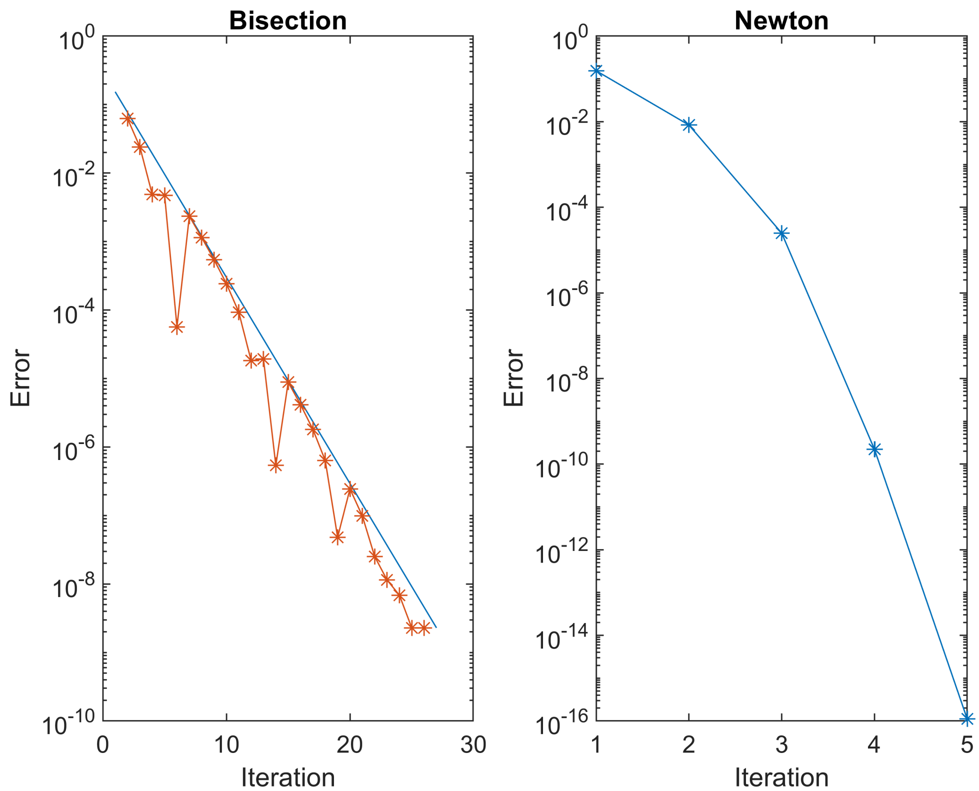
Amar Ramdas - 4461487 - OVM 3

Exercise 1:

```
exact = fzero(inline('0.5*(1+erf(x/sqrt(2))) + exp(x)- 2'), 1);
x0 = 1;
x = x0;
xdiff = 1;
k = 1;
kmax = 100;
tol = 1e-8;

for i=1:20
    dotline(i) = 10*1/2^i;
    nana(i) = f(i-10) - 2/3;
end

% newton
while (xdiff >= tol && k< kmax)
    Fval = 0.5*(1+erf(x/sqrt(2))) + exp(x) - 2;
    Fprime = exp(-0.5*x^2)/sqrt(2*pi) + exp(x);
    increment = Fval/Fprime;
    x=x- increment;
    xnewton(k) = x;
    newterr(k) = abs(xnewton(k)-exact);
    k = k+1;
    xdiff = abs(increment);
end


S = 5290.36; E = 5400; T = 0.211; tau = T; r=0.05;
%starting values
sigmahat = sqrt(2*abs( (log(S/E) + r*T)/T ) );
tol = 1e-8;
sigma_1 = 0;
sigma_2 = 2*sigmahat;
k = 1;
kmax = 100;
sigmadiff=1;
number_itterations=0;
%bisection
while (sigmadiff >= tol && k < kmax)
    d1_1 = (log(S/E) + (r + 0.5*sigma_1^2)*(tau))/(sigma_1*sqrt(tau));
    d2_1 = d1_1 - sigma_1*sqrt(tau);
    N1_1 = 0.5*(1+erf(d1_1/sqrt(2)));
    N2_1 = 0.5*(1+erf(d2_1/sqrt(2)));
    C_1 = S*N1_1-E*exp(-r*(tau))*N2_1 - true;
    d1_2 = (log(S/E) + (r + 0.5*sigma_2^2)*(tau))/(sigma_2*sqrt(tau));
    d2_2 = d1_2 - sigma_2*sqrt(tau);
    N1_2 = 0.5*(1+erf(d1_2/sqrt(2)));
    N2_2 = 0.5*(1+erf(d2_2/sqrt(2)));
```

```
    C_2 = S*N1_2-E*exp(-r*(tau))*N2_2- true;
    sigmadiff=abs(sigma_1-sigma_2);
    sigma_mid=(sigma_1+sigma_2)/2;
    d1_mid = (log(S/E) + (r + 0.5*sigma_mid^2)*(tau))/(sigma_mid*sqrt(tau));
    d2_mid = d1_mid - sigma_mid*sqrt(tau);
    N1_mid = 0.5*(1+erf(d1_mid/sqrt(2)));
    N2_mid = 0.5*(1+erf(d2_mid/sqrt(2)));
    C_mid = S*N1_mid-E*exp(-r*(tau))*N2_mid - true;
    if C_mid*C_1<0
        sigma_2=sigma_mid;
    else
        sigma_1=sigma_mid;
    end
    impvol(k)=(sigma_1+sigma_2)/2 ;
    k=k+1;
    number_itterations=number_itterations+1;
end

iteration=1:number_itterations;
error=zeros(1,k-1);
for i=2:(k-1)
    error(i)=abs(impvol(i)-impvol(k-1));

end
X=(2*sigmahat)*((2.^(iteration+1)).^-1);

figure;
subplot(1,2,1)
plot(iteration,X)
hold on
plot(iteration,error,'*-')
set(gca,'Yscale', 'log');%log scale
ylabel('Error')
xlabel('Iteration')
title('Bisection')
hold on
subplot(1,2,2)
semilogy(newterr, '*-')
ylabel('Error')
xlabel('Iteration')
title('Newton')
```



Exercise 2:

a) Solve exercise P14.1 from Higham's book (page 138).

```
%CH14 Program for Chapter 14
% Computes implied volatility for a European call
%%%%%%%%%%% parameters %%%%%%%%%%
r=0.03; S = 2; E = 2; T = 3; tau = T; sigma_true = 0.3;
[~, ~, P_true, ~] = ch08(S,E,r,sigma_true,tau);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%starting value
sigmahat = sqrt(2*abs( (log(S/E) + r*T)/T ) );

%%%%%% Newton's method %%%%%
tol = 1e-8;
sigma = sigmahat;
sigmadiff = 1;
k = 1;
kmax = 100;
while (sigmadiff >= tol && k< kmax)
    [C, ~, Cvega, P, ~, Pvega] = ch10(S,E,r,sigma,tau);
    increment = (P-P_true)/Pvega;
    sigma = sigma - increment;
    k = k+1;
    sigmadiff = abs(increment);
end
```

```
sigma
```

```
sigma = 0.3000
```

Check whether the Newton-Raphson method also performs well for deep in-the-

money options. Give arguments for your observations

```
%Repeating with very large E
%%%%%%%%%%% parameters %%%%%%%%%%
r=0.03; S = 2; E = 200000; T = 3; tau = T; sigma_true = 0.3;
[C_true, Cdelta, P_true, Pdelta] = ch08(S,E,r,sigma_true,tau);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%starting value
sigmahat = sqrt(2*abs( (log(S/E) + r*T)/T ) );

%%%%%% Newton’s method %%%%%
tol = 1e-8;
sigma = sigmahat;
sigmadiff = 1;
k = 1;
kmax = 100;
while (sigmadiff >= tol && k< kmax)
    [C, Cdelta, Cvega, P, Pdelta, Pvega] = ch10(S,E,r,sigma,tau);
    increment = (P-P_true)/Pvega;
    sigma = sigma - increment;
    k = k+1;
    sigmadiff = abs(increment);
end
sigma
```

```
sigma = 0.8946
```

The sigma changes meaning the volatility/variance increases and it's not the same as sigma_true anymore.

b) Acquire some real option data, either electronically or via a news paper, of a company whose name starts with your initial or the first letter of your surname and create a figure like Figure 14.2. Plot and discuss the implied volatility as the strike price varies.

Got AEX data from euronext.

```
r=0.05;
% sigmahat = sqrt(2*abs((log(S)/E + r(T-t))/(T-t)));

strikeprices = [516.000, 517.000, 518.000, 519.000];
bidprice = [4.150, 3.450, 2.900, 2.4000]
```

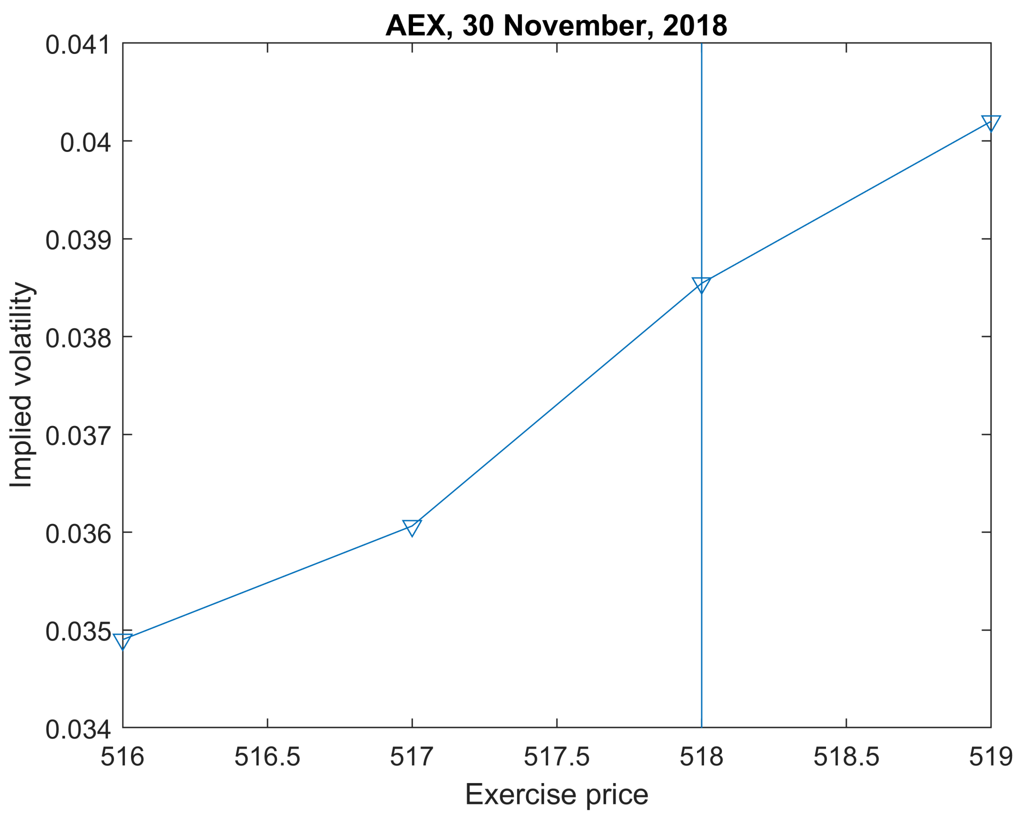
```
bidprice = 1×4
    4.1500    3.4500    2.9000    2.4000
```

```
timemonths = [0.75, 0.75, 0.75, 0.75];
timeyrs = timemonths./12;
```

```
Volatility = blsimpv(518, strikeprices, r, timeyrs, bidprice, 'Limit', 0.5)
```

```
Volatility = 1×4
    0.0349    0.0361    0.0385    0.0402
```

```
figure;
plot(strikeprices, Volatility, 'v-')
xlabel('Exercise price')
ylabel('Implied volatility')
title('AEX, 30 November, 2018')
SP=518;
hold on;
line([SP SP], [0.034 0.041]);
hold off;
```



The volatility seems to increase which seems to be a bit off, however this might be caused by incorrect estimation of the strike date (which is in three weeks) (by converting it to a scalar), as changing these gives different results.

Exercise 3:

Solve Exercise 14.2 from Higham's book page 13

See written part.

```
function y = f(x)
y = N(x) - 2/3;
end

function l = N(w)
l = 1/sqrt(2*pi)*integral(@(x) exp(-(x.^2)/2),-inf, w);
end

function [C, Cdelta, P, Pdelta] = ch08(S,E,r,sigma,tau)
% Program for Chapter 8
% This is a MATLAB function
%
% Input arguments: S = asset price at time t
% E = Exercise price
% r = interest rate
% sigma = volatility
% tau = time to expiry (T-t)
%
% Output arguments: C = call value, Cdelta = delta value of call
% P = Put value, Pdelta = delta value of put
%
% function [C, Cdelta, P, Pdelta] = ch08(S,E,r,sigma,tau)
if tau > 0
    d1 = (log(S/E) + (r + 0.5*sigma^2)*(tau))/(sigma*sqrt(tau));
    d2 = d1 - sigma*sqrt(tau);
    N1 = 0.5*(1+erf(d1/sqrt(2)));
    N2 = 0.5*(1+erf(d2/sqrt(2)));
    C = S*N1-E*exp(-r*(tau))*N2;
    Cdelta = N1;
    P=C+ E*exp(-r*tau) - S;
    Pdelta = Cdelta - 1;
else
    C = max(S-E,0);
    Cdelta = 0.5*(sign(S-E) + 1);
    P = max(E-S,0);
    Pdelta = Cdelta - 1;
end
end

function [C, Cdelta, Cvega, P, Pdelta, Pvega] = ch10(S,E,r,sigma,tau)
% Program for Chapter 10
% This is a MATLAB function
%
% Input arguments: S = asset price at time t
% E = exercise price
% r = interest rate
% sigma = volatility
% tau = time to expiry (T-t)
%
% Output arguments: C = call value, Cdelta = delta value of call
% Cvega = vega value of call
% P = Put value, Pdelta = delta value of put
```

```
% Pvega = vega value of put
%
% function [C, Cdelta, Cvega, P, Pdelta, Pvega] = ch10(S,E,r,sigma,tau)
if tau > 0
    d1 = (log(S/E) + (r + 0.5*sigma^2)*(tau))/(sigma*sqrt(tau));
    d2 = d1 - sigma*sqrt(tau);
    N1 = 0.5*(1+erf(d1/sqrt(2)));
    N2 = 0.5*(1+erf(d2/sqrt(2)));
    C = S*N1-E*exp(-r*(tau))*N2;
    Cdelta = N1;
    Cvega = S*sqrt(tau)*exp(-0.5*d1^2)/sqrt(2*pi);
    P=C+ E*exp(-r*tau) - S;
    Pdelta = Cdelta - 1;
    Pvega = Cvega;
else
    C = max(S-E,0);
    Cdelta = 0.5*(sign(S-E) + 1);
    Cvega = 0;
    P = max(E-S,0);
    Pdelta = Cdelta - 1;
    Pvega = 0;
end
end
```