Increment Theorem:

Let $f(x,\, y)$ **be differentiable at** $(x_0,\, y_0)$**.**

Then we have $f(x_0 + \Delta x,\, y_0 + \Delta y) - f(x_0,\, y_0) = f(x_0,\, y_0)\, \Delta x + fy\,(x_0,\, y_0)\, \Delta y +$ $\varepsilon 1 \Delta x + \varepsilon 2 \Delta y$

where $\varepsilon 1\,(\Delta x,\, \Delta y)\,, \varepsilon 2\,(\Delta x,\, \Delta y) \longrightarrow 0$ **as** $\Delta x \longrightarrow 0$ **and** $\Delta y \longrightarrow 0$**.**

Proof :

Let $H = (\Delta x, \Delta y)$ **Since the function is differentiable at**$(x_0,\, y_0)$**,**

we have $f(x_0 + \Delta x,\, y_0 + \Delta y) - f(x_0,\, y_0) = f(x_0,\, y_0)\, \Delta x + fy\,(x_0,\, y_0)\, \Delta y +$ $\|H\| \varepsilon\,(H)\,, \varepsilon\,(H) \longrightarrow 0 \ as\ H \longrightarrow 0.$

We have to show that $\|H\| \varepsilon\,(H) = \varepsilon 1 \Delta x + \varepsilon 2 \Delta y$ **for some functions** $\varepsilon 1$ **and** $\varepsilon 2$**.**

Note that

$\varepsilon\,(H)\,||H|| = \varepsilon\,(H)\,\|H\|\,(x2 + \Delta y2) = (\Delta x \varepsilon\,(H)\,\|H\|)\,\Delta x + (\Delta y \varepsilon\,(H)\,\|H\|)\,\Delta y$**.**

Define

$\varepsilon 1\,(H) = \Delta x \varepsilon\,(H)\,\|H\|\, and\, \varepsilon 2\,(H) = \Delta y \varepsilon\,(H)\,\|H\|$ **.**

Note that

$|\varepsilon 1\,(H)\,| = |\Delta x \varepsilon(H)\|H\|| \leq |\varepsilon\,(H)\,| \longrightarrow 0\, as\, H \longrightarrow 0.$

Similarly we can show that

$\varepsilon 2\,(H) \longrightarrow 0\, as\, H \longrightarrow 0.$

This proves the result