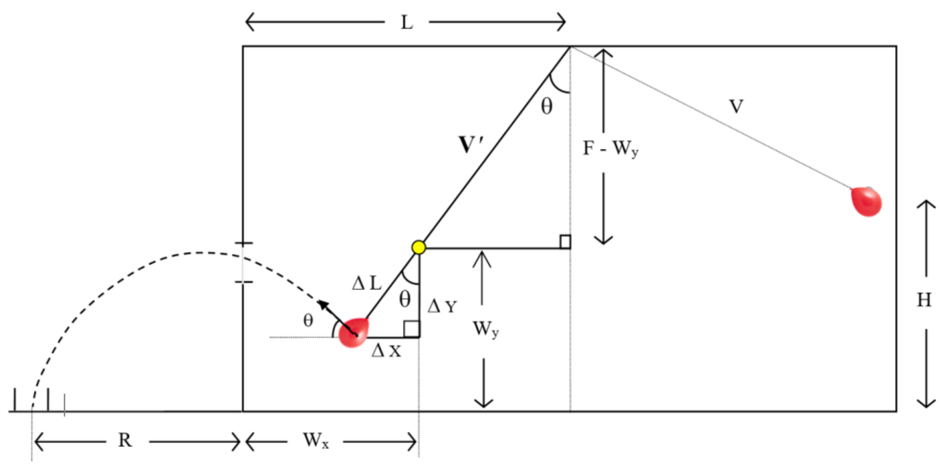
HELPING DOCUMENT

Solving for an exact solution in one try is extremely unlikely. You will need to use an iteration process to "narrow in" on a good solution. This will take time, so make sure you provide plenty of time to complete your calculations. You will have to work hard, but in the process you will develop new physical insight. You will also hone your mathematical skills.

The vine length is defined as the distance from the pivot to the center of the Tarzan water balloon. Thus, you must account for the size of the Tarzan water balloon so that Tarzan does not hit the hot wire. We have had plenty of Tarzan water balloons 'get a haircut' and pop on the hot wire through the years. Due to the low voltages used and the temperature of the hot wire, the water causes no issues when it hits the wire. A generic solution to this level, with explanation, follows.



Please refer to this schematic as needed when looking at the mathematical solution on the following pages.

$$\theta = \tan^{-1}\left[\frac{L - W_x}{F - W_y}\right]$$

This is the vertical angle of the vine when it is cut. It also is the angle of trajectory for the balloon.

$$\mathbf{V'} = \sqrt{(F - W_y)^2 + (L - W_x)^2}$$

$$V = \mathbf{V'} + \Delta L$$

$$\Delta Y = \Delta L \cos\theta$$
$$\Delta X = \Delta L \sin\theta$$

The value **V'** is the distance from the pivot location to the hot wire location. **The true vine length used (i.e., V) should be at least 2" longer than V' to account for the size of the Tarzan water balloon so that it does not hit the hot wire.** Whatever additional length one decides to use, we call this additional length ΔL in our equations at the left.

$$x_o = W_x - \Delta X$$
$$y_o = W_y - \Delta Y$$

This is the position of the balloon when the vine is cut.

$$v = \sqrt{2 g [H - y_o]}$$

This is the speed of the balloon at the time the vine is cut. It can be found from Conservation of Energy.

$$v_{ox} = v \cos\theta$$
$$v_{oy} = v \sin\theta$$

Thus, the x-component and the y-component of velocity can be found.

$$t = \frac{x_o}{v_{ox}}$$

The time it takes for the balloon to reach the cave plane is:

$$\boxed{y_f = y_o + v_{oy} \cdot t - \frac{1}{2} g t^2}$$

The height of the balloon at the time it crosses the cave plane (y_f) can now be solved.

The y_f value should match the cave height. If it does not, you must go back to the beginning of the problem and try changing something – perhaps L or V for example – and solve again. It is a process to narrow in on a correct solution. If the solution is close to the correct cave height, a small change in V (vine length) is simplest to try first since it doesn't change the angle of trajectory (θ).

Now we have the additional calculation for "net location" (**R**). If the catch container has height **N**, then the quadratic equation is solved for time, as shown:

$$y_f = y_o + v_{oy} \cdot t - \tfrac{1}{2} g t^2$$

$$\tfrac{1}{2} g t^2 - v_{oy} \cdot t + (N - y_o) = 0$$

$$t = \frac{v_{oy} \pm \sqrt{v_{oy}^2 - 2g(N - y_o)}}{g}$$

The correct time is then used to calculate the total x-distance the balloon travels...

$$\boxed{x = v_{ox} \cdot t}$$

The distance R is then:

$$\boxed{R = x - x_o}$$

Air resistance will continue to act on the balloon as it passes through the cave and heads for the container at position R. Which type of solution is more likely to be successful here, and why: a medium-speed path with little arcing, or a path with low speed, more height and more arcing?