Simple and Accurate Friction Loss Equation for Plastic Pipe

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Introduction

Engineers are continually searching for simpler calculation methods that do not compromise accuracy. Some simple methods of calculating pipe friction loss have been found, but the more complex Darcy-Weisbach equation is the most universally accepted.

Darcy-Weisbach (D-W) Equation

The Darcy-Weisbach equation for friction loss in smooth pipes was derived from investigating the shear stress of a fluid near the wall of a pipe and by similarity laws (dimensional analysis). Because of its derivation, the equation is considered theoretical. The equation, which follows, includes a dimensionless friction factor f that is a function of the Reynolds number and the roughness of the pipe:

$$h_L = \left(\frac{fl}{d}\right)\left(\frac{v^2}{2g}\right) \quad (1)$$

in which h_L = head loss (L); l = pipe length (L); d = pipe diameter (L); v = average velocity (L/T); g = gravitational constant (L/T^2); and f = friction factor.

It is the calculation of the friction factor f that complicates the D-W equation. There are theoretical methods for estimating the friction factor, but empirical approaches are commonly used. The Prandtl method for smooth pipe is an example of a theoretical method.

Methods of Calculating Friction Factor

Colebrook-White (C-W) Friction Factor

The friction factor f is customarily determined by the implicit combination of the Prandtl and von Karman equations into the C-W equation. The solution is reached either by iteration or by reference to the Moody diagram. These processes are not considered simple. The accuracy of the C-W method has been questioned by some researchers (Smith et al. 1956). Von Bernuth and Wilson (1989) showed data from several sources indicating that the C-W did not work well for small plastic pipe.

Blasius Friction Factor

Blasius proposed a simple equation for estimating the friction loss factor for very smooth pipes (Blasius 1913). This equation is a function only of the Reynolds number and is given as

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$$f = a \cdot \mathsf{R}^b \qquad (2)$$

where f = the friction factor; a and b = empirically determined coefficients; and R = the Reynolds number.

Theory and Verification of Blasius Equation

The theoretical background for the Blasius equation can be found in similarity laws, especially in the principle of incomplete self-similarity in the Reynolds number as explained by Barenblatt and Monin (1979). Their theory is consistent with power laws for velocity profiles in turbulent flow, and the exponent of the Blasius equation matches the one-seventh power law for velocity distribution (Schlichting 1968).

Schlichting (1968) states that the Blasius equation is very accurate for smooth pipes and Reynolds numbers less than 100,000. Watters and Keller (1978) and von Bernuth and Wilson (1989) ahve shown that the Blasius equation works well for small-diameter plastic pipe when the Reynolds number is less than 100,000. They also showed that it is very accurate, indeed, more accurate than the Colebrook-White (C-W) equation.

Use and Accuracy of Blasius Equation

The Blasius equation is not independent of the pipe diameter since it depends upon the Reynolds number. The Blasius equation accuracy is degraded above a 100,000 Reynolds number because at very high Reynolds numbers, it approaches zero, when, in fact, the friction factor approaches some positive value. However, the equation is valid for virtually any pipe size as long as the Reynolds number is limited to 100,000. When the Reynolds number is less than 4,000 in laminar flow or critical zones, the Blasius equation will overestimate the friction factor by as much as a factor of five. For irrigation-pipeline purposes, that is insignificant because the losses would be considered negligible. If flow is assumed to be laminar, f can be estimated by

$$f = \frac{64}{\mathsf{R}} \qquad (3)$$

If laminar flow is not assumed, the Blasius equation gives a conservative estimate of f.

Design limitations on velocity in irrigation pipelines [1.5 m/s (5 ft/sec)] will limit Reynolds numbers to 100,000 for pipe 64 mm (2.5 in.) or smaller. When the velocity is at the design limit, the error for 102-mm (four-inch) pipe with Reynolds number of 157,400 would be −3.6% and for 152-mm (six-inch) pipe with Reynolds number of 236,100 the error would be −5.6%. Assuming that design velocity limits are met, the Blasius equation will yield reasonably accurate estimates of the friction factor for pipe smaller than 80 mm (three inches). The 100,000 Reynolds number limit should be adhered to for pipes larger than 80 mm.

Many practicing engineers hesitate to use the D-W equation and prefer simpler empirical equations not requiring calculation of the Reynolds number and reference to a table of viscosity values. However, failure to correct for viscosity differences can lead to significant error. For example, a 20° C change in temperature would lead to a 11% error in friction loss if viscosity changes are ignored. In spite of the accuracy disadvantages of empirical equations, engineers continue to use them.

Simple Combination Equation

A combination of the Blasius and the D-W equation results in an equation quite similar to the Hazen-Williams equation. Recognizing that the Reynolds number is given by

$$\mathsf{R} = \frac{d \cdot v}{\nu} \quad \text{(4)}$$

where d = the diameter of the pipe (L); v = the mean velocity (L/T); and ν = the viscosity of the fluid (L^2/T), and that

$$Q = \frac{\pi d^2}{4} \cdot v \quad \text{(5)}$$

the following equation can be derived:

$$h_L = \frac{8(4)^b}{\pi^{(2+b)}} l \left(\frac{a}{g}\right) \nu^{-b} \frac{Q^{2+b}}{d^{5+b}} \quad \text{(6)}$$

where h_L = head loss (L); Q = flow rate (L^3/T); and l = pipe length (L).

The values of a and b as determined by Blasius and recently confirmed for PVC pipe by von Bernuth and Wilson (1989) are a = 0.316 and b = −0.25. The following equation results:

$$h_L = K l \nu^{0.25} Q^{1.75} d^{-4.75} \quad \text{(7)}$$

TABLE 1. Corrections to Friction Loss for Deviation from Standard Temperature (20° C or 70° F)

Standard International Units			English Units		
Deviation (° C) (1)	K' ($\times 10^{-4}$) (2)	Correction factor (3)	Deviation (° F) (4)	K' ($\times 10^{-4}$) (5)	Correction factor (6)
−20	8.984	1.155	−38	4.964	1.162
−15	8.629	1.109	−30	4.783	1.120
−10	8.309	1.068	−20	4.589	1.074
−5	8.030	1.032	−10	4.423	1.035
0	7.779	1.000	0	4.272	1.000
+5	7.556	0.971	+10	4.135	0.968
+10	7.352	0.945	+20	4.015	0.940
+20	7.001	0.900	+30	3.904	0.914
+30	6.703	0.862	+40	3.806	0.891
+40	6.449	0.829	+50	3.720	0.871
+50	6.231	0.801	+60	3.640	0.852
+60	6.037	0.776	+70	3.566	0.835
+70	5.873	0.755	+80	3.498	0.819
+80	5.724	0.736	+90	3.434	0.804
—	—	—	+100	3.376	0.790
—	—	—	+110	3.317	0.776
—	—	—	+120	3.266	0.765
—	—	—	+130	3.218	0.753
—	—	—	+142	3.165	0.741

where $K = 2.458 \times 10^{-2}$ for S.I. units, and $K = 7.488 \times 10^{-3}$ for English units.

The viscosity term can be removed from the calculation by assuming a standard temperature (i.e., 20° C or 70° F) and correcting for deviations from standard. Eq. 7 becomes

$$h_L = K' l Q^{1.75} d^{-4.75} \quad \text{(8)}$$

where $K' = 7.779 \times 10^{-4}$ for S.I. units, and $K' = 4.272 \times 10^{-4}$ for English units.

For deviations from standard temperature, Table 1 can be used. Either substitute the listed K' in Eq. 8 or multiply the results of Eq. 8 with standard temperature by the correction factor in Table 1. It is noteworthy that the correction factor represents the error introduced by failing to correct for viscosity changes due to temperature.

Conclusions

The insertion of the Blasius friction factor into the Darcy-Weisbach (D-W) equation results in a combined equation with the following advantages:

1. It is theoretically sound and dimensionally homogeneous. Both the Blasius and the D-W equations have theoretical bases.
2. It is very accurate for plastic pipe when Reynolds numbers are less than 100,000. The Reynolds number limit is nonrestrictive for irrigation-system design using pipes smaller than 80 mm.
3. It is conveniently written in readily available terms: flow rate, length, and diameter.
4. It can be easily corrected for viscosity changes directly or by referring to the included table of correction factors.

Appendix I. References

Barenblatt, G. I., and Monin, A. S. (1979). "Similarity laws for turbulent stratified shear flows." *Arch. Ration. Mech. Anal.*, 70(4), 307–317.

Blasius, H. (1913). "Das Aehnlichkeitsgesetz bei Reibungsavorgangen in Flussigkeiten," *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens,* 131(1), (in German).

Schlichting, H. (1968). *Boundary layer theory.* 6th Ed., McGraw-Hill, New York, N.Y.

Smith, R. V., et al. (1956). "Flow of natural gas through experimental pipe lines and transmission lines." *U.S. Bureau of Mines Monogram 9,* American Gas Association, New York, N.Y.

von Bernuth, R. D., and Wilson, T. (1989). "Friction factors for small diameter plastic pipes." *J. Hydr. Engrg.*, 115(2), ASCE, 183–192.

Watters, G. Z., and Keller, J. (1978). "Trickle irrigation tubing hyraulics." *ASAE Paper #78-2015,* American Society of Agricultural Engineers, St. Joseph, Mo.

Appendix II. Notation

The following symbols are used in this paper:

a = dimensionless coefficient;

b = dimensionless coefficient;
d = pipe diameter (L);
f = dimensionless friction factor;
g = gravitational constant (L/T^2);
h_L = head loss (L);
K = coefficient (T^2/L^2);
l = pipe length (L);
Q = flow rate (L^3/T);
R = Reynolds number;
v = mean velocity in pipe (L/T); and
ν = kinematic viscosity (L^2/T).

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