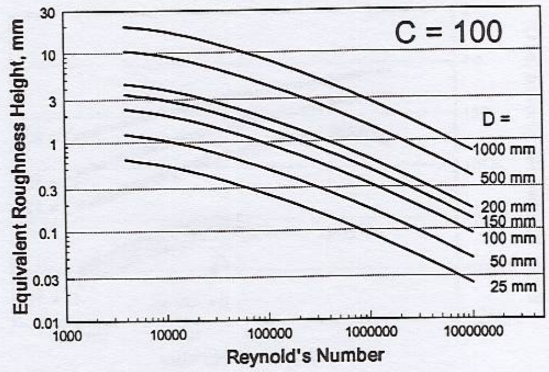

Figure 6–Equivalent roughness heights (e) vs. N_R for Hazen-Williams C = 100 and water temperature = 15°C.

Converting from the Hazen-Williams to the Darcy-Weisbach Equation

In irrigation practice, individuals often "optimize" Hazen-Williams C values to satisfactorily predict friction losses observed in the field or laboratory. These optimized values can be converted into e or e/d ratios to permit application of the Darcy-Weisbach equation. As indicated by equation 17, Hazen-Williams C values do not correlate directly to single values for f or pipe roughness, but vary with N_R and pipe diameter. Figures 5-8 predict the roughness heights equivalent to specific values for C over a range in pipe diameter and turbulence. These figures were computed for water temperature equal to 15°C (59°F) since this is a typical temperature encountered in irrigation. Other figures can be created for other temperatures. The general sensitivity of equivalent roughness height to change in temperature is illustrated in figure 3 and sensitivity of h_f to change in temperature is indicated in the trickle pipe example in following section.

In all cases, roughness height equivalent to a specific value of C increases significantly with increasing pipe diameter and in general, decreases with increasing N_R. The roughness height curves for C = 80, 100, and 120 in figures 5-7 were computed only for $N_R > 4000$ since the

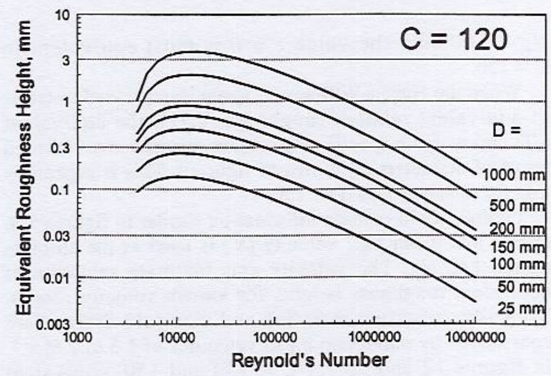

Figure 7–Equivalent roughness heights (e) vs. N_R for for Hazen-Williams C = 120 and water temperature = 15°C.

Hazen-Williams equation is valid for only turbulent flow conditions. In figure 8, where C = 150, equivalent roughness heights are presented only for $N_R > 100{,}000$, since $N_R < 100{,}000$ results in $e \leq 0$. This is due to the under-prediction of friction losses by the Hazen-Williams equation for C = 150 as compared to the Darcy-Weisbach equation with e = 0 and $N_R < 100{,}000$ (fig. 4). For

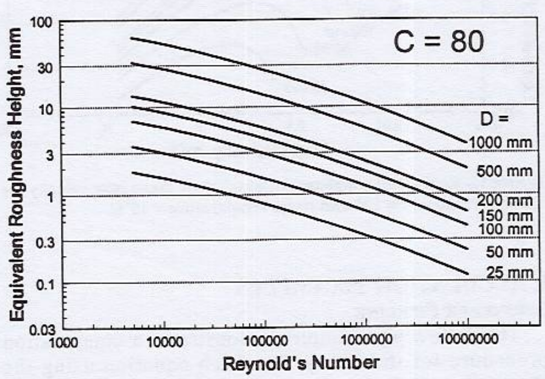

Figure 5–Equivalent roughness heights (e) vs. N_R for Hazen-Williams C = 80 and water temperature = 15°C.

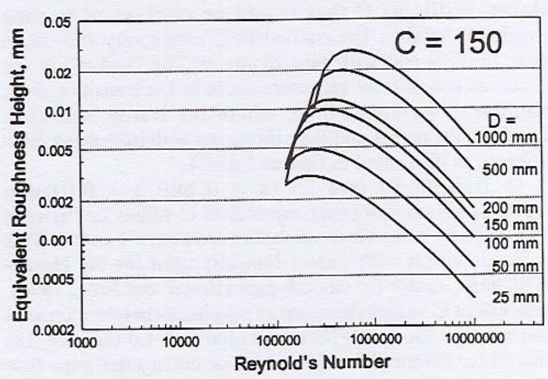

Figure 8–Equivalent roughness heights (e) vs. N_R for Hazen-Williams C = 150 and water temperature = 15°C.

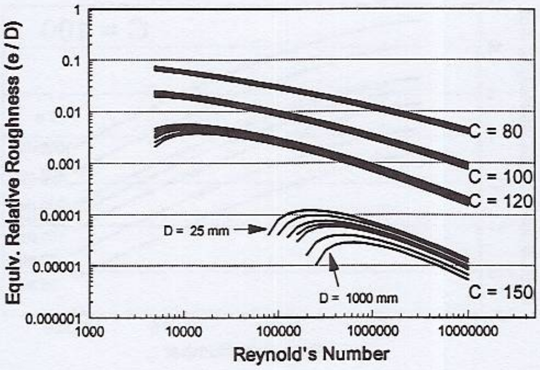

Figure 9–Equivalent relative roughness (e/D) for various Hazen-Williams C values and water temperature = 15°C.

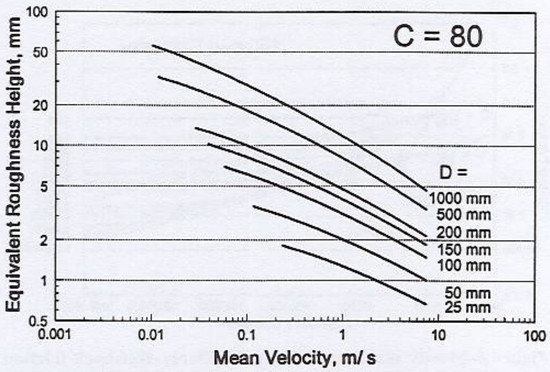

Figure 10–Equivalent roughness heights (e) vs. mean pipe velocity for Hazen-Williams C = 80 and water temperature = 15°C.

N_R < 100,000, the value e = 0 is most equivalent to C = 150.

When the Hazen-Williams C value is expressed in terms of equivalent relative roughness (e/D), the equivalent e/D curves for a specific N_R remain constant over a broad range of diameters as shown in figure 9. This is especially true for C values less than 150.

Figures 10-13 contain information similar to figures 5-8, except that mean pipe velocity (V) is used as the abscissa rather than N_R. The velocity axis facilitate selection of equivalent roughness heights for known velocity ranges. Typically, irrigation mainline and manifold designs are constrained by maximum mean velocities of 1.5 to 2 m s^{-1}. In figures 12 and 13 for C = 120 and 150, equivalent roughness height curves decrease at very low velocities due to the nonlinearity of the Darcy-Weisbach friction factor relative to the Hazen-Williams equation for low N_R in the transition range. Water temperature was assumed to be 15°C. The effect of changing water temperature is illustrated in figure 4.

Computing Hazen-Williams C from Pipe Roughness

Figures 14-19 are presented for determining values of Hazen-Williams C that would be equivalent to pipe roughness heights. The equivalent C values vary with mean pipe velocity and with pipe diameter. The "spikes" in the C curves at low flow velocities occur in the transition from laminar to turbulent flow, where the Darcy-Weisbach f initially decreases and then increases with increasing flow velocity as illustrated in figures 1 and 4.

In figures 14 and 15 (e = 0 and e = 0.01 mm representing smooth pipe), equivalent C values lie between 145 and 155 when flow velocities are above 1 m s^{-1}. This is in agreement with values typically used for the Hazen-Williams C factor for smooth pipe (Brater and King, 1976). Equivalent C values decrease as roughness height increases and average about 110 for rough pipe (e = 1.0 mm, fig. 18) and 60 for extremely rough (pitted or corrugated) pipe (e = 10 mm, fig. 19).

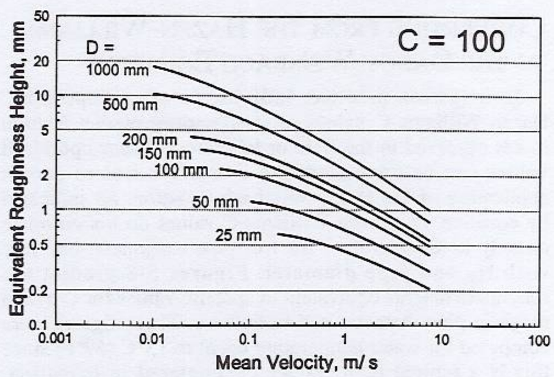

Figure 11–Equivalent roughness heights (e) vs. mean pipe velocity for Hazen-Williams C = 100 and water temperature = 15°C.

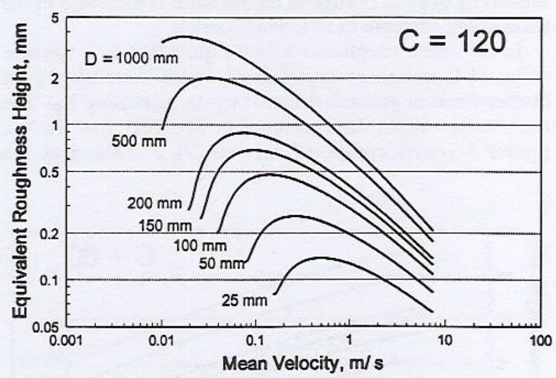

Figure 12–Equivalent roughness heights (e) vs. mean pipe velocity for Hazen-Williams C = 120 and water temperature = 15°C.

Calculation Examples

Sprinkler Pipeline

The following example demonstrates a computation procedure for the Darcy-Weisbach equation using the Churchill equation to compute the friction factor f. The conditions are aluminum sprinkler irrigation pipe where e =

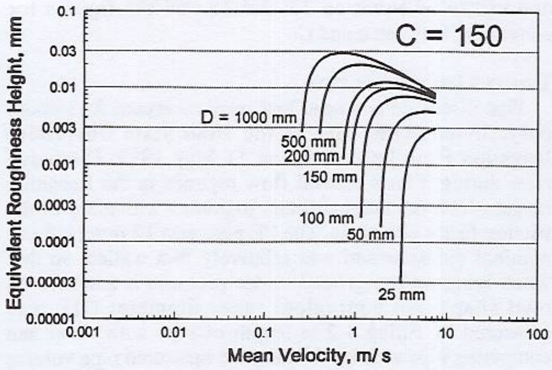

Figure 13–Equivalent roughness heights (e) vs. mean pipe velocity for Hazen-Williams C = 150 and water temperature = 15°C.

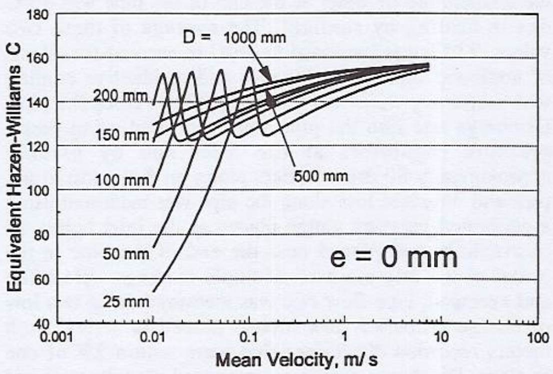

Figure 14–Equivalent Hazen-Williams C factors vs. mean pipe velocity for roughness height (e) = 0 mm and water temperature = 15°C.

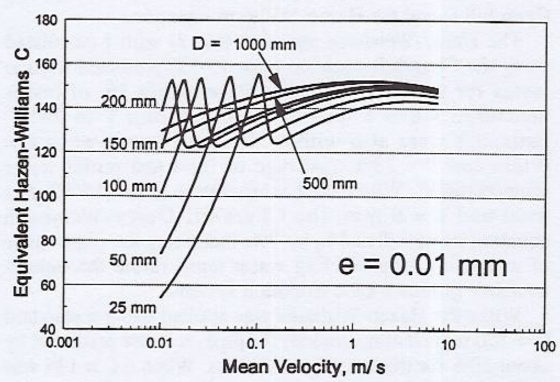

Figure 15–Equivalent Hazen-Williams C factors vs. mean pipe velocity for roughness height (e) = 0.01 mm and water temperature = 15°C.

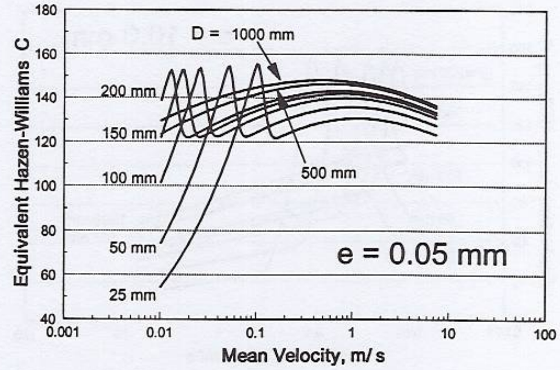

Figure 16–Equivalent Hazen-Williams C factors vs. mean pipe velocity for roughness height (e) = 0.05 mm and water temperature = 15°C.

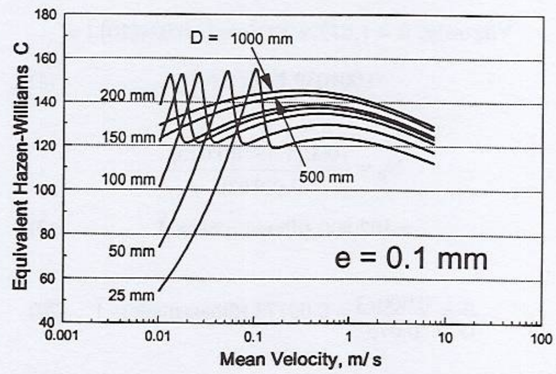

Figure 17–Equivalent Hazen-Williams C factors vs. mean pipe velocity for roughness height (e) = 0.1 mm and water temperature = 15°C.

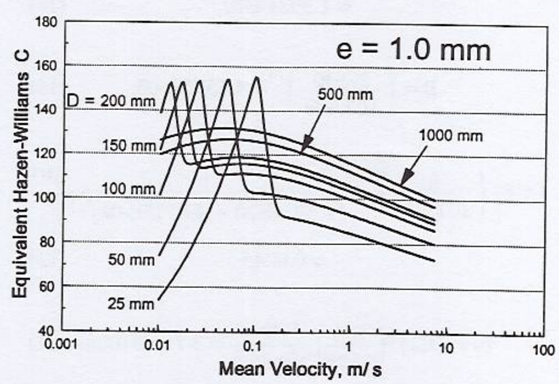

Figure 18–Equivalent Hazen-Williams C factors vs. mean pipe velocity for roughness height (e) = 1.0 mm and water temperature = 15°C.

0.13 mm, D = 75 mm, Q = 6.6 l s^{-1}, and T = 20°C. The following calculations apply:

$$V = \frac{Q}{A} = \frac{(6.6)(0.001 m^3/L)}{0.075^2 \frac{\pi}{4}} = 1.494 \, m \, s^{-1} \qquad (19)$$

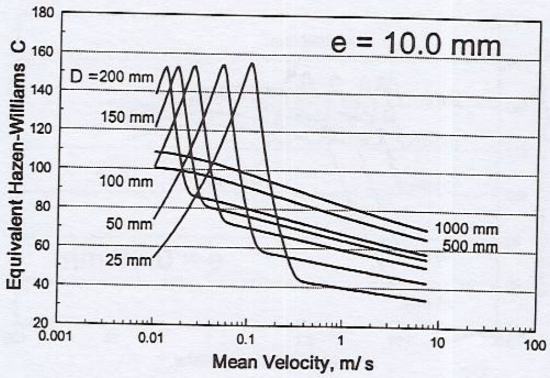

Figure 19–Equivalent Hazen-Williams C factors vs. mean pipe velocity for roughness height (e) = 10 mm and water temperature = 15°C.

$$\text{Viscosity, } \mu = 1.679 \times 10^{-3} \exp[-0.024(20)] = 0.001039 \text{ Kgm}^{-1}\text{s}^{-1} \quad (4)$$

$$N_R = \frac{1000(1.494)(0.075)}{0.001039} = 107{,}800 \text{ (dimensionless)} \quad (3)$$

$$\frac{e}{D} = \frac{0.00013}{0.075} = 0.00173 \text{ (dimensionless)} \quad (20)$$

$$A = \left\{-2.457 \text{ Ln}\left[\left(\frac{7}{107{,}800}\right)^{0.9} + 0.27(0.00173)\right]\right\}^{16} = 1.3024\text{ E20} \quad (14)$$

$$B = \left(\frac{37530}{107{,}800}\right)^{16} = 4.6575\text{E-08} \quad (15)$$

$$f = 8\left[\left(\frac{8}{107{,}800}\right)^{12} + \frac{1}{(1.3024\text{E20} + 4.6575\text{E-08})^{1.5}}\right]^{1/12} = 0.0245 \quad (13)$$

$$h_f = 0.0245\left(\frac{100}{0.075}\right)\frac{1.494^2}{2(9.807)} = 3.71 \text{ m/100m} \quad (2)$$

In this example, the A term (the turbulence term) is large and the B term (the transition term) is small, since the flow is nearly fully turbulent. The equivalent Hazen-Williams C value for 50 mm diameter pipe from figure 17 for e = 0.1 mm and V = 1.5 m s^{-1} is approximately 128. Using this value, the Hazen-Williams equation predicts h_f = 3.69 m/100 m, which is –0.5% different from the value from the Darcy-Weisbach equation, illustrating the validity and utility of equation 17 and 18 and the figures for converting between e and C.

Trickle Irrigation Pipe

Pipe discharge and head loss were measured for smooth polyethylene (PE) pipe at the Utah State University Irrigation Field Laboratory on 11 May 1993. Discharge rates during 3 tests created flow regimes in the transition range, which is a range difficult to predict with many of the friction factor equations. The PE pipe was 12 mm (1/2 in.) nominal diameter and was relatively thin walled, so that when lying on the ground under pressure it assumed an oval shape. An equivalent inner diameter (ID) was measured by filling a 2 m length of pipe with water and computing pipe area by dividing the measured pipe volume by the pipe length. The equivalent measured ID was 10.1 mm. Total pipe length was 175 m.

Temperature of water entering the pipe was 16°C and the temperature of water at the end of the pipe was 30°C due to heating by sunlight. The average of these two values, 23°C, was increased to 25°C to account for effects of nonlinear radiant, convective and conductive cooling with increasing water and pipe temperature along the pipe. Discharge rate into the pipe was controlled using preset pressure regulators at the inlet and by placing approximately 60 drip emitters along an 8 m strip at the pipe end. Pressure loss along the pipe was measured using a calibrated pressure gauge placed at the inlet below all obstructions and placed near the end of the pipe in the center of the drip emitters. Multiple readings were taken and averaged. Pipe flow rate was measured using two low discharge propeller flow meters placed in series. Both meters recorded discharges that were within 2% of one another. Discharge readings compared closely to timed volumetric measurements taken from the emitters. Elevation difference along the pipe was zero. Computations made from the field measurements are listed in table 2 for both the Darcy-Weisbach equation (using Churchill f) and the Hazen-Williams equation.

The Darcy-Weisbach equation (eq. 2) with f calculated using the Churchill equation (eqs. 13-15) predicted friction losses for the three tests that were within 3% of losses measured when e was set to 0 mm and T to 25°C. Estimates were also within 3% of measured when e = 0 mm and T = 23°C (average of inlet and outlet water temperatures). When inlet temperature only (15°C) was used and e = 0 mm, the Churchill-Darcy-Weisbach equation overpredicted h_f by 7%, indicating the importance of measuring or predicting water temperature throughout an above ground trickle irrigation system.

When the Hazen-Williams was applied using a standard C = 150 representing smooth PE pipe, it under predicted by about 20% for the low N_R conditions. When a C = 135 was selected from figure 14 according to the pipe size and mean velocity, friction loss predictions were within 3% of measured values for all three tests (table 2). This illustrates the need to use the Darcy-Weisbach equation with a valid friction factor equation for flow in the transition zone or to find an equivalent C value from figures 14-19 before using the Hazen-Williams equation. The effects of emitter barbs on the friction factor and C-values were not considered in these computations, since the emitters were placed together near the end of the pipe lateral.

Table 2. Comparison of field measurements and computations using the Darcy-Weisbach-Churchill and Hazen-Williams equations for 10.1 mm ID pipe

Parameter	Test 1	Test 2	Test 3
Measured discharge (l s^{-1})	0.0376	0.0471	0.0502
Mean velocity (m s^{-1})	0.469	0.589	0.627
N_R (for T = 25°C)	5130	6430	6850
Meas. h_f (m/100 m)	4.16	6.30	6.84
h_f from Darcy-Weisbach			
(e = 0, T = 25°C) (m/100 m)	4.18* (0.5%)	6.13* (–2.7%)	6.84* (0.0%)
(e = 0, T = 23°C)	4.23* (1.7%)	6.22* (–1.3%)	6.93* (1.3%)
h_f from Hazen-Williams			
(C = 150) (m/100 m)	3.33† (–20%)	5.06† (–20%)	5.69† (–17%)
(C = 135 from fig. 14)	4.05† (–2.6%)	6.15† (–2.4%)	6.92† (1.1%)

* Computed using the Darcy-Weisbach equation (eq. 2) with f computed using the Churchill equation (eq. 13-15). Values in parentheses are % difference from measured.

† Computed using the Hazen-Williams equation (eq. 16). Values in parentheses are % difference from measured.

Conclusions

The Darcy-Weisbach equation is theoretically applicable over all ranges of velocities, temperatures, roughness conditions, and Reynolds numbers, but requires separate determination of a friction factor f. The Churchill equation provides direct calculation of the Darcy-Weisbach friction factor over all ranges of Reynold's numbers and pipe roughness and agrees with the accepted Colebrook-White and Wood equations under conditions of high N_R and with the Poiseuille equation under very low N_R.

The Darcy-Weisbach equation is more accurate than the Hazen-Williams equation for both low velocity and high temperature conditions. The Hazen-Williams C factor, representing a specific relative roughness or roughness height in the fully turbulent region, changes with N_R and V, so that adjustment of C is necessary with changing pipe velocity and diameter.

Figures have been presented for determining roughness heights and relative roughness ratios equivalent to values of Hazen-Williams C-factor. These conversion figures facilitate using the more accurate Darcy-Weisbach equation in place of the Hazen-Williams equation. Other figures determine C-factors that are equivalent to known or measured pipe roughnesses.

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