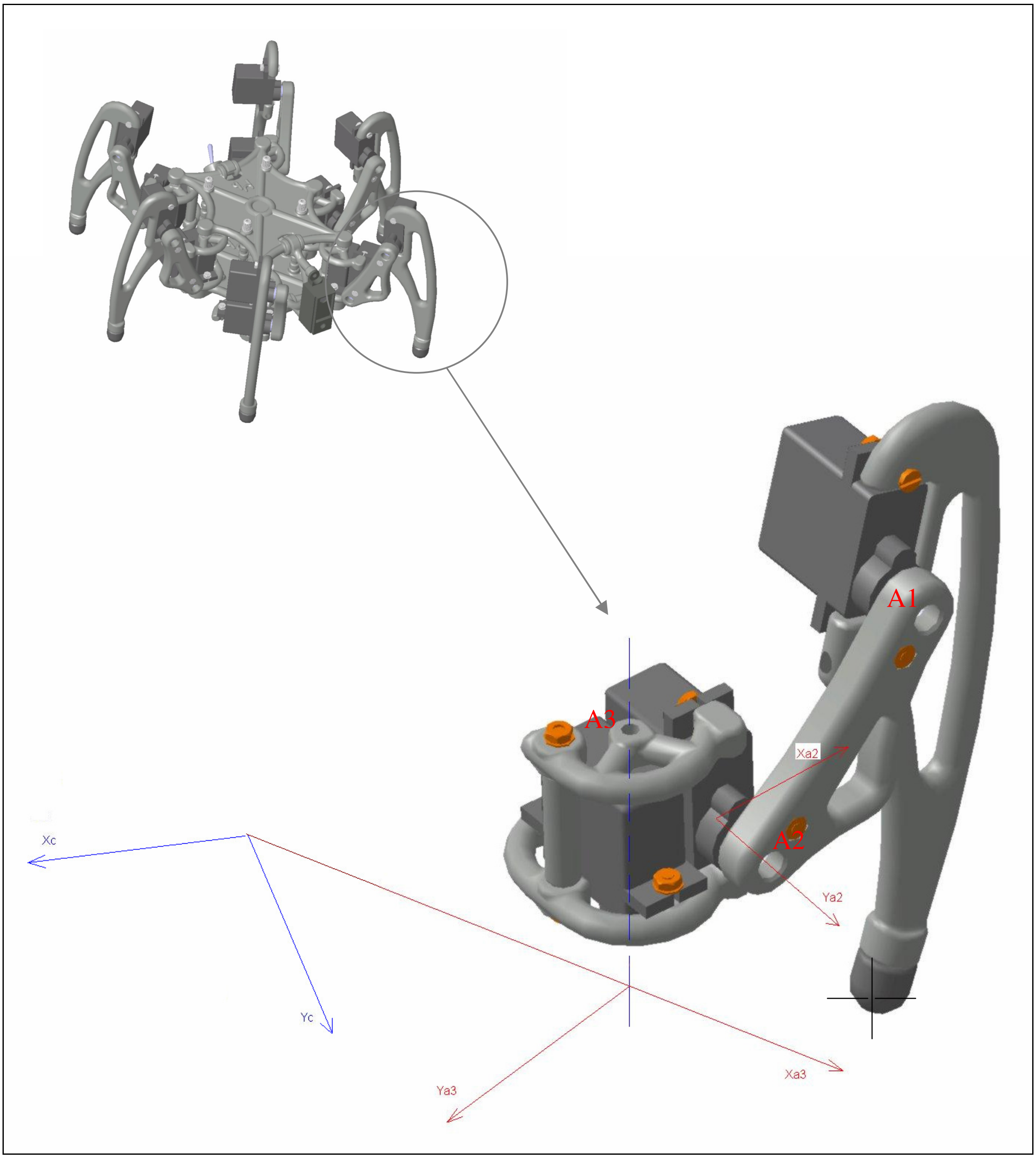
Equations for Hexapod robot control

- ✓ Determine joints angles according to the position of leg tip.
- ✓ A method for linear interpolation (straight walk).
- ✓ A method for circular interpolation (curved walk).
- ✓ A method for body inclination (tilt).

Determine joints angles according to the position of leg tip.

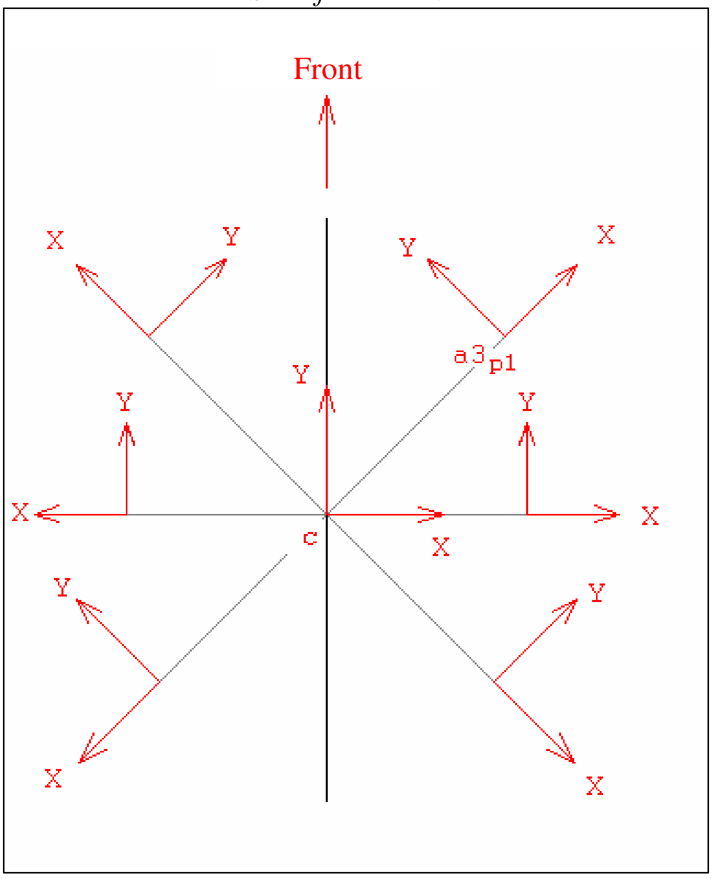
The different coordinate systems



We want to drive the tip of a leg by its coordinates in a Cartesian coordinate system.
For reasons of simplicity, we express the coordinates in a fixed coordinate system centred on the body of the hexapod.
To calculate the angles to be applied to the three joints of the leg we have to use different coordinate systems, either fixed or movable relative to the body.
All angle measurements in formulas are expressed in radians.

- Coordinate systems fixed to the body **c** et **a3** :

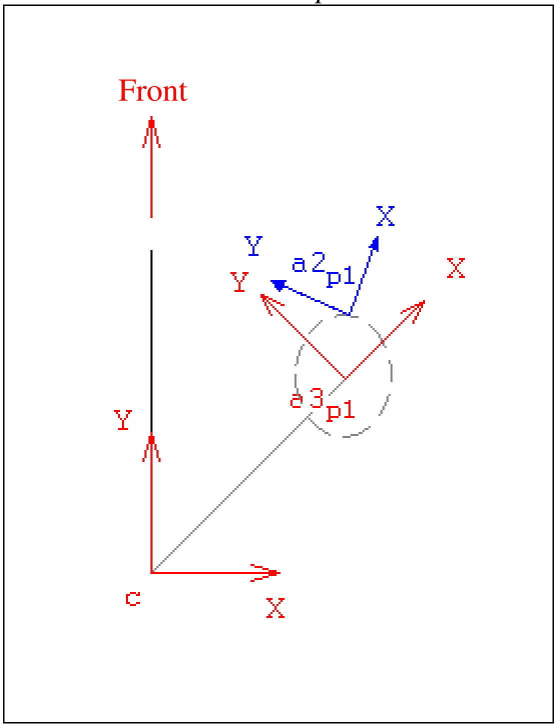
Seen from above



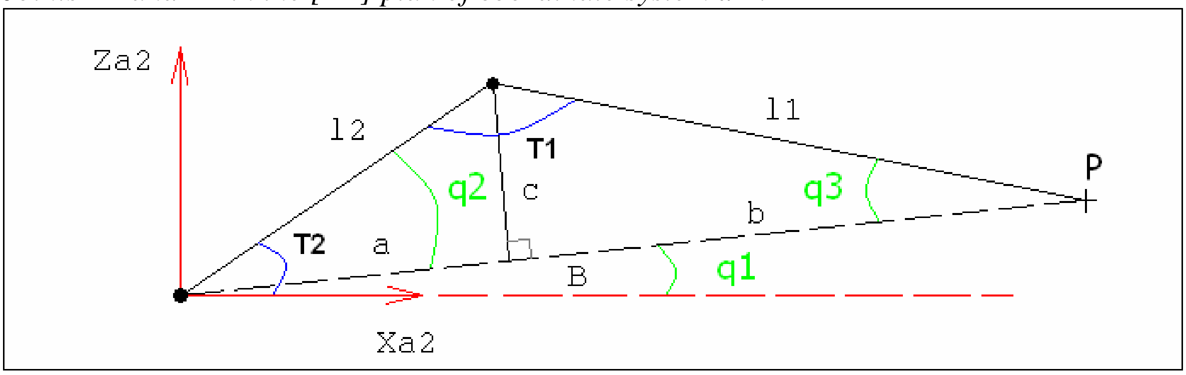
System **c** is centred on the robot body. The plan [XY] of the system coincides with the underside of the body (the lowest point).
Systems **a3px** are centred on **A3** joints of each leg.

- Coordinate systems movable relative to the body **a2**:

Seen top



*Joints A2 and A1 in the [XZ] plan of coordinate system **a2** :*



Given :

- l1et l2 respectively the lengths of the extreme and middle phalanxes.
- T1 angle between the extreme phalanx and middle phalanx.
- T2 angle between X axis and the middle phalanx.
- q1 angle between the axis X and B imaginary line passing through the joint centre A2 and the point P.
- q2 angle between B and the middle phalanx.
- q3 angle between B and the extreme phalanx.

$$B = a + b$$
$$B^2 = Pxa2^2 + Pza2^2$$
$$\cos(q2) = \frac{a}{l2}$$
$$T2 = q1 + q2$$

Find **T2**

$$a^2 = l2^2 - c^2$$
$$b^2 = l1^2 - c^2$$
$$a^2 - b^2 = l2^2 - l1^2$$
$$a^2 - (B - a)^2 = l2^2 - l1^2$$
$$a^2 - (B^2 + a^2 - 2aB) = l2^2 - l1^2$$
$$-B^2 + 2aB = l2^2 - l1^2$$

$$a = \frac{B^2 + l2^2 - l1^2}{2B}$$

$$\left.\begin{aligned} \cos(q2) &= \frac{B^2 + l2^2 - l1^2}{2 * l2 * B} \\ q1 &= \arctan\left(\frac{Pza2}{Pxa2}\right) \end{aligned}\right| \longrightarrow \boxed{T2 = \arccos\left(\frac{B^2 + l2^2 - l1^2}{2 * l2 * B}\right) + \arctan\left(\frac{Pza2}{Pxa2}\right)}$$

Find **T1**

$$T1 = \pi - q2 - q3$$

$$T1 = \pi - \arccos(\frac{a}{l2}) - \arccos(\frac{b}{l1})$$

$$T1 = \pi - (-\arcsin(\frac{a}{l2}) + \arcsin(\frac{b}{l1}) + \pi)$$

$$T1 = \arcsin(\frac{a}{l2}) - \arcsin(\frac{b}{l1})$$

$$\boxed{\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)}$$

$$\cos(T1) = \cos(\arcsin(\frac{a}{l2}) * \arcsin(\frac{b}{l1})) + \frac{ab}{l1l2}$$

$$\cos(T1) = \sqrt{1-\frac{a^2}{l2^2}} * \sqrt{1-\frac{b^2}{l1^2}} + \frac{ab}{l1l2}$$

$$\cos(T1) = \sqrt{\frac{l2^2-a^2}{l2^2}} * \sqrt{\frac{l1^2-b^2}{l1^2}} + \frac{ab}{l1l2}$$

$$\cos(T1) = \sqrt{\frac{c^2}{l2^2}} * \sqrt{\frac{c^2}{l1^2}} + \frac{ab}{l1l2}$$

$$\cos(T1) = \frac{c}{l2} * \frac{c}{l1} + \frac{ab}{l1l2}$$

$$\cos(T1) = \frac{c^2+ab}{l1l2} = \frac{l2^2-a^2+ab}{l1l2}$$

$$\cos(T1) = \frac{l2^2-(a^2-ab)}{l2^2} = -\frac{-l2^2+(a(a+b))}{l1l2}$$

$$\cos(T1) = -\frac{-l2^2+aB}{l1l2} = \frac{l2^2}{l1l2} - \frac{aB}{l1l2}$$

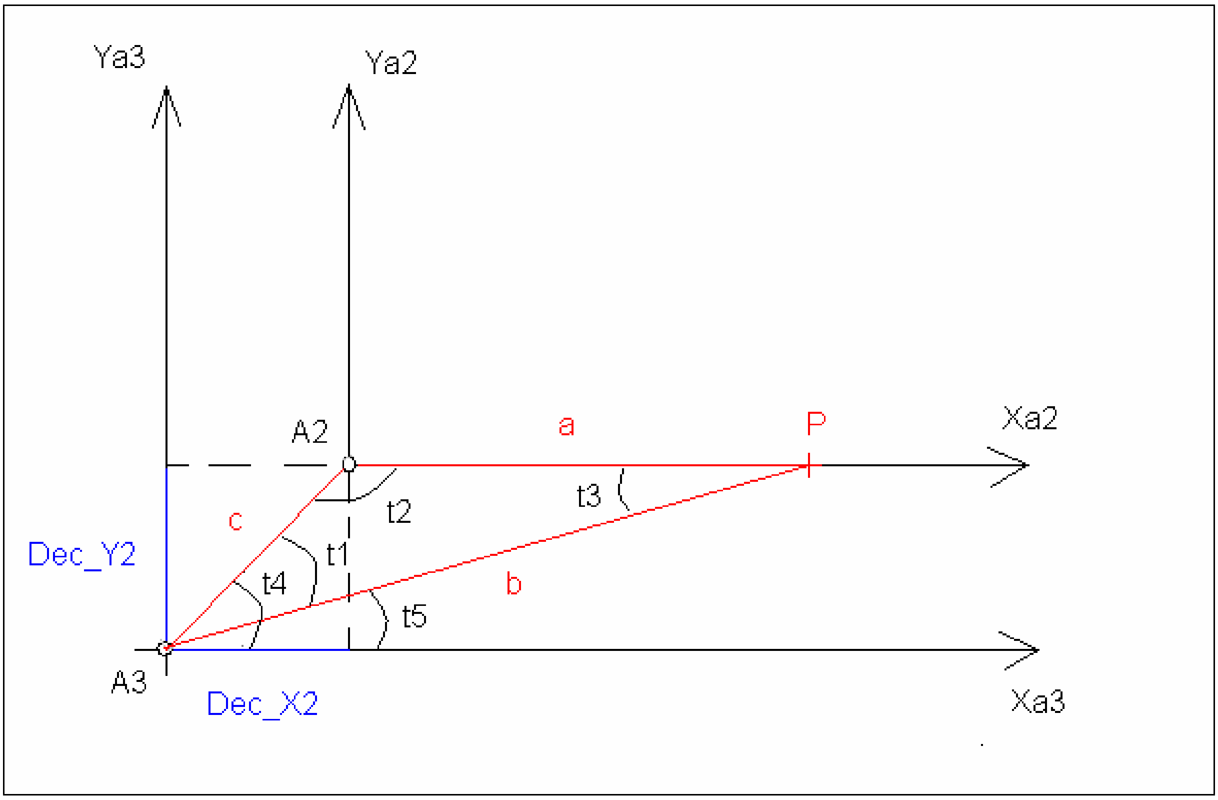
$$\boxed{a = \frac{B^2+l2^2-l1^2}{2B}}$$

$$\cos(T1) = -\frac{B^2+l2^2-l1^2}{2l1l2} + \frac{2l2^2}{2l1l2}$$

$$\cos(T1) = \frac{2l2^2-B^2-l2^2+l1^2}{2l1l2} = \frac{l2^2+l1^2-B^2}{2l1l2}$$

$$\boxed{T1 = \arccos(\frac{l1^2+l2^2-B^2}{2l1l2})}$$

Find joint angle **A3 (T3)**



- **Dec_X2** is the offset on **X** axis, between joints A**3** et **A2**.
- **Dec_Y2** is the offset on **Y** axis, between joints A**3** et **A2**.
- **Dec_Z2** is the offset on **Z** axis, between joints A**3** et **A2 (=0)**.
- Given **a** the projection on **Xa2** of the imaginary line B (seen previously).

$$c = \sqrt{Dec\_X2^2 + Dec\_Y2^2}$$

$$t2 = \frac{\pi}{2} + \arctan(\frac{Dec\_X2}{Dec\_Y2})$$

$$b = \sqrt{Pxa3^2 + Pya3^2}$$

We know that :

$$\frac{a}{\sin(t1)} = \frac{b}{\sin(t2)} = \frac{c}{\sin(t3)}$$ *Sinus proportionality rule.*

So :

$$t3 = \arcsin(\frac{c \times \sin(t2)}{b})$$

$$t1 = \pi - t2 - \arcsin(\frac{c \times \sin(t2)}{b})$$

$$a = \sqrt{b^2 - c^2 \times \sin(t2)^2} + c \times \cos(t2)$$

$$\boxed{Pxa2 = Bx = \sqrt{b^2 - c^2 \times \sin(t2)^2} + c \times \cos(t2)}$$

$$\boxed{Pya2 = 0}$$

$$t4 = \arctan(\frac{Dec\_Y2}{Dec\_X2})$$

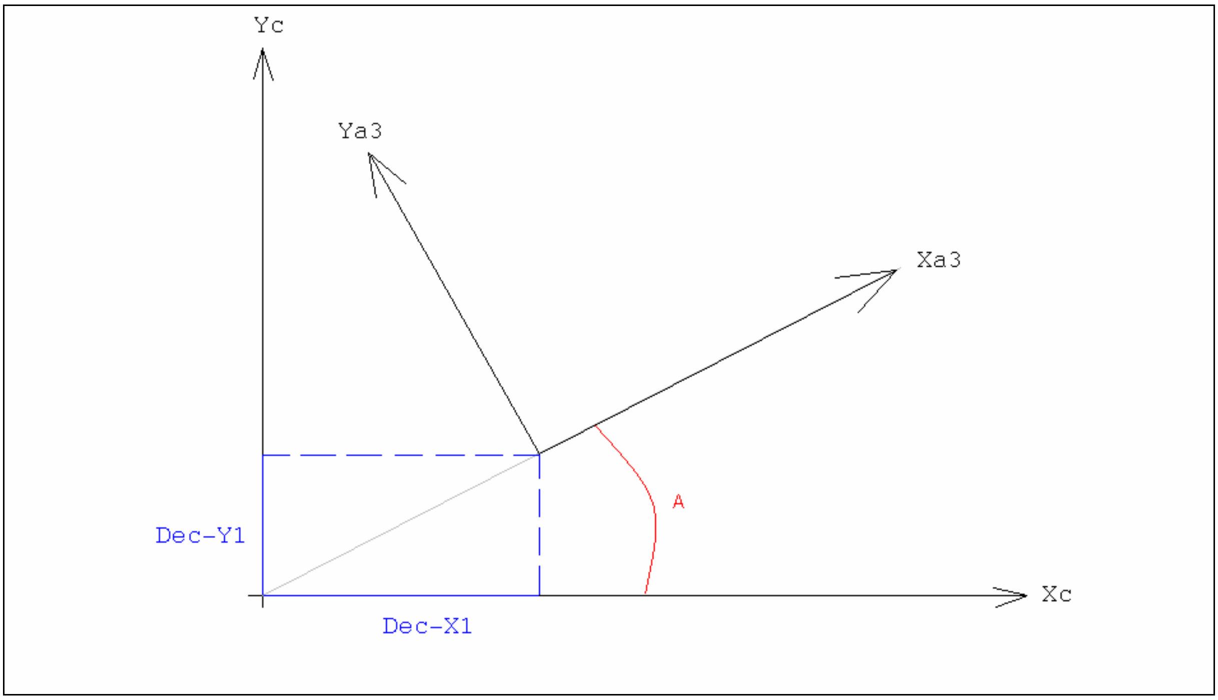
$$t1 = \arcsin(\frac{a \times \sin(t2)}{b})$$

$$t5 = \arctan(\frac{Pya3}{Pxa3})$$

Given T3 the angle of the joint A3, we have :

$$\boxed{T3 = t5 + t1 - t4}$$

Change of coordinate system (system **c** to **a3**)



- System **a3** is centred on **A3** joint.
- System **c** is centred on the hexapod body (**Y** axis pointing to the front of the hexapod).

- **A** is the angle between **X** axis of the body centred system and the **X** axis of the **A3** joint centred system**.**

- **Dec_X1** is the abscissa of **a3** system centre, expressed in the **c** system.
- **Dec_Y1** is the ordinate of **a3** system centre, expressed in the **c** system.
- **Dec_Z1** is the altitude of **a3** system centre, expressed in the **c** system **(=0)**.

A, Dec_X1, Dec_Y1 et **Dec_Z1** are not the same for all legs.

$$Pxa3 = Pxc \times \cos(A) + Pyc \times \sin(A) - \sqrt{DEC\_X1^2 + DEC\_Y1^2}$$
$$Pya3 = -(Pyc \times \cos(A) - Pxc \times \sin(A))$$

Sum up

Coordinate system **c** to **a3** :

$$Pxa3 = Pxc \times \cos(A) + Pyc \times \sin(A) - \sqrt{DEC\_X1^2 + DEC\_Y1^2}$$
$$Pya3 = -(Pyc \times \cos(A) - Pxc \times \sin(A))$$
$$Pza3 = Pzc - Dec\_Z1$$

Coordinate system **a3** to **a2** :

$$b = \sqrt{Dec\_X2^2 + Dec\_Y2^2}$$
$$c = \sqrt{Pxa3^2 + Pya3^2}$$
$$t2 = \frac{\pi}{2} + \arctan(\frac{Dec\_X2}{Dec\_Y2})$$
$$Pxa2 = Bx = \sqrt{b^2 - c^2 \times \sin(t2)^2} + c \times \cos(t2)$$
$$Pya2 = 0$$
$$Pza2 = Pz3 - Dec\_Z2$$

Joints angles :

$$B = \sqrt{Pxa2^2 + Pza2^2}$$

$$T1 = \arccos(\frac{l1^2 + l2^2 - B^2}{2l1l2})$$

$$T2 = \arccos(\frac{B^2 + l2^2 - l1^2}{2 * l2 * B}) + \arctan(\frac{Pza2}{Pxa2})$$

$$T3 = \arctan(\frac{Pya3}{Pxa3}) + \arcsin(\frac{a \times \sin(\frac{\pi}{2} + \arctan(\frac{Dec\_X2}{Dec\_Y2}))}{\sqrt{Pxa3^2 + Pya3^2}}) - \arctan(\frac{Dec\_Y2}{Dec\_X2})$$

A method for linear interpolation (straight walk).

Straight walking consist in moving with a given distance in a given direction with a given step.

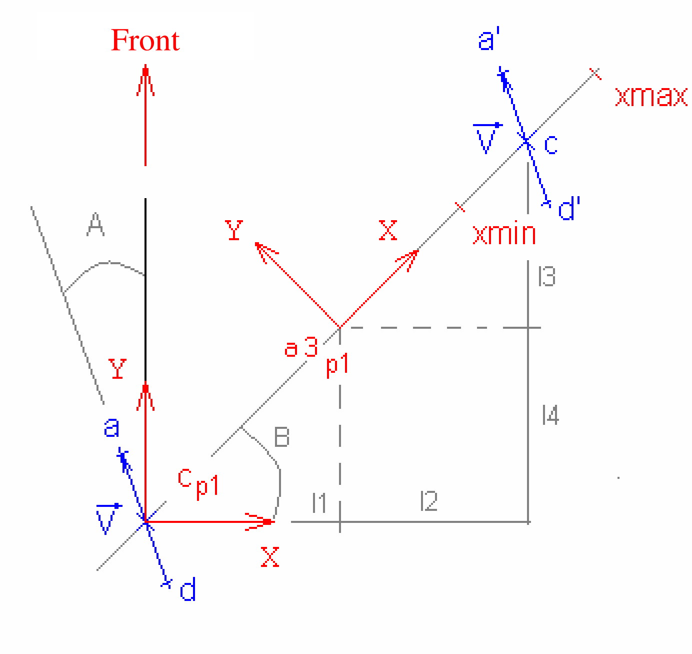
The three method parameters are :

L : The distance.
A : Angle between walking direction and the longitudinal axis of the hexapod (Yc).
P : The unitary step of the leg (linear distance made in one walking cycle).

Another parameter is also used :

G : The ground clearance (body to ground distance).

Example for leg n°1



Given $\vec{V}$ the vector which norm is equal to **L**, and which angle with **Yc** axis is **A** .

$$l1 = Dec\_X1$$

$$l2 = \cos(B) \times (x\min + \frac{x\max - x\min}{2})$$

$$l3 = \sin(B) \times (x\min + \frac{x\max - x\min}{2})$$

$$l4 = Dec\_Y1$$

Coordinates of starting and arrival points **d** et **a** expressed in **cp1** system.

$$ax = -\sin(A) \times \frac{L}{2}$$
$$ay = \cos(A) \times \frac{L}{2}$$
$$dx = -ax$$
$$dy = -ay$$

Coordinates of starting and arrival points **d'** et **a'** translated on **c** point expressed in **cp1** system.

$$a'x = ax + l1 + l2$$
$$a'y = ay + l3 + l4$$
$$d'x = dx + l1 + l2$$
$$d'y = dy + l3 + l4$$

Then, we are able to calculate the values of the different joints for starting and arrival point thanks to the previous formulas.

<u>*Remarks :*</u>

Leg movement vector points in the opposite direction to the wanted body movement vector.

Number of cycles to be performed to reach the distance is equal to $\frac{L}{P}$

In **cp1** system Z coordinate of the leg tip is equal to –G.

A method for circular interpolation (curve walk).

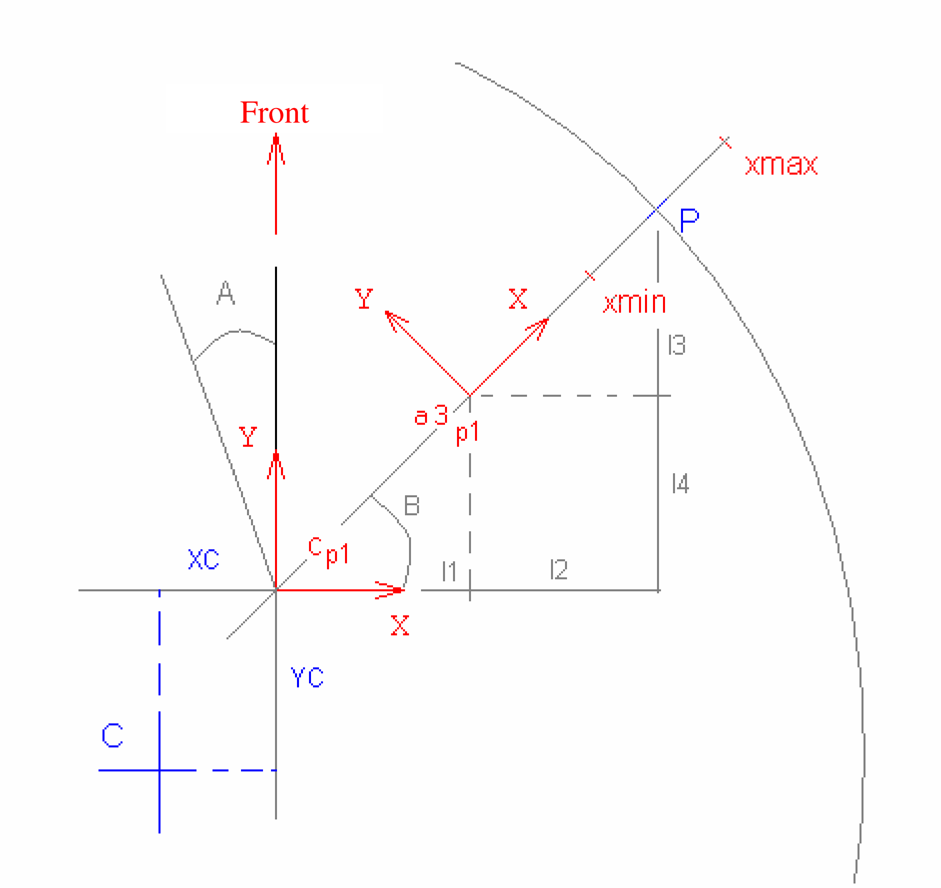
Curve walking consists in moving of a given angle on a circle of given centre with a given step.

The three method parameters are :

A : Rotation angle.
C(xc;yc) : Coordinates of the rotation centre.
P : Unitary step of the leg (linear distance made in one walking cycle).

Another parameter is also used :

G : The ground clearance (body to ground distance).



$$l1 = Dec\_X1$$

$$l2 = \cos(B) \times (x\min + \frac{x\max - x\min}{2})$$

$$l3 = \sin(B) \times (x\min + \frac{x\max - x\min}{2})$$

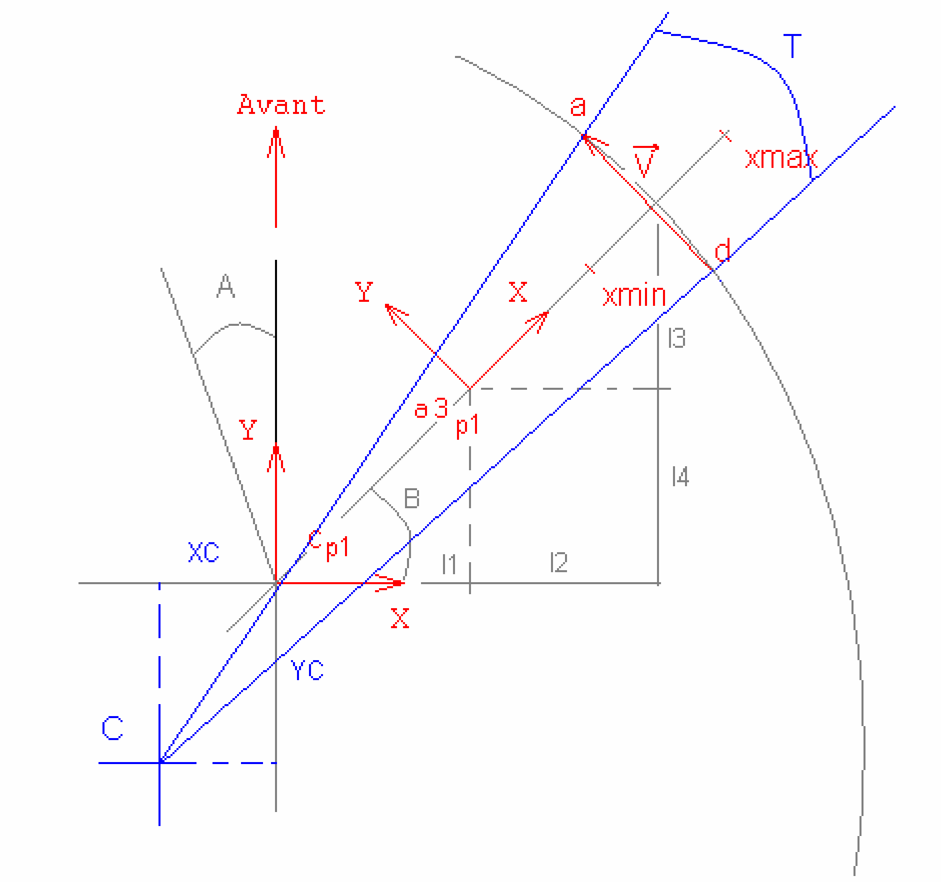
$$l4 = Dec\_Y1$$

Compared to straight walking not all legs have the same movement vector. For a given unitary angular step, the farthest leg from the rotation centre will describe the longest distance in one walking cycle (Equal to **P** in our case).

At first we have to determine which leg is the farthest from the rotation centre.
For each leg we calculate the length of CP segment :

$$CP = \sqrt{(l1 + l2 - XC)^2 + (l3 + l4 - YC)^2}$$

For the leg which have the maximal CP, we calculate the angle **T** represented by points **a** and **d** spaced of a distance **P**, belonging to the circle.



$$Circ = 2 \times \pi \times CP$$

$$T = \frac{P \times 2 \times \pi}{Circ}$$

$$T = \frac{P}{CP}$$

T will be the unitary angular step for all legs in order to respect the unitary linear step **P**.

<u>In the coordinate system centred on the cercle we express :</u>

- The angle between the starting point **d** and **X** axis :

$$Ad = \arctan(\frac{(l3 + l4 - YC)}{(l1 + l2 - XC)}) - \frac{T}{2}$$

- The angle between the arrival point **a** and **X** axis :

$$Aa = \arctan(\frac{(l3 + l4 - YC)}{(l1 + l2 - XC)}) + \frac{T}{2}$$

- **d** and **a** coordinates :

$$Xd = \cos(Ad) \times CP$$
$$Yd = \sin(Ad) \times CP$$
$$Xa = \cos(Aa) \times CP$$
$$Ya = \sin(Aa) \times CP$$

Now we can calculate the coordinates in the body coordinate system of the starting and arrival movement vector points.

$$Xd = (\cos(Ad) \times CP) + XC$$
$$Yd = (\sin(Ad) \times CP) + YC$$
$$Xa = (\cos(Aa) \times CP) + XC$$
$$Ya = (\sin(Aa) \times CP) + YC$$

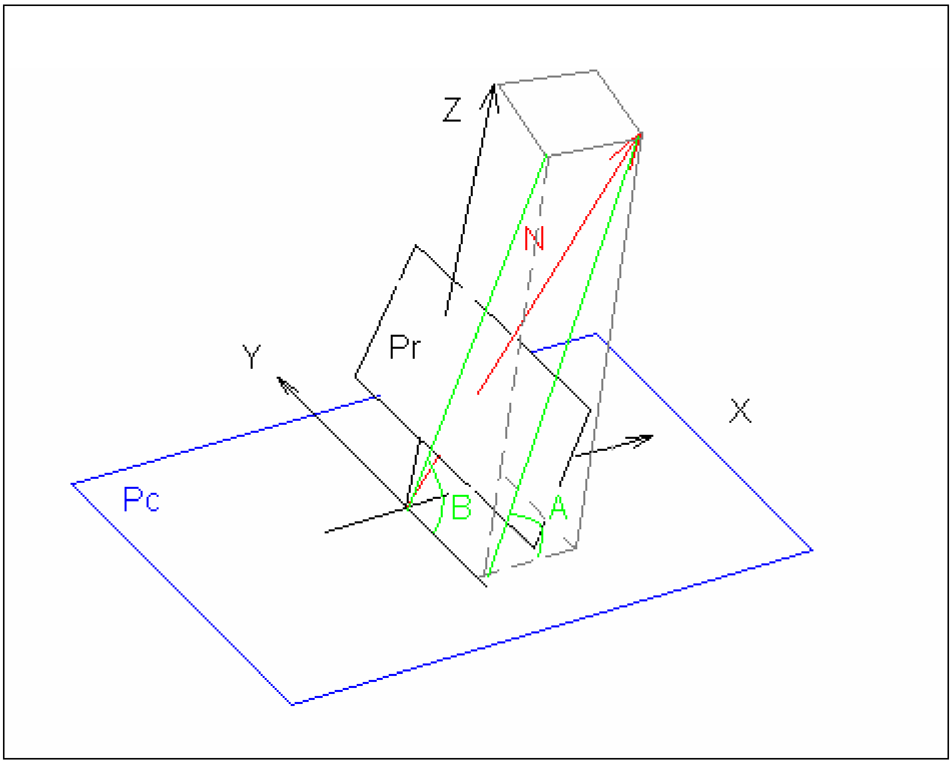
Then, we are able to calculate the values of the different joints for starting and arrival point thanks to the previous formulas.

A method for body inclination (tilt)

Inclination consists in tilting the body of the robot of a given angle A around the longitudinal axis and a given angle B around the transversal axis.
Thus we must have the six leg extremity belonging to the same plane, tilted of the desired value relative to [XY] body plan.

The three method parameters are :

A : rotation angle around Y.
B : rotation angle around X.
G : The ground clearance (body to ground distance).



Calculation of the vector N normal to the Pr plan (tilted plan)

$$\vec{N} = \begin{matrix} X \\ Y \\ Z \end{matrix} \qquad \begin{matrix} X = Z/\tan(A) \\ Y = Z/\tan(B) \\ Z = 1 \end{matrix}$$

Coordinates of the body coordinate system centre projected to the Pr plan, expressed in the body coordinate system.

P = (0;0;-G)

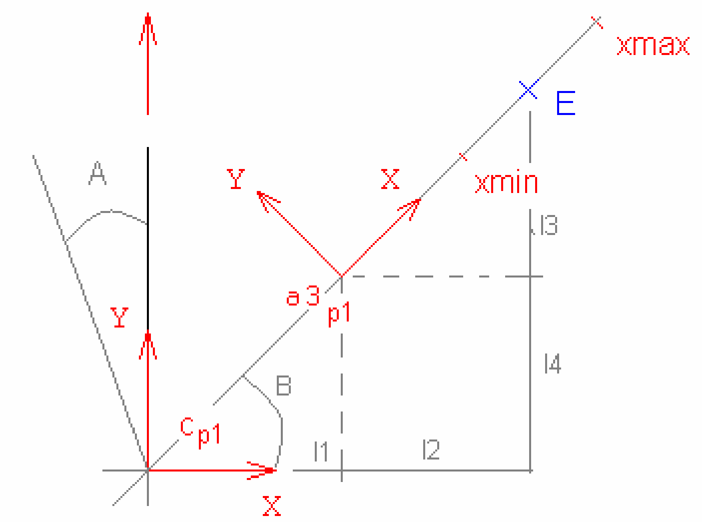
Plane equation.

Given:

N {a,b,c} vector normal to the plan.
A {xa,ya,za} a known point belonging to the plan.
M {x,y,z} a point belonging to the plan.
The plan equation is : ax+by+cz - (axa+bya+cza) = 0

If d=(axa+bya+cza) is the constant part of the equation.
In our case d=-G

Calculation of the coordinates of a leg tip in the body coordinate system.



$$l1 = Dec\_X1$$

$$l2 = \cos(B) \times (x\min + \frac{x\max - x\min}{2})$$

$$l3 = \sin(B) \times (x\min + \frac{x\max - x\min}{2})$$

$$l4 = Dec\_Y1$$

$$Ex = l1 + l2$$

$$Ey = l3 + l4$$

$$Ez = -G - (\frac{Ex}{\tan(A)} - \frac{Ey}{\tan(B)})$$

Then, we are able to calculate the values of the different joints for the starting and arrival points thanks to the previous formulas.