Assignment 1

Real-Time Control Systems

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1 Part 1 | State Space Model

We derived the state-space model of each system. For dual rotary system, states were defined as $[\theta_1, \theta_2, \omega_1, \omega_2]$, the state-space model derived from the system dynamics is as follow:

$$
\begin{aligned}
A &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{J_1} & \frac{k}{J_1} & -\frac{b+\alpha}{J_1} & \frac{d}{J_1} \\ \frac{k}{J_2} & -\frac{k}{J_2} & \frac{d}{J_2} & -\frac{b+\alpha}{J_2} \end{bmatrix} \\
B &= \begin{bmatrix} 0 & 0 & k_m & 0 \end{bmatrix}^T \\
C &= \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}, \quad D = 0
\end{aligned} \tag{1}
$$

For DC motor speed system, states were defined as $[\dot{\theta}_1, i]$, the state-space model is as follow:

$$
\begin{aligned}
A &= \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{J} & -\frac{R}{J} \end{bmatrix} \\
B &= \begin{bmatrix} 0 & \frac{1}{L} \end{bmatrix}^T \\
C &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0
\end{aligned} \tag{2}
$$

2 Part 2 | Basic Analysis

In order to get an initial knowledge about the execution cycles of the applications. We first use a very loose configuration of h and τ: $h_1 = 3ms$ and $\tau_1 = \tau_2 = 2ms$. We designed controller in the feedback feedforward way($u = -Kx + Fr$), with target poles [0.650.650.650.650.65] for Dual Rotary System and [0.980.980.98] for DC Motor Systems.

We put these controllers into PIL simulation (we use `format short` command to make the controller keeps 4-digit) and the PIL simulation gave us the result: $c(DR) = 36950$ and $c(DCM) = 32250$.

Intuitively, the execution cycle is the execution time of assembly code. Under different K and F values, the length and the sequence of the assembly code should keep similar, so is the execution cycle should be similar. Based on this intuition, we did some other experiments with different h and τ combinations. The result is shown in table 1. Note that although in some configurations, the execution cycle is larger than τ setting. We do not regard it as a problem, because we think when calculating the execution cycle, the PIL simulation does not simulate a τ delay.

The table validate our hypothesis.

3 Part 3 | Case Design: $h_1 < 3ms$

Based on the result in last section, we use $c(DR) = 37500$, $c(DCM) = 32000$, $t_{exe}(DR) = 0.9375ms$ and $t_{exe}(DCM) = 0.8ms$, a bit larger than the maximum value in the table 1.

Table 1: Experiments Result of Execution Cycle

System	h/ms	tau/ms	execution cycle	System	h/ms	tau/ms	execution cycle
DR	5	2	37300	DCM	5	2	32325
	3	2	36950		3	2	32250
	3	1	37100		3	1	32250
	2	1	36175		2	1	31150
	2	0.5	36400		2	0.4	31200
	2	0	37150		2	0	31150
	1	0.5	35300		1	0.8	31150

3.1 Design 1

We designed a TDM table configuration as shown in Figure 1. The slots in the figure is introduced in table 2.

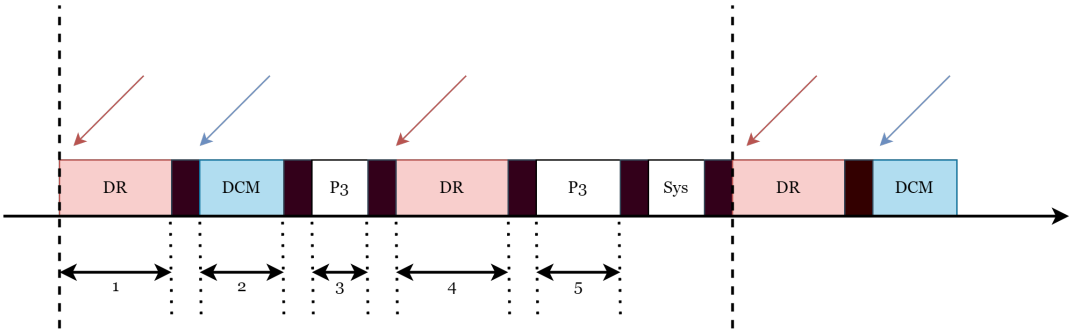


Figure 1: TDM Configuration of Design 1

Table 2: TDM Configuration of Design

Slot	Function	Length (cycle)	Allocation
1	Execution for a whole DR application	37500	DR
2	Execution for a whole DCM application	32000	DCM
3	Execution for Partition 3, actually no work	4500	No work
4	Execution for a whole DR application	37500	DR
5	Execution for Partition 3, actually no work	31500	No work

The h and τ can be computed as follows:

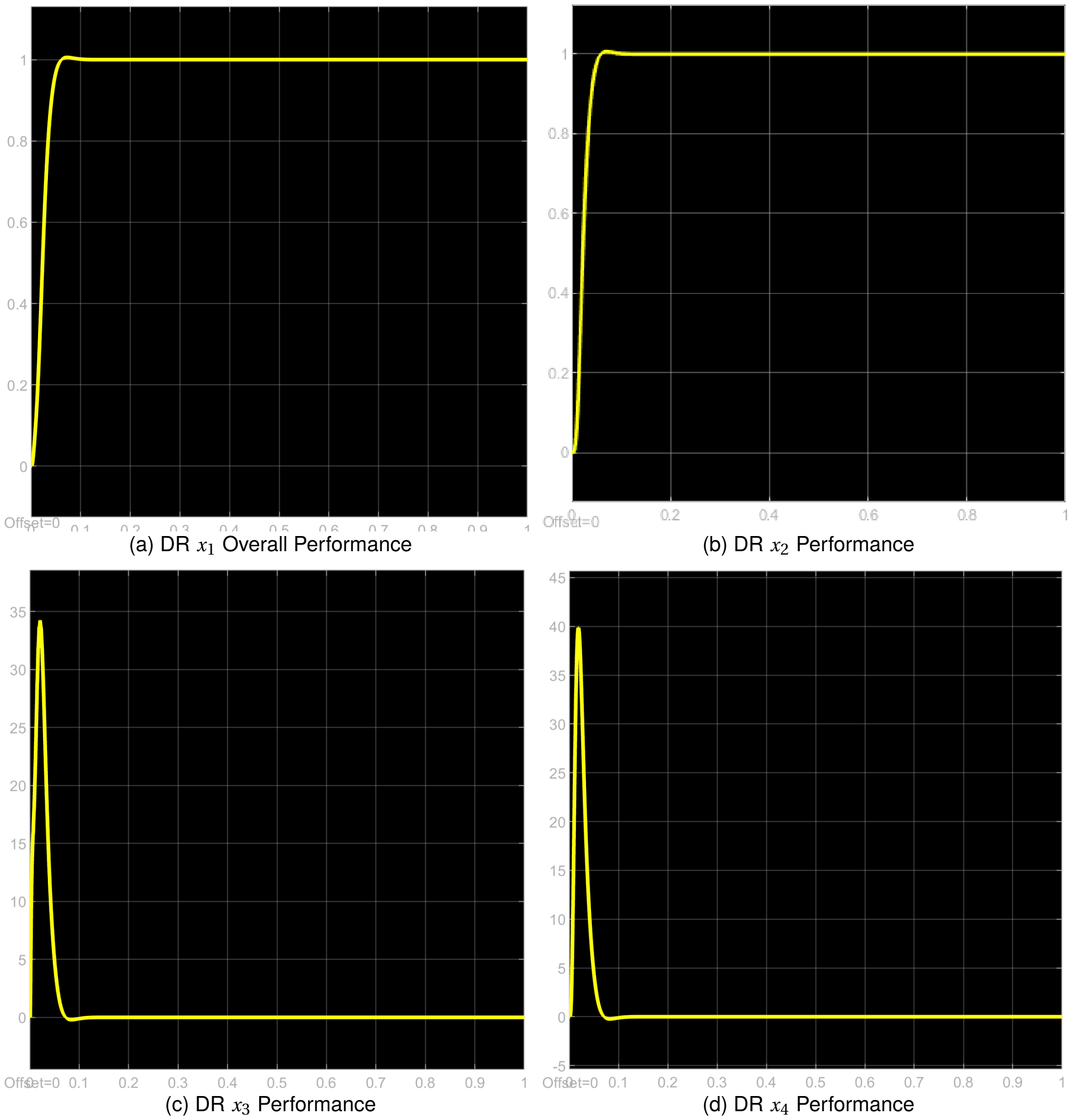
$$\begin{aligned} \text{DCM:} \quad & h_1 = (37500 + 32000 + 4500 + 3 \times 2000)/(40 \times 10^6) \times 10^3 = 2(ms) \\ \text{DCM:} \quad & \tau_1 = (37500)/(40 \times 10^6) \times 10^3 = 0.9375(ms) \\ \text{DR:} \quad & h_2 = 2 \times h_1 = 4(ms) \\ \text{DR:} \quad & \tau_2 = (32000)/(40 \times 10^6) \times 10^3 = 0.8(ms) \end{aligned} \tag{3}$$

Based on above h and τ, we use pole-placement method to design the feedback controller. We set target poles $[0.65, 0.65, 0.65, 0.65, 0.65]$ for Dual Rotary System and $[0.98, 0.98, 0.98]$ for DC Motor Systems. The designed K and F are shown in the following equations. One thing to be noticed is that when designing controller for Dual Rotary System, because $C = [1, 1, 0, 0]$, the output y is actually the sum of x_1 and x_2. Based on the observation that x_1 and x_2 are very symmetrical, after computing F_{DR}, we set the F in the controller as $F = 2F_{DR}$.

$$
\begin{aligned}
K_{DR} = [5.8501, -6.2735, -0.0130, 0.0027, -0.1366]^T \quad F_{DR} = 0.2117 \\
K_{DCM} = [0.2387, -0.0369, 0.9874]^T \quad F_{DCM} = 0.2561
\end{aligned} \tag{4}
$$

MIL Results

For the Dual Rotary System, the value of x_1, x_2, x_3 and x_4 is shown in Figure 2wave of input is shown in Figure 3. The wave of input is shown in Figure 3. The settling time of x_1 is approximately $0.0528s$ and the settling time of x_2 is approximately $0.0528s$.



(a) DR x_1 Overall Performance
(b) DR x_2 Performance
(c) DR x_3 Performance
(d) DR x_4 Performance

Figure 2: Design 1 MIL DR Performance

For the DC Motor System, the value of output and input are shown in Figure 4. The settling time is approximately $1.487s$

HIL Results

For the Dual Rotary system, the value of x_1 and x_2 are shown in Figure 5 and Figure 6. The value of x_3 and x_4 are shown in Figure 7. It can be seen that all of them met the requirements. The settling time is approximately $1.52s$

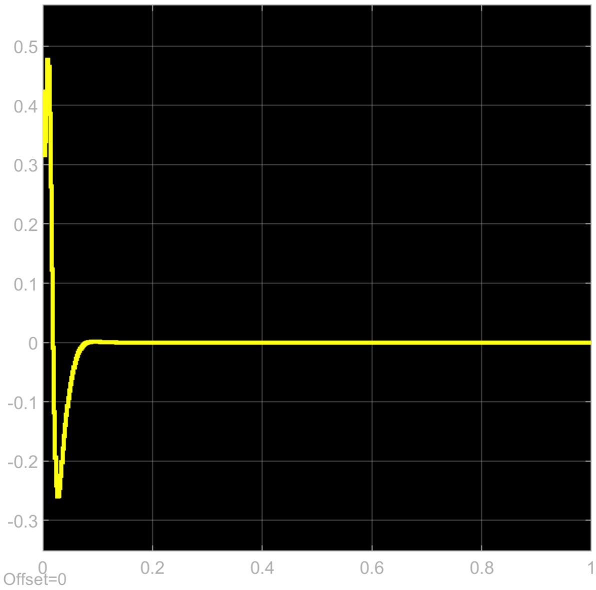
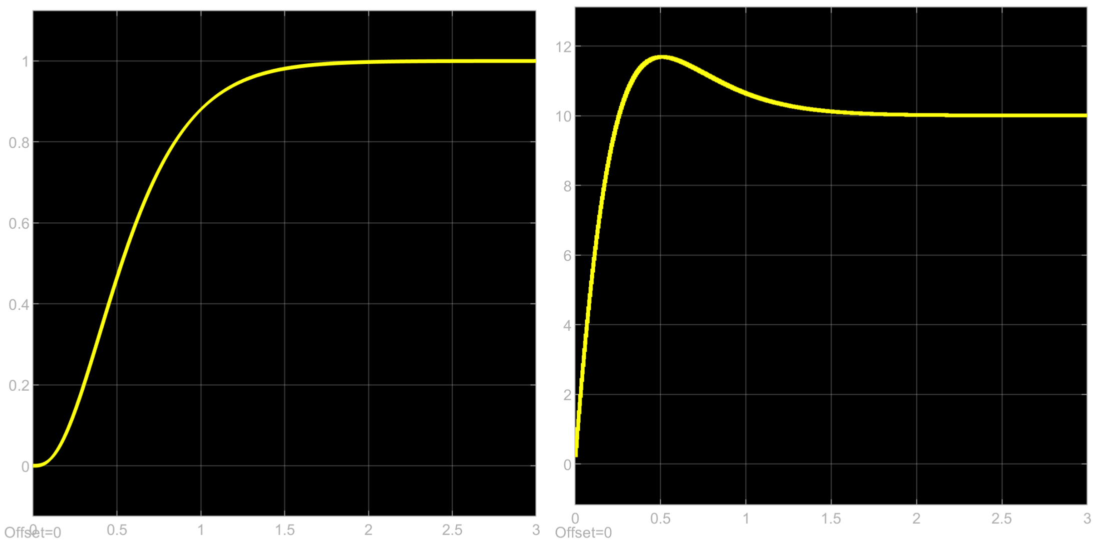
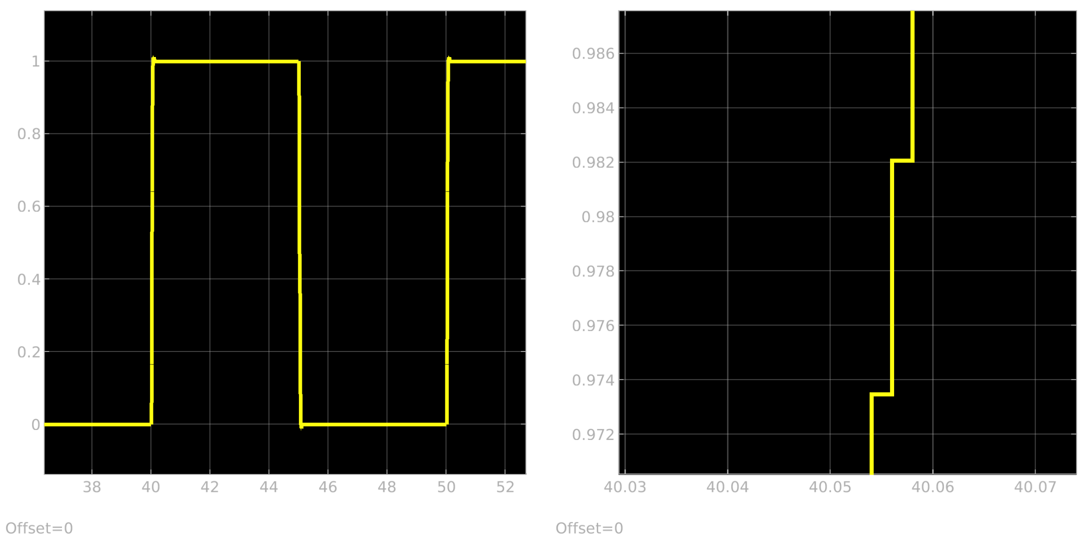


Figure 3: Design 1 MIL DR Input



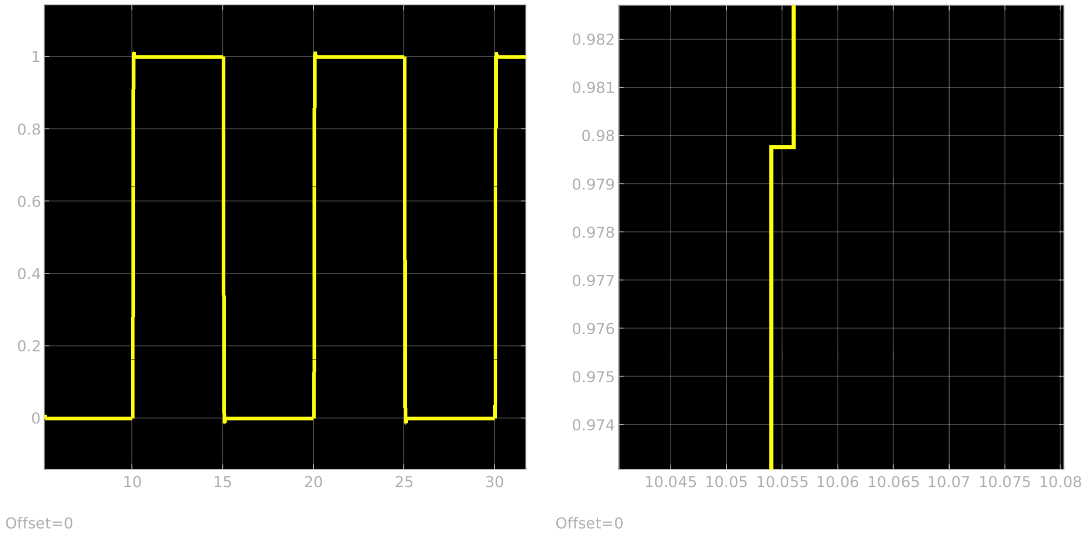
(a) Design 1 MIL DC Motor Output (b) Design 1 MIL DC Motor Input

Figure 4: Design 1 MIL DC Motor Performance



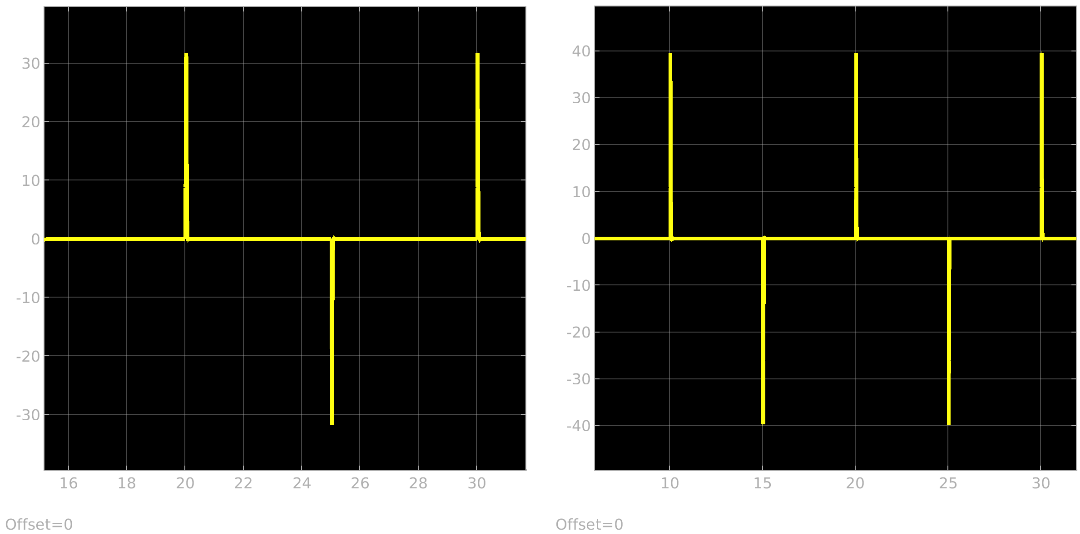
(a) DR x_1 Overall Performance (b) DR x_1 Settling Time

Figure 5: Design 1 DR x_1 HIL Performance



(a) DR x_2 Overall Performance (b) DR x_2 Settling Time

Figure 6: Design 1 DR x_2 HIL Performance



(a) DR x_3 Overall Performance (b) DR x_4 Overall Performance

Figure 7: Design 1 TDM 1 DR x_3 x_4 HIL Performance

The input of the Dual Rotary System is shown in Figure 8. The input also meet the requirements. The settling time is approximately $0.056s$

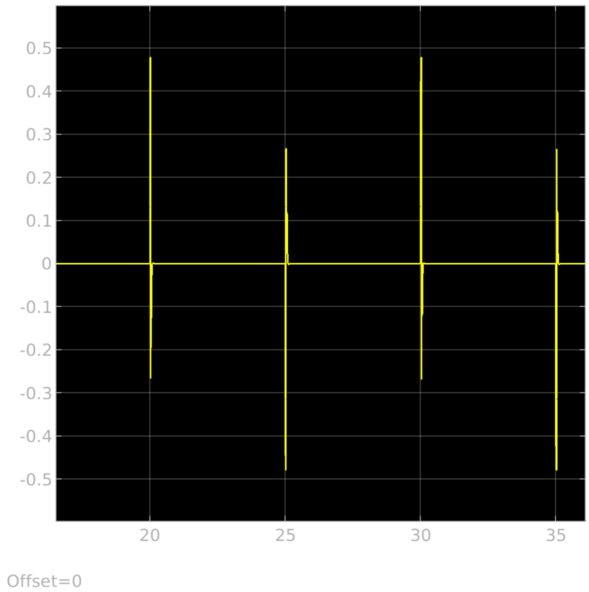
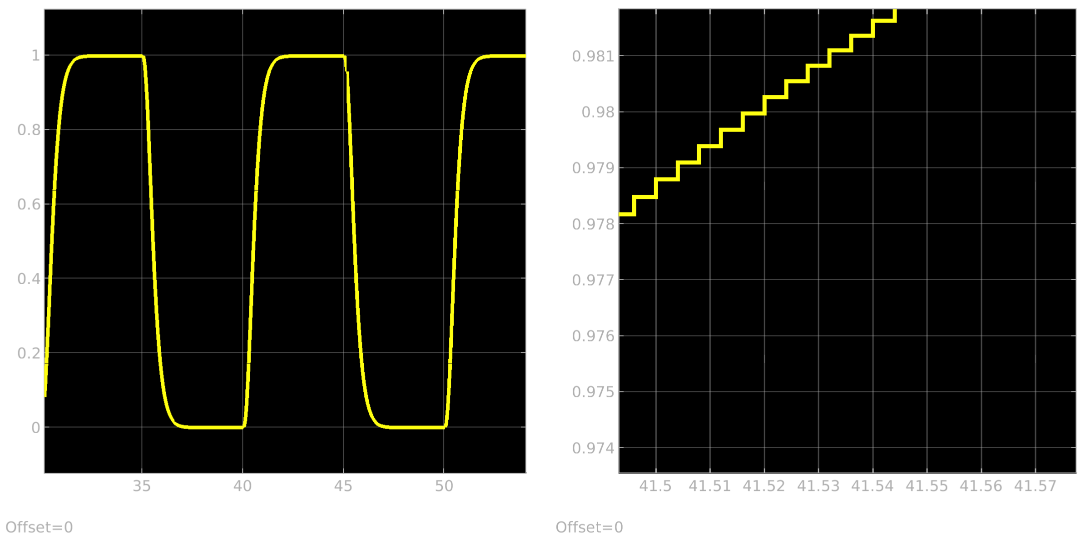


Figure 8: Design 1 DR HIL input

The output of the DC Motor is shown in Figure 9. The input of the DC Motor system is shown in Figure 10. The settling time is approximately $1.52s$. The DC Motor System meet the requirements.



(a) DCM Output Overall Performance (b) DCM Output Settling Time

Figure 9: Design 1 DCM Output HIL Performance

We again check the PIL result, and finally the system can achieve h and τ as follows:

$$\begin{aligned} \text{DR:} \quad & \tau_1 = 35925/(40 \times 10^6) \times 10^3 = 0.895(ms); \quad h_1 = 2(ms); \\ \text{DCM:} \quad & \tau_2 = 30876/(40 \times 10^6) \times 10^3 = 0.770(ms) \quad h_2 = 4(ms); \end{aligned} \tag{5}$$

Comparison

From the comparison of HIL and MIL results, we did not find a significant difference between them. The main difference is the settling time in HIL is a little later than that in MIL, around 6%. It may be because the τ we assumed for designing the controller is a little longer final τ we observed in the PIL simulation. The error is smaller than 5% percent, which is a relatively small difference and which guarantees the performance of the HIL simulation.

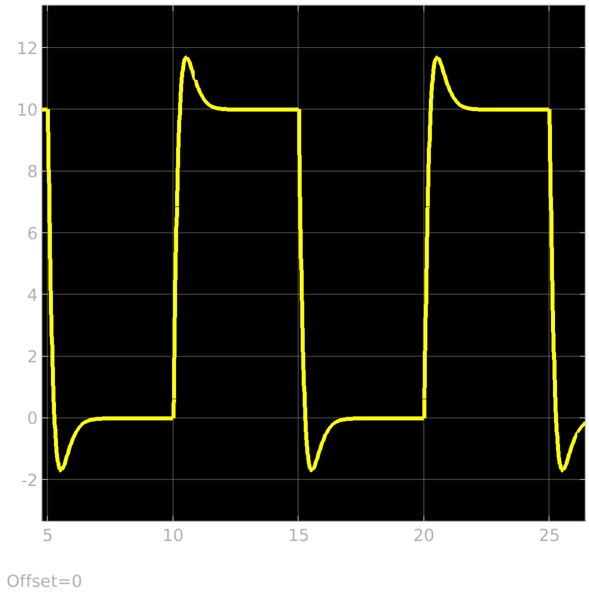


Figure 10: Design 1 DCM HIL input

3.2 Design 2

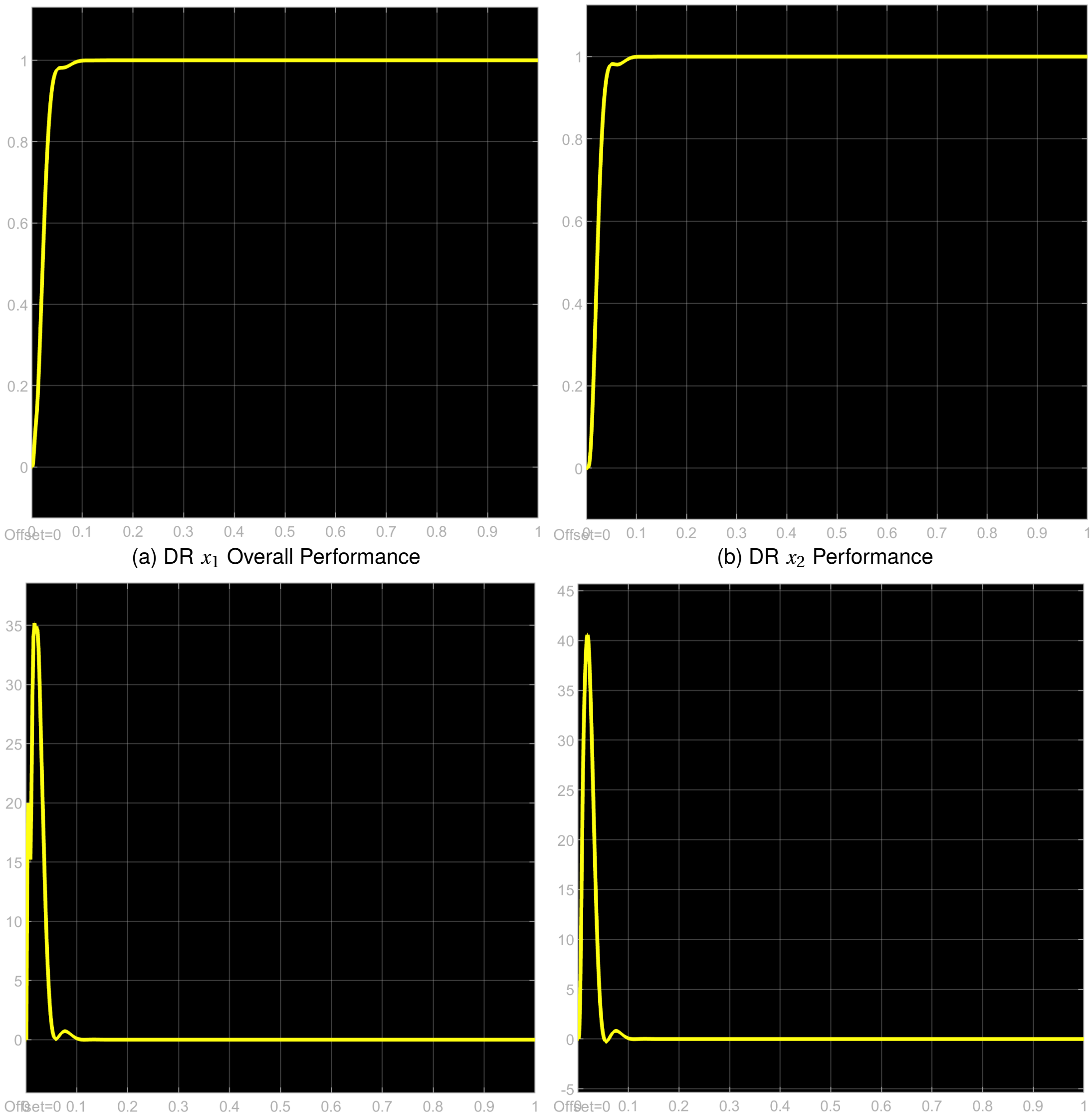
In Design 2, we consider a system without sensor-to-actuator delay. We reuse the sampling time $h_1 = 2(ms)$ as in Design 1. We still use the pole-placement method to design the controller. We first test with the same pole configuration as in Design 1. The outputs can still be perfectly tracked and the settling time still meets the requirements. However, the input of the two system will exceed the required bound. Based on that, we change the pole setting: $[0.7, 0.7, 0.7, 0.7]$ for DR system and $[0.985, 0.985, 0.985]$ for DC Motor System. The calculated controller is shown as follows. One thing to be noticed is that when designing the controller for Dual Rotary System, because $C = [1, 1, 0, 0]$, the output y is the sum of x_1 and x_2. Based on the observation that x_1 and x_2 are very symmetrical, after computing F_{DR}, we set the F in the controller as $F = 2F_{DR}$.

$$\begin{aligned} K_{DR} = [2.2150, -2.8680, -0.0315, 0.0161]^T \quad F_{DR} = 0.3265 \\ K_{DCM} = [-19.1293, 2.1938]^T \quad F_{DCM} = 7.2015 \end{aligned} \tag{6}$$

MIL Results

For the Dual Rotary System, the value of x_1, x_2, x_3 and x_4 is shown in Figure 11. The wave of input is shown in Figure 12. The settling time of x_1 is approximately $0.0525s$ and the settling time of x_2 is approximately $0.0525s$. Which is smaller than the settling time in Design 1.

For the DC Motor System, the value of output and input are shown in Figure 13. The settling time is $1.544s$.



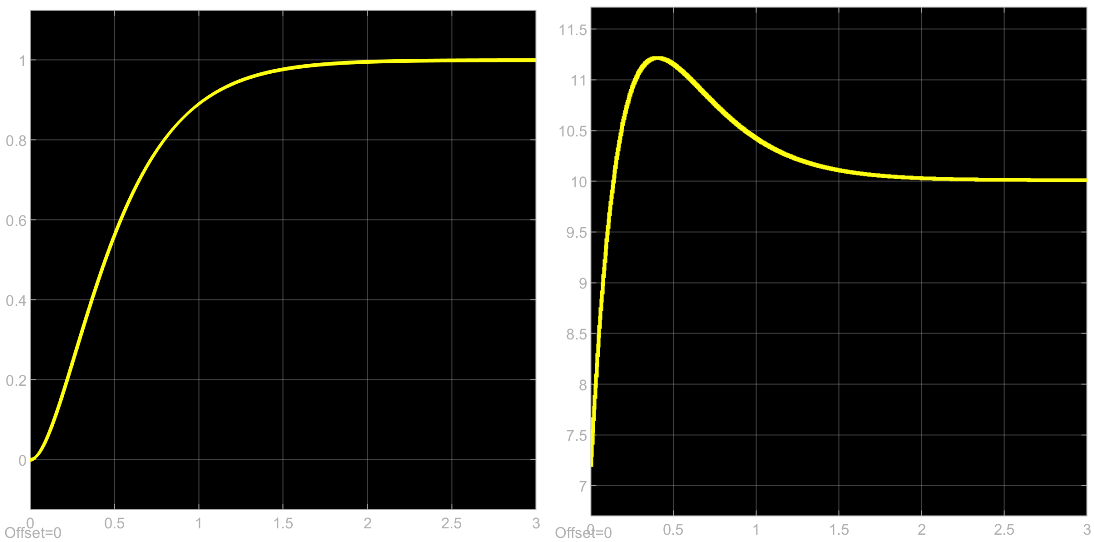
(a) DR x_1 Overall Performance

(b) DR x_2 Performance

(c) DR x_3 Performance

(d) DR x_4 Performance

Figure 11: Design 2 MIL DR Performance



(a) DCM Output

(b) DCM Input

Figure 13: Design 2 MIL DCM Performance

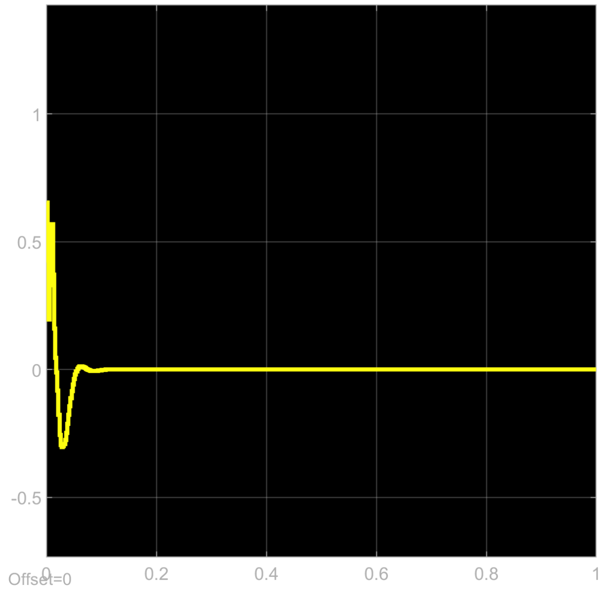


Figure 12: Design 2 MIL DR Input

Comparison

Based on the design process and the system performance of design 1 and design 2, several observations can be made.

The first thing is that if we keep the same pole settings, the settling time of design 2 is smaller than design 1. For the Dual Rotary System, even though poles in design 2 are further away from origins, we can still get a smaller settling time. This may be because when a controller without considering the sensor-to-actuator delay works in a delayed scenario, changes in the input of the system will lag behind the system state's change. This means the system input may still be large even though the system state is closer to the reference point and it will cause the system to get close to the reference point faster.

At the same time, the larger-than-expected input will make states like $\dot{\theta}$ become larger, which in turn leads to a larger input signal. This can illustrate the second observation that when using the same pole configuration to design a controller without considering sensor-to-actuator delay and then utilizing it in a system with delay, the input signal may be amplified and then exceed the requirement. That means in such a scenario, we need to prepare more free margin for each signal and use poles further away from origins.

Besides, the curve of the Dual Rotary system in design 2 is less smooth than that in design 1. This may also due to the changes of the system is lagging behind the system's state changes.

4 Part 4 | Case Design: $h_1 > 3ms$

We used the same configuration $c(DR) = 37500$, $c(DCM) = 32000$, $t_{exe}(DR) = 0.9375ms$ and $t_{exe}(DCM) = 0.8ms$ for following design. We chose sampling time as 4ms and 8ms for Dual Rotary System and DC Motor Speed System respectively.

4.1 design 3

We set TDM table for design 3, shown in the below table 3. Since the sampling time is enlarged while tasks' execution time remains the same, we can rearrange slot distribution as shown in the figure14, also not violate the sampling time.

$$
\begin{aligned}
\text{DCM:}\quad & h_1 = (37500 + 118500 + 2 \times 2000) / (40 \times 10^6) \times 10^3 = 4(ms) \\
\text{DCM:}\quad & \tau_1 = (37500) / (40 \times 10^6) \times 10^3 = 0.9375(ms) \\
\text{DR:}\quad & h_2 = (37500 + 32000 + 77500 + 4 \times 2000 + 5000) / (40 \times 10^6) = 2 \times h_1 = 8(ms) \\
\text{DR:}\quad & \tau_2 = (32000) / (40 \times 10^6) \times 10^3 = 0.8(ms)
\end{aligned} \tag{7}
$$

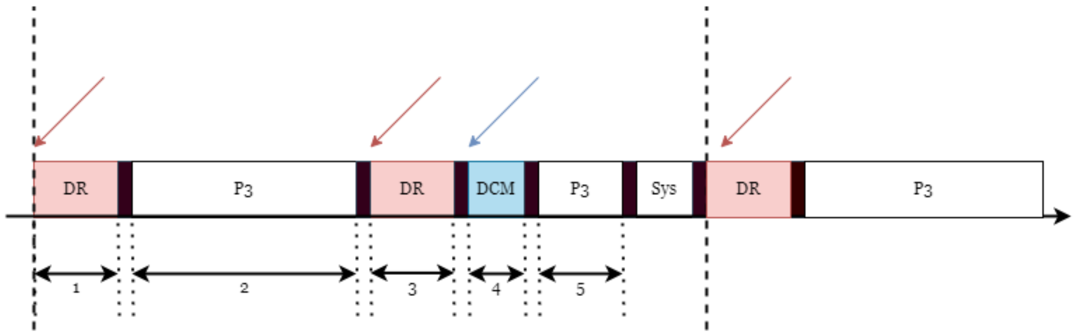


Figure 14: TDM Configuration 1 of Design 3

Table 3: Design 3 TDM Configuration of Design 3:

Slot	Function	Length (cycle)	Allocation
1	Execution for a whole DR application	37500	DR
2	Execution for Partition 3, actually no work	119500	No work
3	Execution for a whole DR application	37500	DR
4	Execution for a whole DCM application	32000	DCM
5	Execution for Partition 3, actually no work	77500	No work

After adjusting the sampling time, we follow the poles placement method and set poles as [0.4 0.4 0.4 0.4 0.4] for the Dual Rotary System, and [0.963 0.963 0.963] for the DC Motor Speed System. The feedback vector and feedforward gain are as follow:

$$
\begin{aligned}
K_{DR} &= \begin{bmatrix} 5.4512 & -5.9141 & -0.0084 & -0.0028 & -0.1089 \end{bmatrix}, & F_{DR} &= 0.4629 \\
K_{DCM} &= \begin{bmatrix} 0.5808 & -0.0825 & 0.9829 \end{bmatrix}, & F_{DCM} &= 0.4145
\end{aligned} \tag{8}
$$

MIL Results

For the Dual Rotary System, the wave of input is shown in Figure 15. The value of x_1, x_2, x_3 and x_4 is shown in Figure 16. The settling time of x_1 is $0.0516s$ and the settling time of x_2 is $0.053s$. All requirements are met.

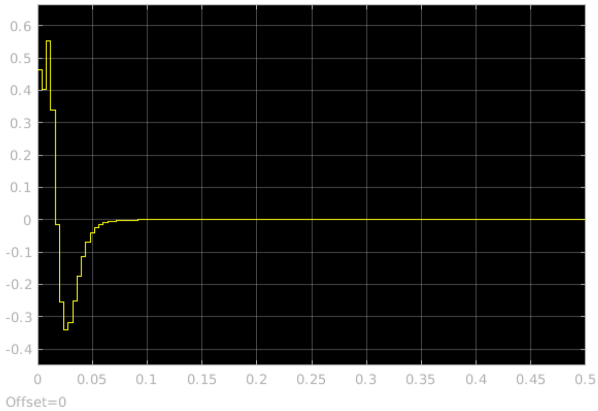
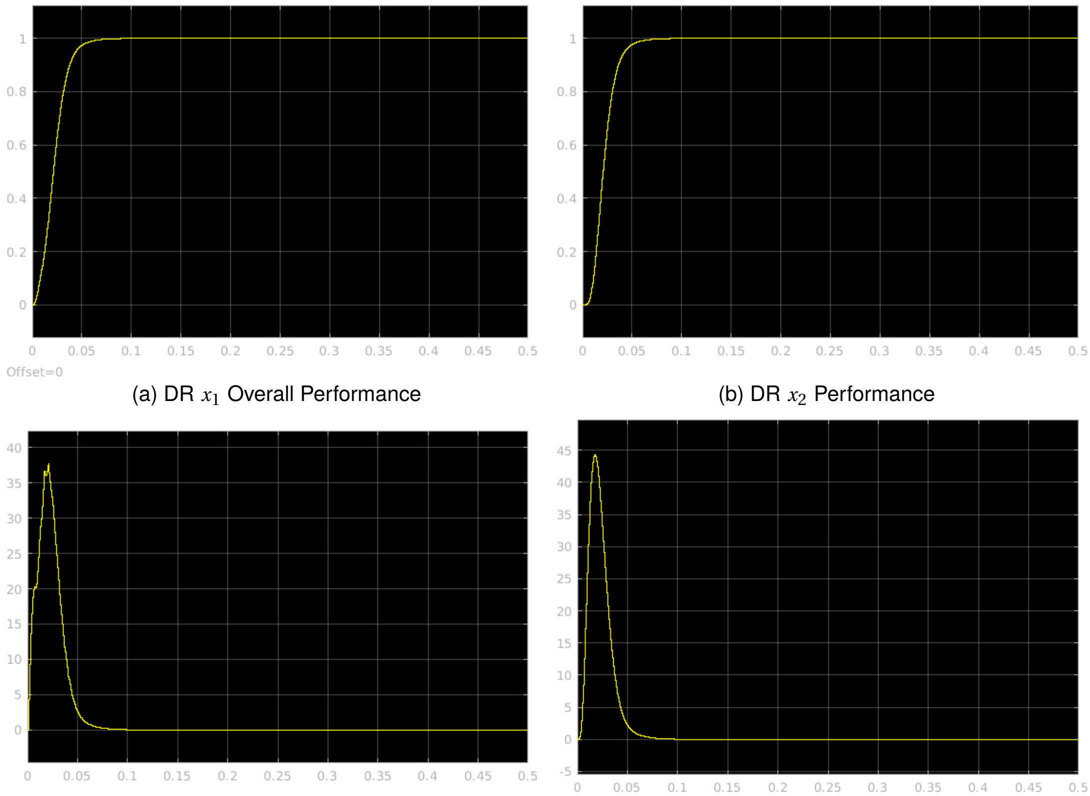


Figure 15: Design 3 MIL DR Input



(a) DR x_1 Overall Performance

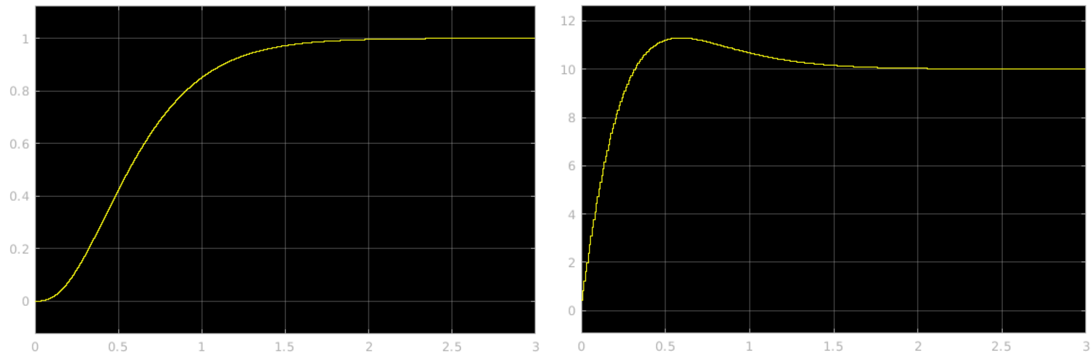
(b) DR x_2 Performance

(c) DR x_3 Performance

(d) DR x_4 Performance

Figure 16: Design 3 MIL DR Performance

For the DC Motor System, the value of output and input are shown in Figure 17. The settling time is approximately $1.592s$. All requirements are met.

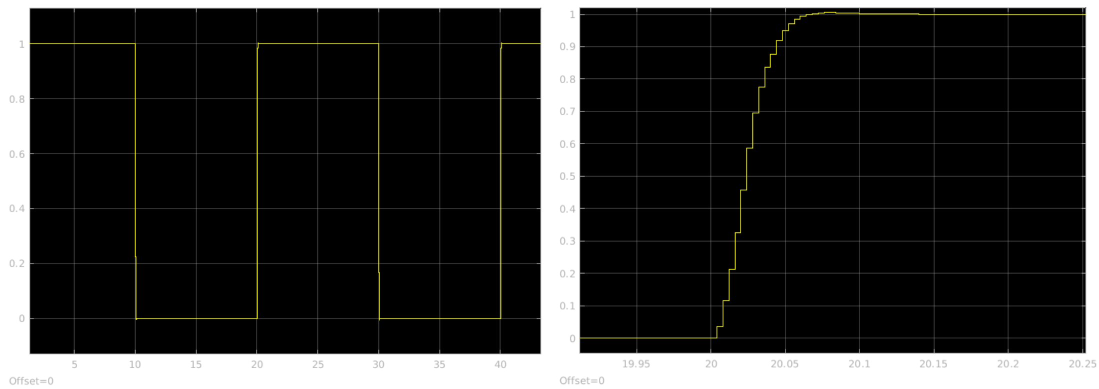


(a) Design 3 MIL DC Motor Output (b) Design 3 MIL DC Motor Input

Figure 17: Design 3 MIL DC Motor Performance

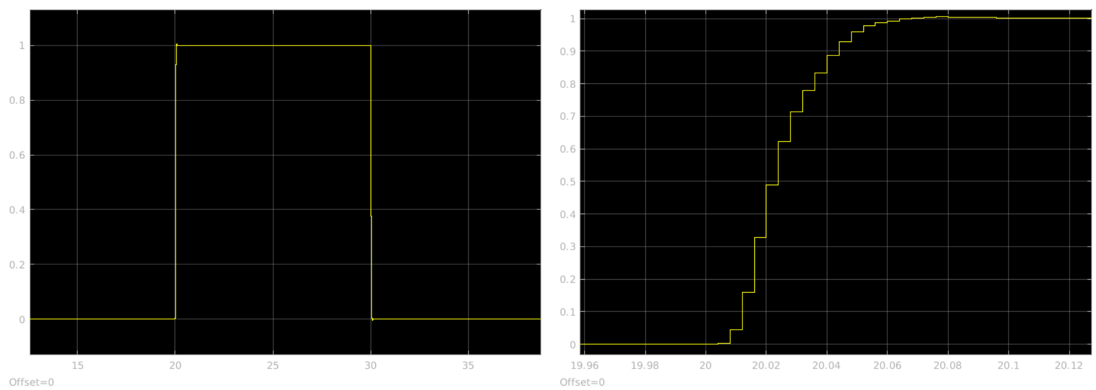
HIL Results

For the Dual Rotary system, the waveform of x_1 and x_2 are shown in Figure 18 and Figure 19. The waveform of x_3 and x_4 are shown in Figure 20. The settling time of x_1 and x_2 are $0.056s$, $0.056s$, respectively.



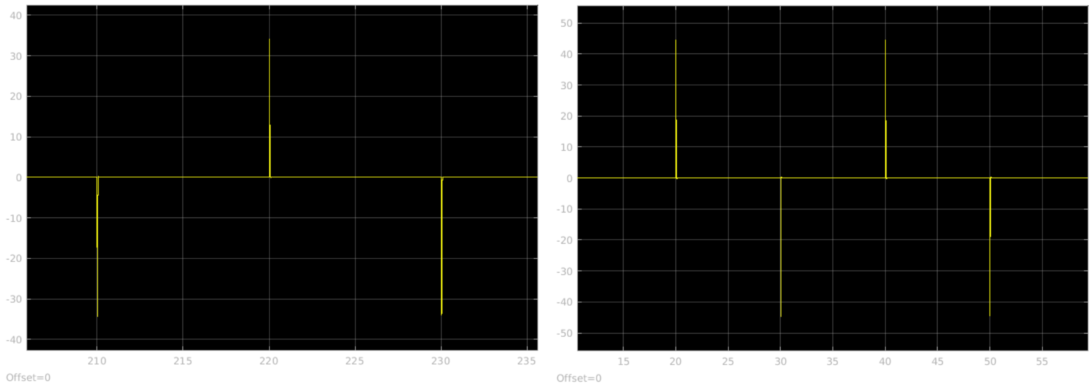
(a) DR x_1 Overall Performance (b) DR x_1 Settling Time

Figure 18: Design 3 TDM 1 DR x_1 Performance



(a) DR x_2 Overall Performance (b) DR x_2 Settling Time

Figure 19: Design 3 TDM 1 DR x_2 Performance



(a) DR x_3 Overall Performance

(b) DR x_4 Overall Performance

Figure 20: Design 3 TDM 1 DR x_3 x_4 Performance

The input of the Dual Rotary System is shown in Figure 21. The input is less than 1, it also meets the requirements.

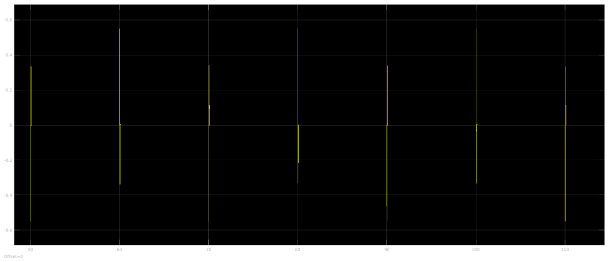
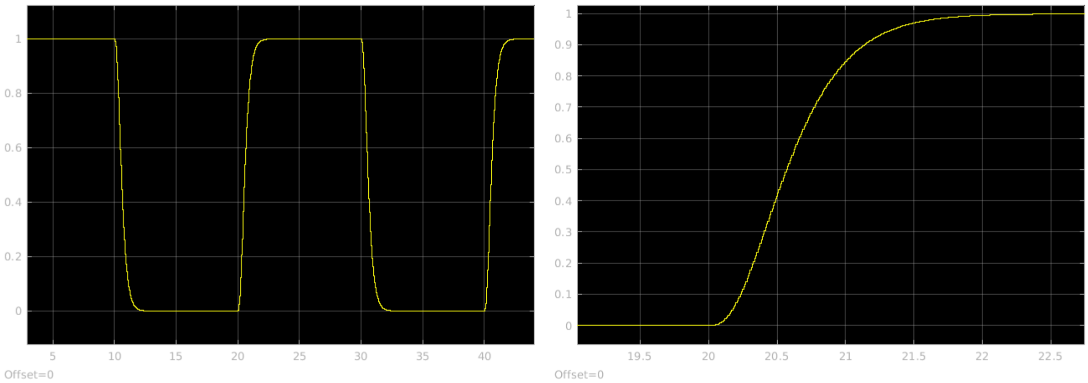


Figure 21: Design 3 TDM 1 DR input

The output of the DC Motor is shown in Figure 22. The input of the DC Motor system is shown in Figure 23. The DC Motor System meets the requirements.



(a) DCM Output Overall Performance

(b) DCM Output Settling Time

Figure 22: DCM Output Performance of Design 3, under TDM 2 configuration

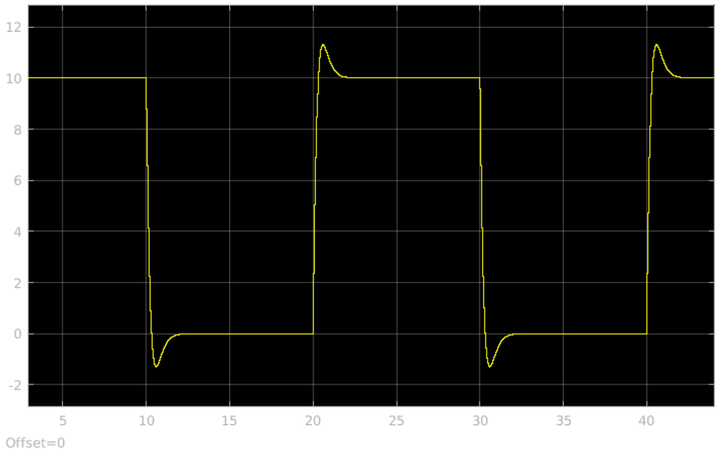


Figure 23: DCM input of Design 3, under TDM 2 configuration

4.2 design 4

In Design 4, we use the sampling time defined in Design 3 and consider the system has no delay. We also test the performance of the same poles placement in Design 3. Some requirements can not be met, in the Dual Rotary System, the value of x_4 will exceed 50, in the DC Motor Speed System, the control input will also exceed its boundary. After re-tuning, the poles settings are $[0.45, 0.45, 0.45, 0.45]$ and $[0.968, 0.968]$ for Dual Rotary System and DC Motor System, respectively.

$$\begin{aligned} K_{DR} &= \begin{bmatrix} 3.9253 & -4.4670 & -0.0171 & 0.0046 \end{bmatrix}, & F_{DR} &= 0.5447 \\ K_{DCM} &= \begin{bmatrix} -17.1693 & 1.8789 \end{bmatrix}, & F_{DCM} &= 8.3910 \end{aligned} \tag{9}$$

MIL Results

For the Dual Rotary System, the value of x_1, x_2, x_3 and x_4 is shown in Figure 25. The wave of input is shown in Figure 24. The settling time of x_1 is $0.052s$ and the settling time of x_2 is $0.052s$.

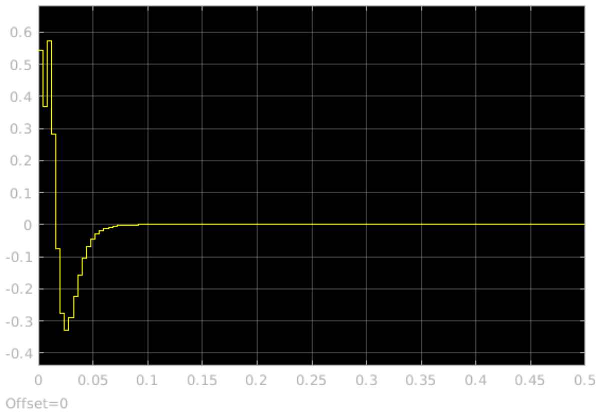
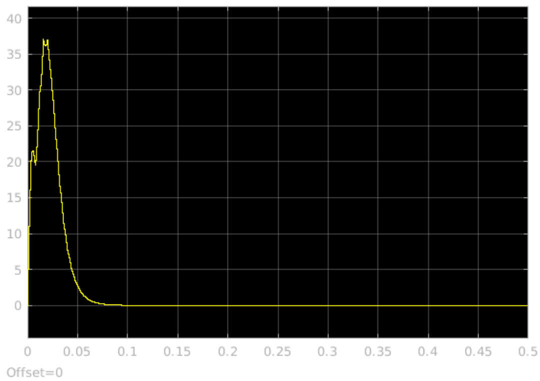
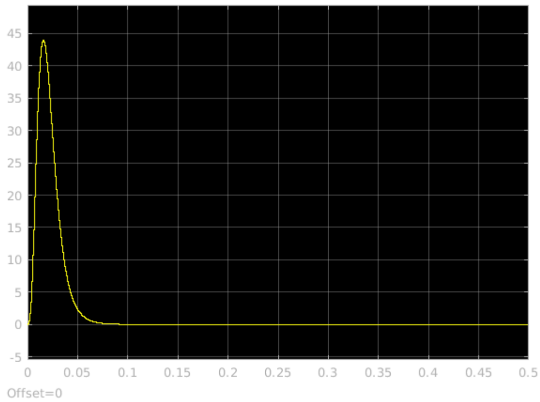


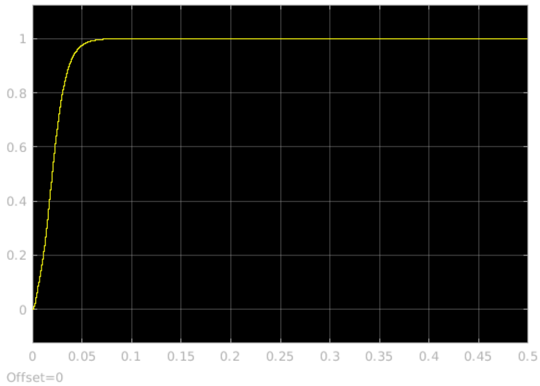
Figure 24: Design 4 MIL DR Input



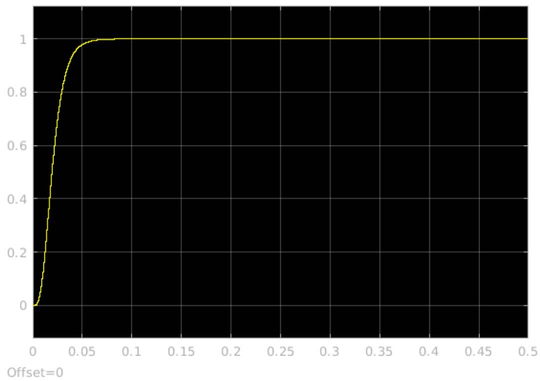
(a) DR x_1 Overall Performance



(b) DR x_2 Performance



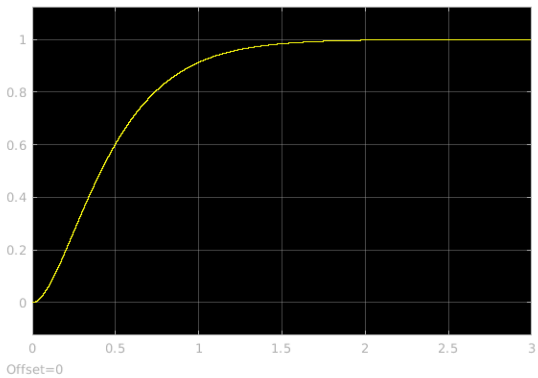
(c) DR x_3 Performance



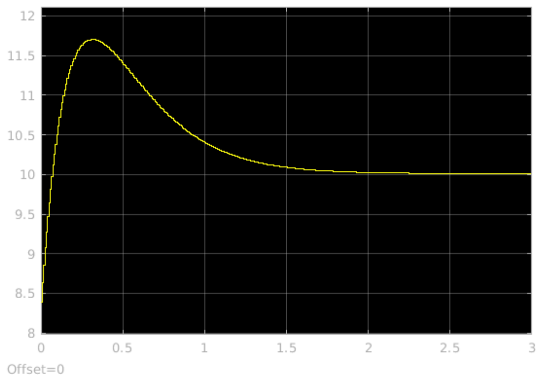
(d) DR x_4 Performance

Figure 25: Design 4 MIL DR Performance

For the DC Motor System, the value of output and input are shown in Figure 26. The settling time is 1.435s



(a) DCM Output



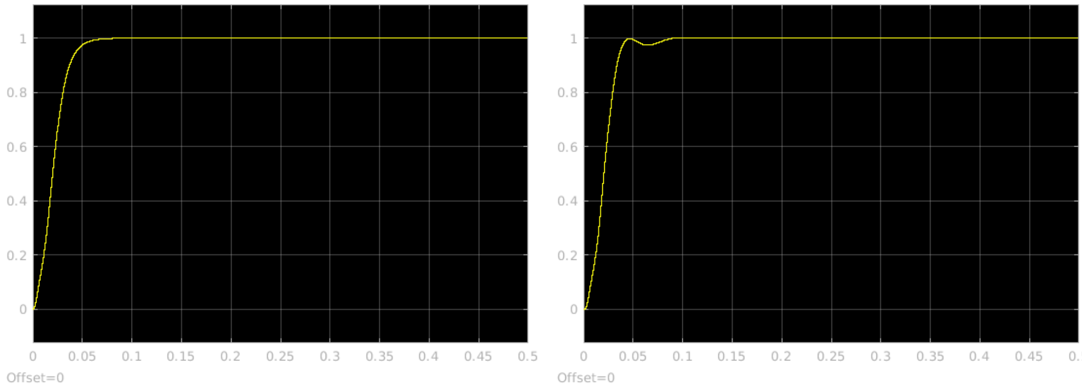
(b) DCM Input

Figure 26: Design 4 MIL DCM Performance

Comparison

Based on the HIL and MIL performance of Design 3, we find no significant difference between them, which might verify the correctness of our modeling and design.

Besides the result from the above section, we also test the result if we design the controller ignoring the delay τ.



(a) Designed without delay, simulated without delay (b) Designed without delay, simulated with delay

Figure 27: Design comparison under different simulation(with/without delay)

From figure 27, in this specific case, we design the controller without considering delay term, and we first simulate the model without introducing delay, as in the left part, then simulate it with delay, as in the right part. We observe that the system output of the delay system can reach 2% reference faster (it's less than 0.05s, while the non-delay system reaches 2% reference after 0.05s), but it also has an oscillation, leading the settling time longer than the requirement.

Our explanation for that phenomenon is, because of the delay lagging, even when the output reaches the reference, the control input would not go to 0, this might cause an oscillation.

5 Part 5 | Conclusion

Based on the design process and the system performance between design 1 and design 2, and between design 3 and design 4, several observations can be made:

If we fixed sampling period h and ignore sensor-to-actuator delay τ when designing the controller, the controller will get a input with higher magnitude if we use the controller in a system with sensor-to-actuator delay, which means we should use a pole configuration closer to the unit cycle.

If we fix sensor-to-actuator delay τ, and change sampling time h. With a smaller sampling time, the system will get a input with higher magnitude if we use the same poles configuration, which means we should use a pole configuration closer to the unit cycle.

Overall, the influence of sensor-to-actuator delay is really important if our physical system has some strict constraints on input and some states. We should always try to take it into consideration when designing control systems.