

Quantum Error Mitigation

[arXiv:2210.00921](https://arxiv.org/abs/2210.00921)

Zero-Noise Extrapolation (**ZNE**)

Zero-noise extrapolation was first introduced in [2, 3] and works by intentionally increasing (scaling) the noise of a quantum computation to then extrapolate back to the zero-noise limit. More specifically, let ρ be a state prepared by a quantum circuit and $E^\dagger = E$ be an observable. We wish to estimate $\text{Tr}[\rho E] \equiv \langle E \rangle$ as though we had an ideal (noiseless) quantum computer, but there is a base noise level γ_0 which prevents us from doing so. For example, γ_0 could be the strength of a depolarizing channel in the circuit. The idea of zero-noise extrapolation is to compute

$$\langle E(\gamma_i) \rangle = \langle E(\lambda_i \gamma_0) \rangle \quad (2)$$

where (real) coefficients $\lambda_i \geq 1$ scale the base noise γ_0 of the quantum computer. After this, a curve is fit to the data collected via Eq. (2) which is then extrapolated to the zero-noise limit. This produces an estimate of the noiseless expectation value $\langle E \rangle$.

Mitiq: [arXiv:2009.04417](https://arxiv.org/abs/2009.04417)

[2] <https://doi.org/10.1103/PhysRevLett.119.180509>

[3] <https://doi.org/10.1103/PhysRevX.7.021050>

4.1.1 Unitary Folding

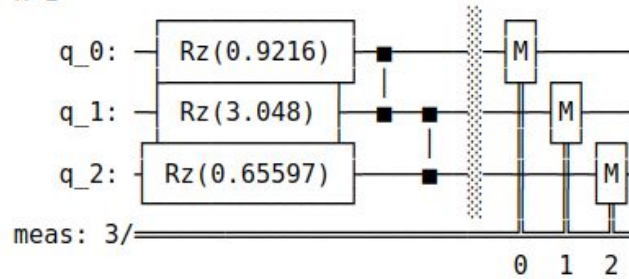
Unitary folding works by mapping gates (or groups of gates) G to

$$G \mapsto GG^\dagger G. \quad (3)$$

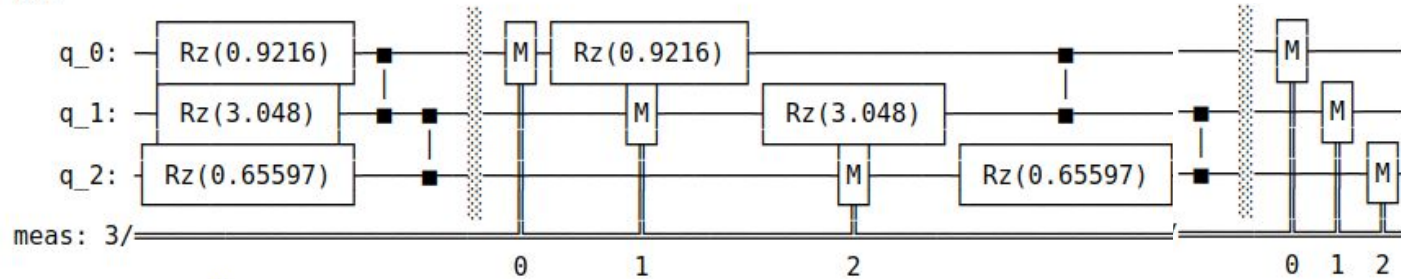
This leaves the ideal effect of the circuit invariant but increases its depth. If G is a gate of the circuit, we refer to the process as *local folding*. If G is the entire circuit, we call it *global folding*.

Zero-Noise Extrapolation (**ZNE**)

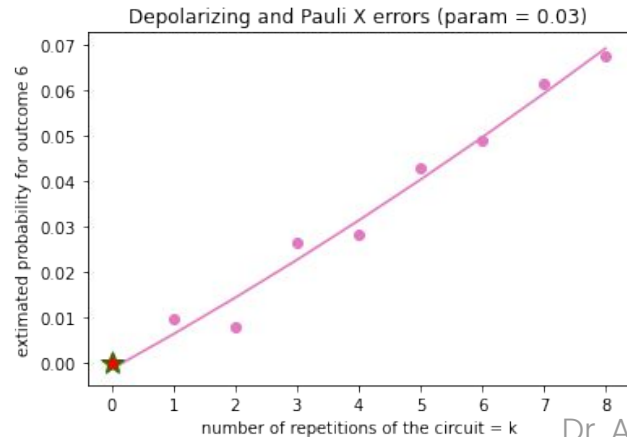
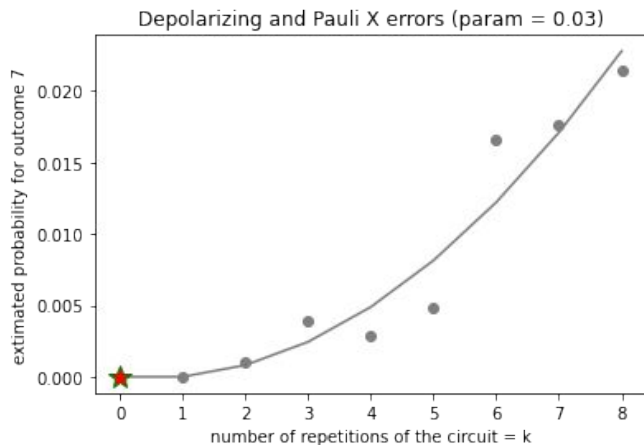
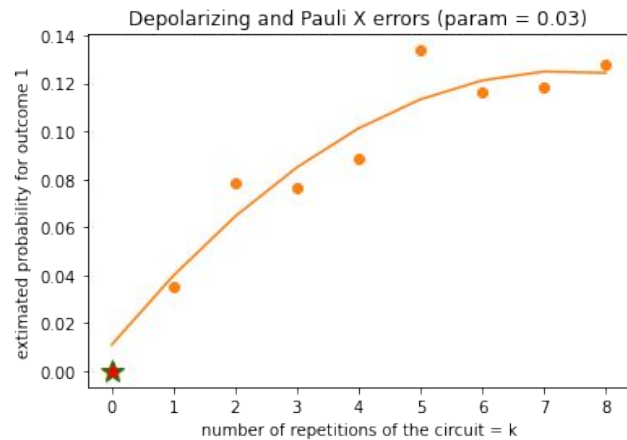
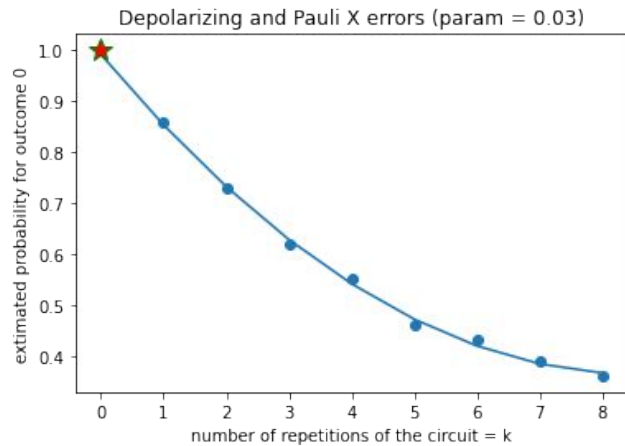
k=1



k=2



Zero-Noise Extrapolation (**ZNE**)



A key step of PEC is to represent each ideal unitary gate \mathcal{G} in a circuit as an average over a set of noisy gates which are physically implementable $\{\mathcal{O}_\alpha\}$, weighted by a real quasi-probability distribution $\eta(\alpha)$:

$$\mathcal{G} = \sum_{\alpha} \eta(\alpha) \mathcal{O}_{\alpha}, \quad (6)$$

where $\sum_{\alpha} \eta(\alpha) = 1$ (trace-preserving condition). The calligraphic operators \mathcal{G} and $\{\mathcal{O}_\alpha\}$ should be considered as linear super-operators acting on density matrices and not on state vectors [2, 4]. If a representation like Eq. (6) is known for each ideal gate of a circuit, then any ideal expectation value can be estimated as a Monte Carlo average over different noisy circuits, each one sampled according to the quasi-probability distributions associated to the ideal gates [2, 4]. The real coefficients $\eta(\alpha)$ which appear in Eq. (6) can be negative for some values of α and, because of this negativity, the required number of Monte Carlo samples can be large [2, 4]. In principle, assuming a perfect tomographic knowledge of the

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[4] <https://doi.org/10.1103/PhysRevX.8.031027>

Clifford Data Regression (**CDR**)

The CDR uses near-Clifford quantum circuit data to learn noise effects an expectation value.

1. Construct the training circuits corresponding to states $\{\rho_i^{\text{train}}, i = 1, \dots, n\}$ by replacing non-Clifford gates in the circuit of interest by Clifford gates.
2. For each training circuit ρ_i^{train} evaluate classically a noiseless expectation value of E , $y_i = \text{Tr} \rho_i E$, and its noisy expectation value x_i using a quantum computer.
3. Fit exact and noisy expectation values of the training circuits $\{(x_i, y_i)\}$ with a linear model $y = ax + b$.
4. Use the fitted model to mitigate $\langle E(\gamma_0) \rangle$

$$\langle E \rangle^{\text{mitigated}} = a \langle E(\gamma_0) \rangle + b.$$

variable noise Clifford Data Regression (**vnCDR**)

1. Prepare the training circuits $\{\rho_i^{\text{train}}, i = 1, \dots, n\}$ using Clifford substitutions, following the same procedure for CDR.
2. For each training circuit ρ_i^{train} evaluate classically a noiseless expectation value of E , $y_i = \text{Tr} \rho_i E$, and its noisy expectation values $x_{i,l}$ using a quantum computer with several noise rates $\lambda_l \gamma_0$, $\lambda_l \geq 1, l = 1, \dots, m$.
3. Fit the expectation values of the training circuits with a linear ansatz given by $y = f(x_1, x_2, \dots, x_m)$. Where

$$f(x_1, x_2, \dots, x_m) = \sum_{l=1}^m x_l a_l + b . \quad (7)$$

4. Use the fitted ansatz to correct the noisy expectation values of E :

$$\langle E \rangle^{\text{mitigated}} = f(\langle E(\lambda_1 \gamma_0) \rangle, \langle E(\lambda_2 \gamma_0) \rangle, \dots, \langle E(\lambda_m \gamma_0) \rangle) .$$

Thank you!