

# Autonomous Navigation of a Small Boat Using IMU/GPS/Digital Compass Integration

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**Abstract**— This article shows how the problem of autonomous navigation of a small boat was formulated and solved. The boat is a catamaran equipped with two water wheels driven by DC motors. A look-up table controller is used to turn on and off the DC motors. Firstly it is shown how the Kalman filter algorithm was applied to estimate in real-time the boat position and heading, using the measurements from a low-cost IMU (Inertial Measurement Unit), a standard GPS receiver and a digital compass. Then a mathematical model of the boat and simulation results for the sensor integration problem and for the boat controller, are discussed. Finally, the article shows how the proposed solution for the autonomous navigation problem was implemented and tested using an embedded computer and the sensors (IMU, GPS receptor and digital compass) aboard the boat.

**Keywords**—autonomous navigation; Kalman filter; Unmanned Surface Vehicle; Inertial navigation; INS/GPS integration.

## I. INTRODUCTION

Navigation can be seen as the theory and practice of directing the course of a vehicle from a starting point toward a desired position. In this context, the accurate determination of the current position is very important and this problem has been solved throughout history using some great inventions such as the magnetic compass and the sextant and more recently radar and GPS [1]. Terrestrial navigation is essentially different from water surface navigation. Terrestrial navigation can be implemented with a map, a magnetic compass and an identifiable reference point on the map. However, in the case of navigation in open waters, in most situations, there are no reference points.

Inertial navigation systems (INS) have been used since the Second World War to calculate, without using external references, the position, velocity and orientation (attitude) of a moving vehicle in a 3D space [2]. Basically, a computer reads the outputs of set of accelerometers and gyrometers attached to the vehicle and performs double integration to determine the vehicle coordinates. However, an INS suffers from integration drift, which means that small errors in the accelerometers and gyrometers accumulate over time and the position error will increase without bound.

GPS (Global Positioning System) is a space-based satellite navigation system that became operational in 1994 and covers

any part of the world [2]. A GPS receiver on the ground determines its distance to at least four GPS satellites and calculates the vehicle position. However, GPS can suffer from satellite loss of signal, are sensitive to jamming and does not provide attitude information.

INS and GPS are complementary navigation systems and can be integrated to produce a more reliable and accurate positioning system. The literature reports different GPS/INS integration configurations considering a vehicle moving in a 3D space [2] [3]. This article presents a novel GPS/INS integration configuration designed specifically for a vehicle moving in a 2D space, such as a nautical vehicle. This configuration allows a simpler formulation than the general case which is therefore easier to implement.

This article presents the navigation system developed for a low-cost USV (Unmanned Surface Vehicle). Fig. 1 shows the vehicle (a catamaran type boat with two wheels driven by small DC motors) and the main components of the navigation system. The control station, which is typically stationary on land, is shown on the top left. It consists of: a) a standard personal computer, which remotely controls the boat, and b) a radio modem that acts as the communication link. The nautical platform is shown on the right and has a module that contains a radio modem, an onboard single-board computer, a standard GPS receiver, a low-cost IMU, and the power circuit for the DC motors. The IMU contains includes 3 linear accelerometers, 3 gyrometers and 3 magnetometers.

Once the platform was built, for cost reasons, it was decided that the software executed at the onboard computer was to be as simple as possible. Therefore, its main tasks are: a) send to the control station the sensors readings, and b) receive only 3 possible commands from the control station for each motor: turn it forward/turn it backwards/turn it off. As a result the software of the control station needs to solve the navigation problem which can be divided into two problems: a) process the onboard sensors readings to estimate the boat current position and its heading; and b) decide which commands should be sent to the boat such it tracks a sequence of waypoints as close as possible.

This article is organized as follows: Section II shows how the noisy information provided by the onboard sensors can be integrated using a Kalman Filter. Section III presents a mathematical model for the boat and in Section IV the boat

control system is described. Section V presents the implementation details and experimental results. Finally, conclusions are drawn in Section VI.

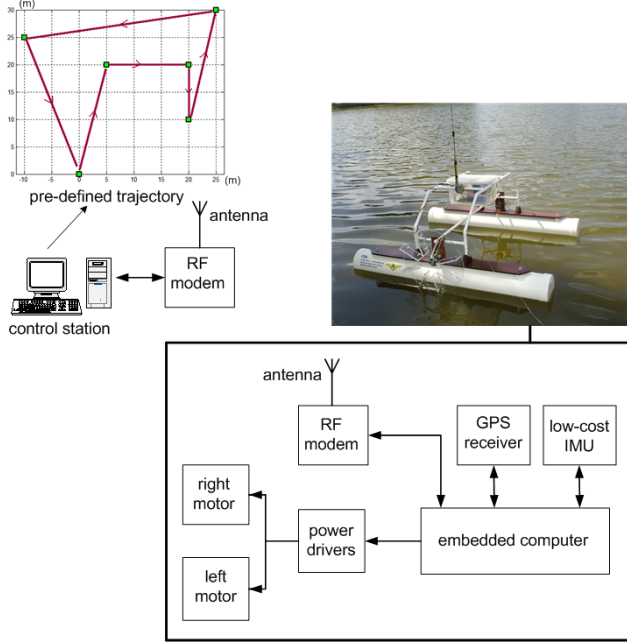


Figure 1 – USV navigation system.

## II. IMU/GPS/DIGITAL COMPASS INTEGRATION USING KALMAN FILTERING

For the boat autonomous navigation one needs to consider only three degrees of freedom: its 2D (x,y) position in the Earth frame and the heading angle (rotation around the z-axis) [3] [4].

Let  $X^V(t)$  denote the true state of the vehicle, where:

$$X^V(t) = \begin{bmatrix} x_1^V(t) \\ x_2^V(t) \\ x_3^V(t) \\ x_4^V(t) \\ x_5^V(t) \end{bmatrix} = \begin{bmatrix} V_{xe}(t) \\ V_{ye}(t) \\ p_x(t) \\ p_y(t) \\ \psi(t) \end{bmatrix} \quad (1)$$

and:

- $V_{xe}(t), p_x(t)$  = velocity and position of the vehicle in the x-axis of the  $e$ -frame (Earth-referenced);
- $V_{ye}(t), p_y(t)$  = velocity and position of the vehicle in the y-axis of the  $e$ -frame;
- $\psi(t)$  = heading (yaw) angle of the boat.

To convert the accelerations from the vehicle frame to the  $e$ -frame, one can use:

$$\begin{bmatrix} a_{xe}(t) \\ a_{ye}(t) \end{bmatrix} = \begin{bmatrix} \cos \psi(t) & -\sin \psi(t) \\ \sin \psi(t) & \cos \psi(t) \end{bmatrix} \begin{bmatrix} a_{xb}(t) \\ a_{yb}(t) \end{bmatrix} \quad (2)$$

where:

- $a_{xe}, a_{ye}$  = true vehicle acceleration in the x- and y-axes of the  $e$ -frame;
- $a_{xb}, a_{yb}$  = true vehicle acceleration in the x- and y-axes of the inertial platform installed on the boat.

Since  $\dot{V}_{xe}(t) = a_{xe}(t)$ ,  $\dot{V}_{ye}(t) = a_{ye}(t)$ ,  $\dot{p}_x(t) = V_{xe}(t)$  and  $\dot{p}_y(t) = V_{ye}(t)$ , from eq. (1) then:

$$\dot{X}^V(t) = \begin{bmatrix} \dot{x}_1^V \\ \dot{x}_2^V \\ \dot{x}_3^V \\ \dot{x}_4^V \\ \dot{x}_5^V \end{bmatrix} = \begin{bmatrix} \cos(x_5^V) u_1^V - \sin(x_5^V) u_2^V \\ \sin(x_5^V) u_1^V + \cos(x_5^V) u_2^V \\ x_1^V \\ x_2^V \\ u_3^V \end{bmatrix} \quad (3)$$

where  $U^V(t) = \begin{bmatrix} u_1^V(t) \\ u_2^V(t) \\ u_3^V(t) \end{bmatrix} = \begin{bmatrix} a_{xb}(t) \\ a_{yb}(t) \\ w_{zb}(t) \end{bmatrix}$ ,  $Y^V(t) = \begin{bmatrix} p_x(t) \\ p_y(t) \\ \psi(t) \end{bmatrix}$ , and

$w_{zb}(t)$  is the true vehicle rotation rate around its z-axis.

Corresponding to the vectors  $X^V(t)$ ,  $U^V(t)$  and  $Y^V(t)$ , one can define for the INS (Inertial Navigation System),

$$X^{INS}(t) = \begin{bmatrix} V_{xe}^{INS} \\ V_{ye}^{INS} \\ p_x^{INS} \\ p_y^{INS} \\ \psi^{INS} \end{bmatrix}, \quad U^{INS}(t) = \begin{bmatrix} a_{xb}^{INS} \\ a_{yb}^{INS} \\ w_{zb}^{INS} \end{bmatrix}, \quad Y^{INS}(t) = \begin{bmatrix} p_x^{INS} \\ p_y^{INS} \\ \psi^{INS} \end{bmatrix}$$

where  $a_{xb}^{INS}$ ,  $a_{yb}^{INS}$  and  $w_{zb}^{INS}$  denote respectively the measured values of the accelerations in the x- and y-axes and the rotation rate around the z-axis of the IMU installed on the boat. These measurements are provided by the IMU linear accelerometers and gyrometer. It is assumed that these values are related to the true values by the following equations:

$$\begin{aligned} a_{xb}^{INS}(t) &= a_{xb}(t) + bias_{ax}(t) + noise_{ax}^{INS}(t) \\ a_{yb}^{INS}(t) &= a_{yb}(t) + bias_{ay}(t) + noise_{ay}^{INS}(t) \\ w_{zb}^{INS}(t) &= w_z(t) + bias_{wz}(t) + noise_{wz}^{INS}(t) \end{aligned} \quad (4)$$

where:

- $bias_{ax}(t)$ ,  $bias_{ay}(t)$ ,  $bias_{wz}(t)$  denote respectively the bias of the linear accelerometers in the x- and y-axes and the gyrometer in the z-axis,
- $noise_{ax}^{INS}(t)$ ,  $noise_{ay}^{INS}(t)$ ,  $noise_{wz}^{INS}(t)$  denote the noise for the same devices.

The state error vector  $X^E(t)$ , the input error vector  $U^E(t)$  and the measurement error vector  $Y^E(t)$  can be defined as:

$$X^E(t) = \begin{bmatrix} X^{INS}(t) - X^V(t) \\ Bias(t) \end{bmatrix} \quad (5)$$

$$U^E(t) = U^{INS}(t) - U^V(t) \quad (6)$$

$$Y^E(t) = Y^{INS}(t) - Y^{GC}(t) \quad (7)$$

where:

- $Y^{GC}(t) = \begin{bmatrix} p_x^{GPS}(t) \\ p_y^{GPS}(t) \\ \psi^{COMP}(t) \end{bmatrix}$  and  $Bias(t) = \begin{bmatrix} bias_{ax}(t) \\ bias_{ay}(t) \\ bias_{wz}(t) \end{bmatrix}$ ,
- $p_x^{GPS}(t)$ ,  $p_y^{GPS}(t)$  are the (x,y) vehicle coordinates in the  $e$ -frame provided by the vehicle onboard GPS receptor, and
- $\psi^{COMP}(t)$  is the vehicle heading angle provided by the onboard digital compass.

It is assumed that:

$$\begin{aligned} p_x^{GPS}(t) &= p_x(t) + noise_x^{GPS}(t) = x_3^V(t) + noise_x^{GPS}(t) \\ p_y^{GPS}(t) &= p_y(t) + noise_y^{GPS}(t) = x_4^V(t) + noise_y^{GPS}(t) \\ \psi^{COMP}(t) &= \psi(t) + noise_{\psi}^{COMP}(t) = x_5^V(t) + noise_{\psi}^{COMP}(t) \end{aligned} \quad (8)$$

where  $noise_x^{GPS}(t)$ ,  $noise_y^{GPS}(t)$  and  $noise_{\psi}^{COMP}(t)$  denote respectively the GPS and compass noises.

As in eq. (3), one can write:

$$\dot{X}^{INS}(t) = \begin{bmatrix} \cos(x_5^{INS}) u_1^{INS} - \sin(x_5^{INS}) u_2^{INS} \\ \sin(x_5^{INS}) u_1^{INS} + \cos(x_5^{INS}) u_2^{INS} \\ x_1^{INS} \\ x_2^{INS} \\ u_3^{INS} \end{bmatrix} \quad (9)$$

Defining:

$$\begin{aligned} \bullet \quad N^{INS}(t) &= \begin{bmatrix} noise_{ax}(t) \\ noise_{ay}(t) \\ noise_{wz}(t) \end{bmatrix} = \begin{bmatrix} n_1^{INS}(t) \\ n_2^{INS}(t) \\ n_3^{INS}(t) \end{bmatrix} \\ \bullet \quad N^{GC}(t) &= - \begin{bmatrix} noise_x^{GPS}(t) \\ noise_y^{GPS}(t) \\ noise_{\psi}^{COMP}(t) \end{bmatrix} \end{aligned}$$

and assuming that for  $\forall t$ :

$$\bullet \quad \frac{dBias(t)}{dt} = \mathbf{0}_{3 \times 1} \text{ (biases are constant),}$$

- $x_5^{INS}(t) \approx x_5^V(t)$  (the heading angle provided by the INS has a small error), it is possible to show that:

$$\dot{X}^E(t) = \begin{bmatrix} \cos(x_5^{INS})[x_6^E + n_1^{INS}] - \sin(x_5^{INS})[x_7^E + n_2^{INS}] \\ \sin(x_5^{INS})[x_6^E + n_1^{INS}] + \cos(x_5^{INS})[x_7^E + n_2^{INS}] \\ x_1^E \\ x_2^E \\ x_8^E + n_3^{INS} \\ \mathbf{0}_{3 \times 1} \end{bmatrix} \quad (10)$$

$$U^E(t) = X_{6:8}^E(t) + N^{INS}(t) \quad (11)$$

$$Y^E(t) = [\mathbf{0}_{3 \times 2} \quad \mathbf{I}_{3 \times 3} \quad \mathbf{0}_{3 \times 3}] X^E(t) + N^{GC}(t) \quad (12)$$

$$\dot{X}^E(t) = A^E(t) X^E(t) + B^E(t) N^{INS}(t) \quad (13)$$

where:

$$A^E(t) = \begin{bmatrix} \mathbf{0}_{2 \times 2} & & C(t) & \mathbf{0}_{4 \times 1} \\ \mathbf{I}_{2 \times 2} & \mathbf{0}_{8 \times 3} & & \mathbf{1} \\ \mathbf{0}_{4 \times 2} & & \mathbf{0}_{6 \times 2} & \mathbf{0}_{3 \times 1} \end{bmatrix} \quad (14)$$

$$B^E(t) = \begin{bmatrix} C(t) & \mathbf{0}_{4 \times 1} \\ \mathbf{0}_{6 \times 2} & \mathbf{1} \\ & \mathbf{0}_{3 \times 1} \end{bmatrix} \quad (15)$$

$$C(t) = \begin{bmatrix} \cos(x_5^{INS}(t)) & -\sin(x_5^{INS}(t)) \\ \sin(x_5^{INS}(t)) & \cos(x_5^{INS}(t)) \end{bmatrix} \quad (16)$$

The discrete-time version of eq. (13) can be written as:

$$X^E[(k+1)T] = A_d^E(kT) X^E(kT) + B_d^E(kT) N^{INS}(kT) \quad (17)$$

where:

- $A_d^E(kT) = \exp[A^E(kT)T]$ ;
- $B_d^E(kT) = \left[ \int_{kT}^{(k+1)T} \exp(A^E(kT)t) dt \right] B^E(kT)$ ;
- $T$  = sampling period.

Under the previous assumptions, eq. (14) shows that the INS error dynamics can be assumed to be a time-variant linear system. It is therefore proposed that: a) using the GPS and digital compass readings, a Kalman filter [5] implementation can be used to estimate the biases of the IMU accelerometers and gyrometer, b) using these bias estimates, the IMU readings are then corrected and the INS equations produce more accurate estimates for the vehicle position and heading. Experiments have shown that, using the INS equations without correcting the IMU readings, the INS position and heading errors will rapidly diverge, that is, increase without bounds.

The proposed solution to estimate the boat position and heading is as follows [6]:

### Step 1: INS State and Kalman Filter Initialization:

The user must provide:

$$X_0^{INS-} = \begin{bmatrix} V_{xe}^{INS}(0) \\ V_{ye}^{INS}(0) \\ p_x^{INS}(0) \\ p_y^{INS}(0) \\ \psi^{INS}(0) \end{bmatrix} = \text{Initial INS state vector} \quad (18)$$

$$\hat{Bias}_0^- = \begin{bmatrix} \hat{bias}_{ax}(0) \\ \hat{bias}_{ay}(0) \\ \hat{bias}_{wz}(0) \end{bmatrix} = \text{Initial IMU bias estimate} \quad (19)$$

$$P_0^- = \text{diag}\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8\} \quad (20)$$

where  $P_0^-$  is the initial state error covariance matrix.

### Step 2: INS State and Kalman Filter Update

At time  $t = kT$  ( $k = 0, 1, 2, \dots$ ) read the GPS receiver and digital compass data and calculate:

$$Y_k^E = X_k^{INS-} [3:5] - \begin{bmatrix} p_{x,k}^{GPS} \\ p_{y,k}^{GPS} \\ \psi_k^{COMP} \end{bmatrix} \quad (21)$$

$$G_k = P_k^- H^T [H P_k^- H^T + R_k]^{-1} \quad (22)$$

$$P_k^+ = [I_{8 \times 8} - G_k H] P_k^- \quad (23)$$

$$\hat{X}_k^E = \begin{bmatrix} \mathbf{0}_{5 \times 1} \\ \hat{Bias}_k^- \end{bmatrix} + G_k Y_k^E \quad (24)$$

$$X_k^{INS+} = X_k^{INS-} - \hat{X}_k^E [1:5] \quad (25)$$

$$\hat{Bias}_k^+ = \hat{X}_k^E [6:8] \quad (26)$$

where  $H = [\mathbf{0}_{3 \times 2} \quad I_{3 \times 3} \quad \mathbf{0}_{3 \times 3}]$ ,  $R_k$  is the covariance matrix of the GPS and digital compass noise vector  $N^{GC}(kT)$ , and the following notation was used:

- $X_k$  denotes  $X(t)$  at time  $t = kT$ ,
- $p_{x,k}^{GPS}$  denotes  $p_x^{GPS}(t)$  at time  $t = kT$ ,
- $X_k[N]$  denotes the  $n$ -th component of  $X_k$ ,
- $X_k[N:M]$  denotes the column vector  $[X_k[N], \dots, X_k[M]]^T$ ,
- $\hat{X}_k$  denotes the estimate of  $X_k$ .

### Step 3: INS State and Kalman Filter Propagation

At time  $t = kT$  read the IMU data and calculate:

$$U_k^{INS} = \begin{bmatrix} a_{xb,k}^{INS} \\ a_{yb,k}^{INS} \\ w_{zb,k}^{INS} \end{bmatrix} - \hat{Bias}_k \quad (27)$$

Calculate the next INS state  $X_{k+1}^{INS}$  by numerically integrating eq. (9) from  $t = kT$  to  $(k+1)T$ :

$$\dot{X}^{INS}(t) = f(X^{INS}(t), U^{INS}(t)) \quad (28)$$

Set  $\hat{Bias}_{k+1}^- = \hat{Bias}_k^+$  and calculate:

$$P_{k+1}^- = A_d^E P_k^+ [A_d^E]^T + B_d^E Q_k [B_d^E]^T + \beta P_0^- \quad (29)$$

where  $A_d^E = A_d^E(kT)$ ,  $B_d^E = B_d^E(kT)$ ,  $\beta$  is a small positive scalar used to avoid the divergence of the Kalman Filter, and  $Q_k$  is the covariance matrix of the IMU noise vector  $N^{INS}(kT)$ .

### III. BOAT MODELLING

In order to simulate the autonomous navigation system, a dynamic model of the boat is needed [7-9]. Fig. 2 shows the forces that act on the boat and two coordinate systems, one fixed on Earth ( $e$ -frame) with axes  $(X_0, Y_0)$  and another with rotating axes  $(\bar{x}, \bar{y})$  with are aligned with the boat axes.

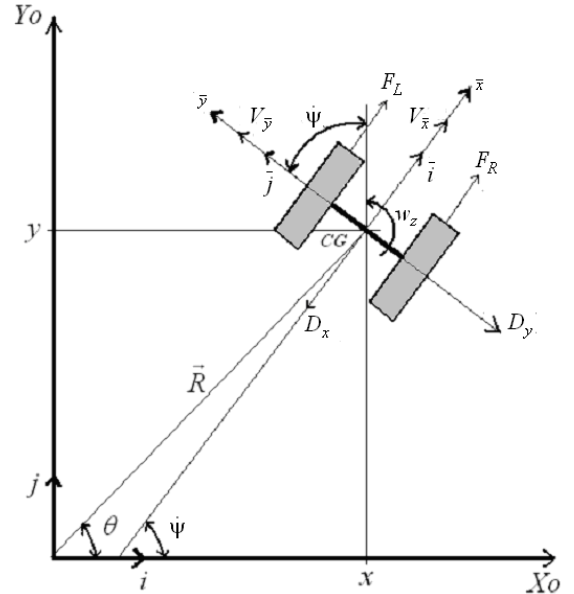


Figure 2 – Coordinate systems and forces acting on the boat.

The vehicle velocity vector and its derivative can be written as:

$$\vec{V} = V_{\bar{x}} \vec{i} + V_{\bar{y}} \vec{j} = V_x \vec{i} + V_y \vec{j} \quad (30)$$

$$\frac{d\vec{V}}{dt} = \dot{V}_{\bar{x}} \vec{i} + \dot{V}_{\bar{y}} \vec{j} + V_{\bar{x}} \left( \frac{d\vec{i}}{dt} \right) + V_{\bar{y}} \left( \frac{d\vec{j}}{dt} \right) \quad (31)$$

and using  $\frac{d\vec{i}}{dt} = \dot{\psi}\vec{j} = w_z\vec{j}$ ,  $\frac{d\vec{j}}{dt} = -\dot{\psi}\vec{i} = -w_z\vec{i}$ , then:

$$\begin{aligned}\sum \vec{F} &= m \frac{d\vec{V}}{dt} = m(\dot{V}_{\bar{x}}\vec{i} + \dot{V}_{\bar{y}}\vec{j} + V_{\bar{x}}w_z\vec{j} - V_{\bar{y}}w_z\vec{i}) \\ &= (F_R + F_L)\vec{i} - (D_x)\vec{i} - (D_y)\vec{j}\end{aligned}\quad (32)$$

where  $m$  is the boat mass,  $F_R$  and  $F_L$  are the right and left propeller force,  $D_x$  and  $D_y$  are the drag force in the  $\bar{x}$  and  $\bar{y}$  directions. So, in the  $\bar{x}$  axis:

$$m(\dot{V}_{\bar{x}} - w_z V_{\bar{y}}) = F_R + F_L - D_x \quad (33)$$

and in the  $\bar{y}$  axis:

$$m(\dot{V}_{\bar{y}} + w_z V_{\bar{x}}) = -D_y \quad (34)$$

In the  $\bar{z}$  axis:

$$J \dot{w}_z = \sum T = T_{rot} + D_z \quad (35)$$

where  $J$  is the boat inertial moment,  $T_{rot} = (F_R - F_L)d$ ,  $D_z$  is the drag torque in the  $\bar{z}$  axis,  $d$  is the distance between the boat CG and a propeller [10].

A quadratic model was assumed for the drag forces:  $D_x = K_x(V_{\bar{x}})^2$ ,  $D_y = K_y(V_{\bar{y}})^2$ ,  $D_z = K_z w_z^2$ .

The above equations can be rearranged and grouped as the following dynamical model:

$$\begin{bmatrix} \dot{V}_{\bar{x}} \\ \dot{V}_{\bar{y}} \\ \dot{p}_x \\ \dot{p}_y \\ \dot{\psi} \\ \dot{w}_z \end{bmatrix} = \begin{bmatrix} w_z V_{\bar{y}} + \text{sign}(V_{\bar{x}})(F_R + F_L - D_x)/m \\ -w_z V_{\bar{x}} - \text{sign}(V_{\bar{y}})K_y V_{\bar{y}}^2/m \\ V_{\bar{x}} \cos \psi - V_{\bar{y}} \sin \psi \\ V_{\bar{x}} \sin \psi + V_{\bar{y}} \cos \psi \\ w_z \\ ((F_R - F_L)d - \text{sign}(w_z)K_z w_z^2)/J \end{bmatrix} \quad (36)$$

Experiments were performed to determine the following model parameters:  $m = 14.81$  kg,  $J = 3.37$  kg/m<sup>2</sup>,  $K_x = 7.025$  Ns<sup>2</sup>/m<sup>2</sup>,  $\max(|F_R|) = \max(|F_L|) = 1.4$  N,  $d = 0.56$  m. The following parameters were estimated:  $K_y = K_z = 5K_x = 35.12$  Ns<sup>2</sup>/m<sup>2</sup>.

#### IV. BOAT CONTROL SYSTEM

The remote computer receives the readings of the boat sensors and calculates the boat position and heading as explained in the previous section. Considering the desired boat trajectory, the remote computer decides the actions of the boat actuators (2 DC motors) and sends commands, via its radio modem, to the electronic drivers of each DC motor in the boat.

For simplicity, each DC motor has only 3 possible states: turned off, turned on rotating forward, turned on rotating backwards.

Fig. 3 shows the control system diagram. The control strategy was implemented as a lookup table (shown in Fig. 4). The table inputs are  $e_\psi(t)$  (the heading error) and  $e_p(t)$  (the position error). The table outputs are  $\{M_L, M_R\}$ , the commands received by the electronic drivers of the left and right DC motor respectively. The table inputs are calculated as follows:

$$\psi^d(t) = \text{atan2}(p_y^d(t) - p_y(t), p_x^d(t) - p_x(t)) \quad (37)$$

$$e_\psi(t) = \psi^d(t) - \psi(t) \quad (38)$$

$$e_p(t) = \sqrt{(p_x^d(t) - p_x(t))^2 + (p_y^d(t) - p_y(t))^2} \quad (39)$$

where  $(p_x^d(t), p_y^d(t))$  are the (x,y) coordinates of the desired boat position in the earth frame. Since the boat is symmetrical around its main axis, if the absolute value of the heading error  $e_\psi(t)$  is greater than 90°, a simpler and faster maneuver can be executed by applying the following redefinitions:

$$e_p(t) = -e_p(t) \quad (40)$$

$$\text{if } e_\psi(t) > 90^\circ: e_\psi(t) = e_\psi(t) - 90^\circ \quad (41)$$

$$\text{if } e_\psi(t) < -90^\circ: e_\psi(t) = e_\psi(t) + 90^\circ \quad (42)$$

The set of possible commands for each motor is  $\{-1, 0, 1\}$  with the following corresponding motor action: {turn it backwards, turn it off, turn it forward}.

A desired trajectory is formed by a sequence of waypoints. Each waypoint is specified by its (x,y) coordinates. Each waypoint is selected sequentially. If the boat is sufficiently close to the current waypoint, it is assumed that it has been reached and the next waypoint in the sequence is selected. The table presented in Fig. 4 shows that a waypoint is assumed reached when the position error is less than 3 m and the heading error is less than 5°.

Figure 6 shows the simulation results using the boat dynamic model derived in the previous section and the proposed control strategy. The first waypoint, the last waypoint and the boat initial position are (0,0). The boat initial heading is 0° (horizontal axis). The true boat position and heading were supposed to be available for this simulation.

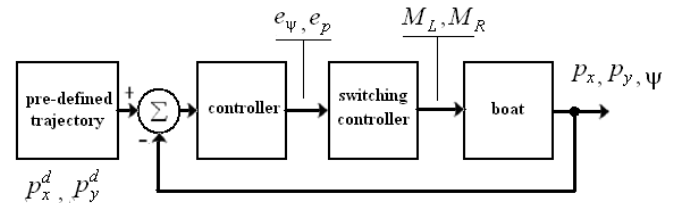


Figure 3. The boat control system.

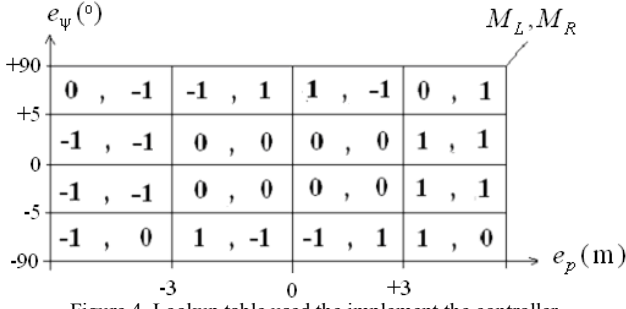


Figure 4. Lookup table used to implement the controller.

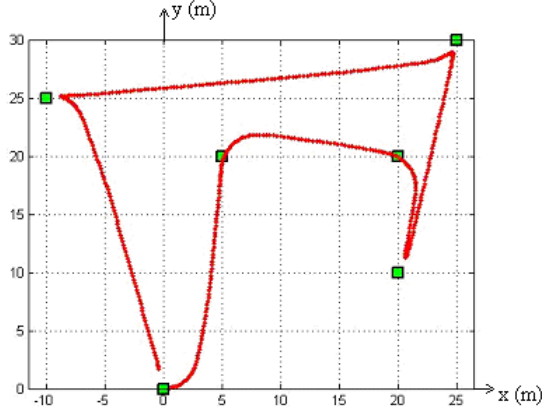


Figure 6. Simulation results for proposed control strategy.

## V. IMPLEMENTATION DETAILS AND EXPERIMENTAL RESULTS

The boat is a 1.2 m x 1.2 m catamaran with the following main electronic components (Fig. 7): a) FlashTCP, a DOS-based single board computer manufactured by JK Microsystems, b) IMU: model 3DM-GX1 manufactured by MicroStrain, c) GPS receptor: model A1021 manufactured by TYCO, d) 12 V 4.5 Ah standard battery, e) a 9600 bps 100 mW standard radio modem. The software that runs on the onboard computer was written in C. The digital compass was implemented by processing the output of the 3 IMU magnetometers. The onboard sensors are read once every second.

The Kalman filter algorithm was implemented using state and measurement variables in the SI system. The following constant values were used:

- $\beta = 0.005$ ,
- $P_0^- = \left[ \text{diag} \left\{ 10, 10, 10, 10, \frac{90\pi}{180}, 5, 5, \frac{25\pi}{180} \right\} \right]^2$ ,
- $R = \left[ \text{diag} \left\{ 1, 1, \frac{\pi}{180} \right\} \right]^2$ ,
- $Q = \left[ \text{diag} \left\{ 0.1, 0.1, \frac{0.1\pi}{180} \right\} \right]^2$ .

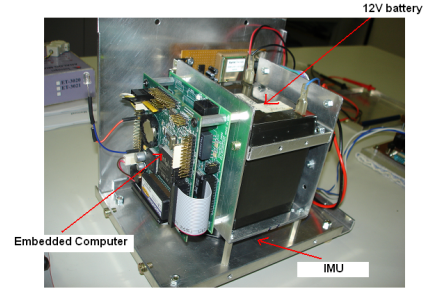


Figure 7. The boat electronic module.

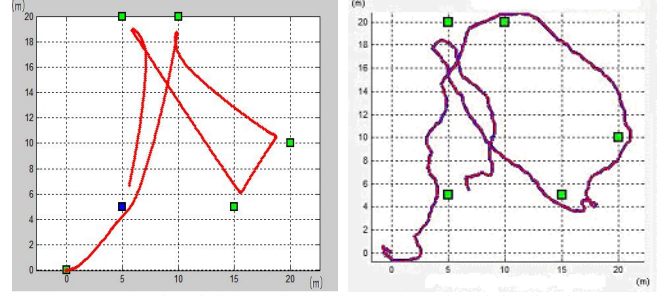


Figure 8. Simulated (left) and experimental results (right) for the same sequence of waypoints.

The remote computer is a standard laptop running MATLAB. It receives the sensor data, executes the Kalman filter algorithm to estimate the boat position and heading. Then the lookup table controller decides which commands should be sent to the boat to turn on/off its motors.

For comparison purposes, Fig. 8 shows the simulated and experimental results for the same sequence of waypoints. The first waypoint is at the origin of the coordinate system. In the simulation results (Fig. 8 left) the controller receives the true boat position and heading and it shows that the controller performed as expected. In the experimental results (Fig. 8 right) the real boat was placed on a lake and was subjected to strong wind conditions and the boat controller receives the estimated values of the boat position and heading provided by the Kalman filter algorithm. Therefore the boat trajectory is not the same as in the simulated case.

## VI. CONCLUSIONS

This article shows how a Kalman filter can be used to integrate an IMU, a GPS receiver and a digital compass and estimate the boat position and heading. A lookup table is used to decide the commands for the boat actuators such that it tracks a sequence of waypoints autonomously. Simulation and experimental results were presented and showed that the proposed autonomous navigation system operated as designed for the experimental platform.

Possible lines for further research are: use of a cheaper IMU unit, tuning of the Kalman filter covariance matrices, derivation of performance indices using true error measurements, experiments using this solution to develop a small autonomous car.

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