A model coaxial photonic crystal for studying band structures, dispersion, field localization, and superluminal effects

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We use widely available electronic components and discuss a system in which the dispersion relation of a wave propagating in a periodic medium can be studied. Important effects related to periodic media are observed, including the energy band gap, field localization, and superluminal wave packet tunneling.
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I. INTRODUCTION

Although the study of band structures in solids is an important component of every physics curriculum, it is not a topic that yields easily to experimentation. Band structures arise from properties of solids at the atomic level, and therefore advanced experimental techniques are required to study them. As a result, most universities provide little or no means for their students to do experiments that illustrate these important concepts. Even the calculation of band structures from first principles requires fairly advanced mathematical tools, typically acquired only at the graduate level.

This paper describes an experiment that introduces undergraduate students to the concepts of dispersion relations and band gaps using widely available electronic components. At the same time, it demonstrates the phenomena of wave localization and superluminal tunneling, two important effects in solid state physics and optics.

The basic idea is to observe the propagation of electric waves in a conductor with periodic impedance, a "coaxial photonic crystal," and use it as an analogy for electron waves in an atomic crystal. Spatially periodic media such as photonic crystals, dielectric mirrors, and fiber Bragg gratings, exhibit bands of frequencies that do not propagate because of multiple reflections and destructive interference. These band gaps are similar to the ones observed in semiconductors and other crystals. The waves in each case are fundamentally different, electromagnetic versus matter waves, but the wave equations directing their behavior are mathematically equivalent,[1] except that electromagnetic fields are vectors and matter waves are described by a scalar.

In recent years, model coaxial photonic crystals have been used to duplicate a wide range of effects observed in conventional, nanometer-scale photonic crystals. Watson *et al.* showed the role of impurities (in this case, segments of mismatching length) in a periodic structure made with barrel and T coaxial connectors[2] and cable segments.[3] Periodic coaxial systems also have been used as a tool to study superluminal effects[4,5] and simulate optical multilayer devices such as Fabry–Perot resonators.[6] Superluminality occurs when the group velocity of a wave exceeds the speed of light in vacuum, and is possible only in a medium with anomalous dispersion (for which the refractive index decreases with increasing frequency),[7] a condition that is encountered near the band gap frequency of a photonic crystal. Coaxial photonic crystals are more attractive for this purpose than their optical counterparts because they are easier to work with and allow the phase and amplitude to be measured anywhere inside the structure. The time delay for pulse propagation also is measurable with typical electronic instruments. In contrast, in some earlier work on optical superluminality, time delays of the order of 1 fs had to be measured to demonstrate the effect.[8] When a pulse enters a periodic medium with anomalous dispersion, the peak of the transmitted pulse can sometimes exit the crystal before the peak of the incoming pulse enters the medium, thereby giving rise to a negative delay and group velocity. This effect was reported in coaxial crystals by Munday *et al.*[9] Finally, Poirier *et al.* showed that by including nonlinear semiconductor devices with the periodic cable structure, interesting nonlinear effects in signal transmission are obtained.[10]

This paper complements previous work[2–5,9,10] by focusing on the band structure and dispersive properties of a coaxial photonic crystal while giving an introductory level discussion of the theory and measurements of field localization and superluminal propagation.

II. EXPERIMENTAL ARRANGEMENT

A series of coaxial cables with alternating impedance is used to form a macroscopic lattice where multiple reflections and the interference of electrical signals can occur. Twelve unit cells, each made of 5 m of coaxial cable RG-58/U (50 Ω impedance) and 5 m of RG-59/U (75 Ω), were connected in a row, for a total length of 120 m. A periodic structure of this length acts essentially as a one-dimensional photonic crystal for radio frequencies (MHz). Both cable types have a phase velocity $v_\phi = 0.66c_0$, where c_0 is the speed of light in vacuum. Cable segments are joined with T connectors so that the electric signal may be measured at several places along the line. (Barrel and T connectors may be interchanged without affecting the properties of the system.) A standard wave generator capable of frequencies of up to 15 MHz (HP 33120A) is used to transmit sinusoidal or arbitrarily shaped waves, and a multi-channel digital oscilloscope measures the signal phase and the amplitude at various points in the structure. A 50 Ω termination is connected to the end of the system to avoid end reflections. The number and length of the unit cells used in the system may vary, but limitations are imposed by the cable attenuation (especially at high frequencies), the low transmission in the band gap, and the highest frequency accessible with the wave generator.

Group velocity measurement is done with the wave generator programmed to produce a sine wave with a Gaussian envelope with a duration of a few μs. (Pulses also can be obtained from the beat of two closely tuned frequencies.[9])

The oscilloscope measures the time delay between the input and output pulses through the crystal, from which the group velocity can be deduced.

III. THEORY

A. Dispersion relation

To predict the propagation of an arbitrary signal in a medium, it is necessary to know its dispersion relation or $\omega(k)$, where ω is the frequency of a wave with wavenumber k. The standard procedure for calculating $\omega(k)$ in photonic crystals is to find the eigenmodes of Maxwell's wave equation with a spatial periodic condition imposed on the dielectric constant.[1] However, this approach is not suitable here because we are considering a finite structure, and therefore the problem is better treated with a theory based on an effective index of refraction.[11] This theory is based on the idea that the phase difference between two points in a wave can be ascribed to an effective index of the medium, even though the medium may be inhomogeneous. The phase shift between the outgoing and incoming waves can be related to the overall index of refraction, and it is expected that the latter will depend on the frequency because the phase is greatly influenced by multiple reflections.

The problem of finding $\omega(k)$ is better posed by finding $k(\omega)$, because ω is the independent parameter, the wave generator frequency. We write

$$k(\omega)=\frac{n(\omega)\omega}{c_0}, \tag{1}$$

where $n(\omega)$ is the effective index of refraction of the medium. From $n(\omega)$ we may define an effective phase velocity $c=c_0/n$ (which is not necessarily equal to the phase velocity in a cable segment or the phase velocity in vacuum c_0). The task is then reduced to finding $n(\omega)$ from the phase shift $\Delta\phi$ of the wave after it travels a distance d in the medium:

$$\Delta\phi=kd=\frac{n(\omega)\omega}{c_0}d. \tag{2}$$

The phase shift is experimentally obtained by simultaneously measuring the wave at two separate points. If τ is the time delay between two successive peaks or two similarly related points on the wave, then

$$\Delta\phi=2\pi\frac{\tau}{T}=\tau\omega, \tag{3}$$

where T is the period of the wave. Equation (3) works provided that the two points are close enough so that the phase shift is smaller than 2π. If this is not the case, the proper multiple of 2π must be added to $\Delta\phi$. If we combine Eqs. (1)–(3), we obtain

$$k(\omega)=\frac{\tau\omega}{d}. \tag{4}$$

Equation (4) suggests a simple experimental way for measuring the band structure and dispersion.

B. Band gap

A wave is resonant with a periodic medium at the Brillouin zone boundary where the wave vector matches the lattice wave vector, or $k=\pm\pi/a$, where a is the lattice constant (in this case the unit cell length). This condition satisfies the Bragg condition for diffraction, which is the same for both electrons in solid crystals[12] and photons in photonic crystals.[1] The corresponding band gaps occur at frequencies

$$\omega_{\mathrm{bg}}=\pm\frac{\pi c}{a}, \tag{5}$$

and at every multiple of $2\omega_{\mathrm{bg}}$. In our experiment, with a phase velocity of about $0.66c_0$ and $a=10$ m, the first band gap is expected near 10 MHz, the second one at 30 MHz, etc. The unit cell length should be chosen so that ω_{bg} is smaller than the highest attainable generator frequency.

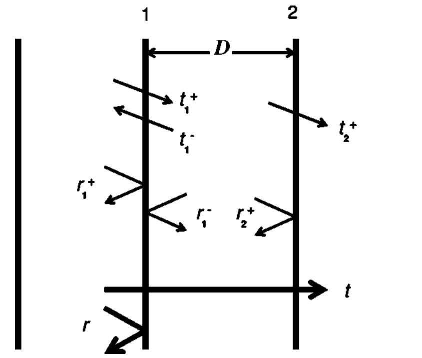

Fig. 1. The transmission and reflection through a cable segment located inside a periodic system are calculated from the multilayer theory. The superscripts + and − refer to waves traveling to the right and to the left, respectively, whereas the subscripts 1 and 2 refer to the first or second interfaces.

C. Transmission spectrum and group velocity

The transmittance spectrum of a periodic cable is calculated from the multi-layer theory.[13] Although the standard approach uses transfer matrices,[6] we will use an equally convenient but simpler recursive method that can easily be implemented in a computer program.

We start by calculating the Fresnel coefficients of reflection and transmission[13] at each segment end from the known cable impedances. For a wave traveling from a medium i to a medium j, they are $r_{ij}=(z_i-z_j)/(z_i+z_j)$ and $t_{ij}=2z_i/(z_i+z_j)$, where $z_{i,j}$ is the impedance of each medium. With z for one medium equal to 50 Ω and for the other $z=75\ \Omega$, $r_{ij}=\pm 0.2$ and $t_{ij}=0.8$ or 1.2.

Next, we determine the signal transmission through a given cable segment inside the coaxial crystal. For a segment of length D bounded by interfaces 1 and 2, the transmission and reflection coefficients through it are

$$t=\frac{t_1^+t_2^+e^{i\delta-\kappa D}}{1-r_1^-r_2^+e^{2i\delta-2\kappa D}}, \tag{6}$$

$$r=r_1^++\frac{t_1^+t_1^-r_2^+e^{2i\delta-2\kappa D}}{1-r_1^-r_2^+e^{2i\delta-2\kappa D}}, \tag{7}$$

where $\delta=\omega D/v_\phi$ is the phase shift and κ is the attenuation coefficient in the cable. The Fresnel coefficients must be calculated with the appropriate direction at each interface. The superscripts + and − correspond to the forward and backward components of the wave, respectively. Figure 1 shows the definitions of the Fresnel coefficients and other parameters. It should be pointed out that, in general, κ is not constant, but varies with frequency, and the best results are obtained when this dependence for each cable is included in Eqs. (6) and (7).

To obtain the transmission through the entire periodic cable, Eqs. (6) and (7) are first applied to the very last cable segment, and then are applied recursively back to the first segment. In other words, once t and r are found for one segment, they become t_2^+ and r_2^+, respectively, for the previous segment, and so on. For the end segment (the first one to be calculated), the transmitted impedance is the impedance of the oscilloscope or the terminator (50 Ω).

The transmission through the overall structure is a complex number from which the phase $\Delta\phi$ of the transmitted wave is obtained. If we write $t=|t|e^{i\phi_t}$, then

$$\phi_t = \arctan\frac{\mathrm{Im}\,t}{\mathrm{Re}\,t}, \tag{8}$$

and

$$\Delta\phi = \phi_t + m\pi, \tag{9}$$

where $m=0,1,2...$. Because $\Delta\phi\rightarrow 0$ as $\omega\rightarrow 0$, the exact phase can be calculated at any frequency by setting $m=0$ at $\omega=0$ and by adding π to every cycle of ϕ_t as ω increases. From $\Delta\phi$, the effective index of refraction of the coaxial crystal is obtained using Eq. (2), and from this index the dispersion relation is obtained via Eq. (1). From the definition of the group velocity, $v_g = d\omega/dk$, it is easy to derive the alternate equation

$$v_g = \frac{c_0}{n(\omega)+\omega\, dn(\omega)/d\omega}. \tag{10}$$

Note that if $n(\omega)+\omega dn(\omega)/d\omega < 1$, the group velocity is superluminal. This condition can only be satisfied when $dn(\omega)/d\omega < 0$, that is, in a region of anomalous dispersion.

D. Field distribution inside the periodic medium

It is useful and educational to numerically simulate the wave inside the coaxial crystal using a mechanical model. A series of masses coupled with springs accurately duplicates dispersion, band gaps, and other features of photonic crystals, because the wave equation for the mechanical wave is mathematically identical to Maxwell's wave equation. Furthermore, the model applies very well to nonsinusoidal waves such as pulses and gives the spatial and temporal evolution in addition to the transmission spectrum.

Consider an array of identical and equally spaced masses m connected to each other with identical springs with a Hooke's constant K. The masses are restricted to move longitudinally along the x axis. Newton's law of motion applied to one mass (labeled i) yields the equation

$$m\frac{d^2\psi_i}{dt^2} = K(\psi_{i+1} - 2\psi_i + \psi_{i-1}), \tag{11}$$

where ψ_i is the displacement of mass i relative to its equilibrium position (and the labels $i\pm 1$ correspond to its nearest neighbors). Equation (11) can easily be solved numerically for a large number of masses and arbitrary boundary conditions. For example, by forcing the first mass to move in a sinusoidal fashion, a harmonic wave is excited and carried collectively by the ψ_i's with a given phase velocity, thereby simulating a plane wave propagating in a homogeneous medium. The phase velocity is calculated in the following way. If Δl is the space between each mass at equilibrium and $\Delta l\rightarrow 0$ (or if sufficiently small relative to the wavelength of the wave), then ψ_i can be replaced by $\psi(x,t)$ and the right-hand side of Eq. (11) is approximated by a second-order derivative in x:

$$\frac{\partial^2\psi}{\partial t^2} = \frac{K}{m}\Delta l^2 \frac{\partial^2\psi}{\partial x^2}. \tag{12}$$

Equation (12) is a mechanical wave equation with phase velocity $v=\sqrt{K\Delta l^2/m}$ and is analogous to the electromagnetic wave equation, except that it applies to a longitudinal, scalar wave.[14] If we define $\rho = m/\Delta l$, the mass per unit of length in the mass-spring array, the phase velocity becomes

$$v = \frac{1}{\rho}\sqrt{Km}. \tag{13}$$

The mechanical model is interesting because it mimics the behavior of electromagnetic waves, and can be applied to an arbitrary medium and wave shape. For example, the crossing of an electromagnetic wave from one medium to another and the resulting partial reflection is simulated with the spring-mass model by tweaking K or m. When the mechanical wave crosses from a region with a phase velocity v_i to another one with velocity v_j, it experiences partial reflection and transmission for the same reason as the electromagnetic case: the field and frequency of the wave must be continuous across the interface. The corresponding Fresnel coefficients are, by analogy to the optical case,

$$r_{ij} = \frac{1/v_i - 1/v_j}{1/v_i + 1/v_j} \tag{14}$$

and

$$t_{ij} = \frac{2/v_i}{1/v_i + 1/v_j}, \tag{15}$$

which can be matched to the Fresnel coefficients at the interface of coaxial segments. By putting several mass-spring arrays in a row with alternating phase velocity, the mechanical model closely reproduces almost all features associated with one-dimensional photonic crystals, including band gap effects, anomalous dispersion, and superluminal tunneling. Moreover, the model is useful for determining the amplitude of the wave anywhere in the periodic structure and studying the localization of the field.

In order for the numerical solution of Eq. (11) to produce a steady state solution, the system must dissipate energy, and so it is convenient to add a small damping force on each of the masses [that is, a term $-A\partial\psi_i/dt$ on the right-hand side of Eq. (11)]. If no such force is added, the mass-spring system steadily increases its oscillation amplitude.

IV. RESULTS

Before calculating the transmission spectrum of the coaxial photonic crystal, the attenuation coefficient κ in plain cables (a few tens of meters each) was measured in the 0–15 MHz frequency range. The purpose of this measurement is to

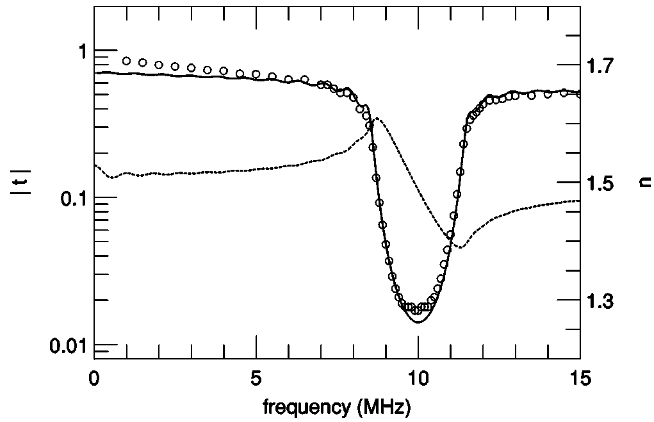

Fig. 2. Measured (circles) and calculated (solid curve) transmission through 120 m of periodic coaxial cable. The dashed curve is the effective index of refraction extracted from the calculated value of t.

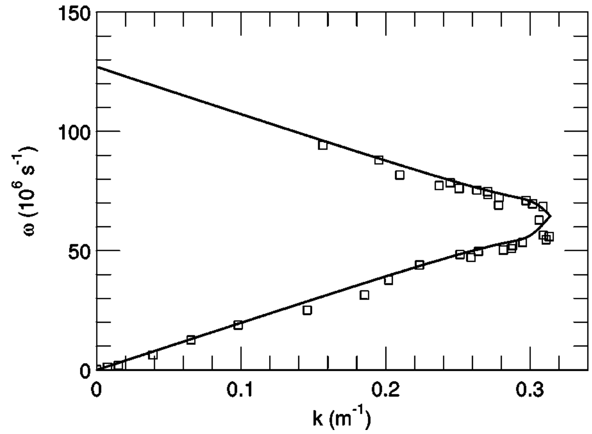

Fig. 3. The theoretical curve and data points for the dispersion relation and band structure of the 120 m coaxial photonic crystal. At the Brillouin zone boundary, the linear relationship between k and ω breaks down.

use accurate values of κ in Eqs. (6) and (7). The frequency dependence of the attenuation coefficient was experimentally found to be

$$\kappa(\omega)=-\ln(-1.7\times10^{-9}\omega+0.9928)/18.6 \tag{16}$$

in RG-58/U cables, and

$$\kappa(\omega)=-\ln(-1.3\times10^{-9}\omega+0.9328)/31.5 \tag{17}$$

in RG-59/U cables, where ω is the frequency of the wave in rad/s. To obtain these relations, sinusoidal electric waves were launched in a cable of length D and the ratio between the output and input amplitudes $A_{\mathrm{out}}/A_{\mathrm{in}}$ was measured at different frequencies. The value of κ was then extracted from the relation $A_{\mathrm{out}}/A_{\mathrm{in}}=\exp(-\kappa D)$, the usual Beer's law of attenuation.[15] Finally, a logarithmic fit was applied to the relation $\kappa(\omega)$ to obtain Eqs. (16) and (17).

Figure 2 shows the calculated and measured transmission spectra using these relations for $\kappa(\omega)$ and a sinusoidal wave crossing the 120 m system. Here $|t|$ represents the ratio between the input and output signal amplitudes. No parameter was adjusted for the theory, and the calculation is based on the cable segment lengths and the specified phase velocities and impedances. The data are in good agreement with theory, and the deep band gap is centered at 10 MHz. The effective index of refraction was calculated from the complex value of t (see Sec. III C). Note that the dispersion is anomalous between 9 and 11 MHz.

Dispersion relations are obtained using Eq. (1) and the results of Fig. 2. The calculated bands folded at the first Brillouin zone ($k=0.314\ \mathrm{m}^{-1}$) for a coaxial crystal of 120 m in length are plotted in Fig. 3. The data points were obtained from the phase difference method we have discussed [and Eqs. (3) and (4)] by measuring the wave at two points separated by 10 m (one unit cell). As can be seen, the linear dependence between ω and k breaks down at the Brillouin zone boundary and becomes approximately parabolic as in semiconductor band structures. In this resonant region the effective index of refraction varies most rapidly with frequency (see Fig. 2). Unlike ideal, infinite crystals where a sharp jump in frequency occurs at the zone boundary, the transition in this case is gradual, and the dispersion is obtained inside the gap because the transmitted signal is still measurable. Although the dispersion curve is calculated for frequencies up to 20 MHz, the data were limited to the highest experimentally attainable frequency, 15 MHz.

In a periodic medium, multiple reflection can be a strong process causing the electric field to be localized, a phenomenon that was pointed out in the early discovery of photonic crystals.[16] Because of this scattering-like phenomenon, the electric field decreases exponentially with the distance to its emitter, even when the constituent material is transparent. In a microscopic structure such as a photonic crystal, this localization effect is not directly observable, and hence there is interest in a macroscopic periodic system like this one where the electric field distribution can be fully characterized. Figure 4 shows the electric signal amplitude as a function of the distance from the wave generator for a sine wave at 10 MHz. The standing wave at the anti-nodes (maxima) fits the exponentially decaying curve expected from theory. As expected, localization does not occur as strongly at frequencies away from the band gap. In fact, the slow amplitude decay at 5 MHz is mostly due to attenuation in the cable itself rather than the band gap.

The mass-spring model result in Fig. 4 is for 40 masses per cable segment and yields qualitatively satisfying results for the amplitude distribution. However, there is a $\pi/2$ phase

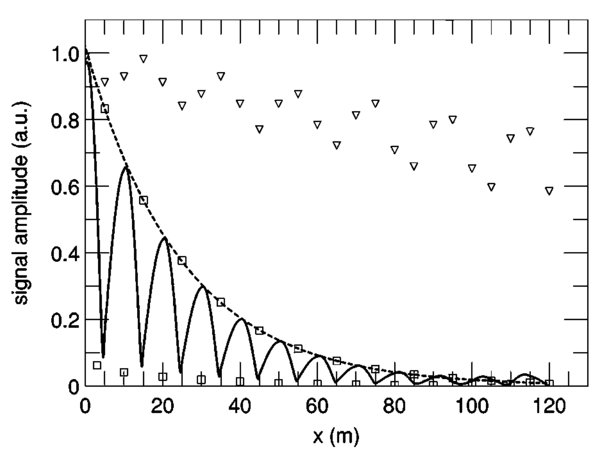

Fig. 4. The electric field amplitude inside the cable for a sinusoidal signal at 10 MHz (squares) and 5 MHz (triangles). The solid curve is the result from the mass-spring model, and the dashed line is the best fit to the function exp $(-Ax)$ with $A=0.04$.

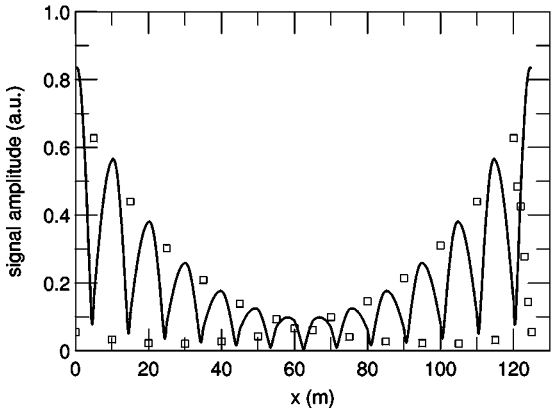

Fig. 5. The electric field distribution in a symmetrical closed loop with a signal at a frequency of 10 MHz. The data points represent the field amplitude measured locally, and the solid curve is the simulation results using the mass-spring model. The two end points, $x=0$ and 125 m, are joined.

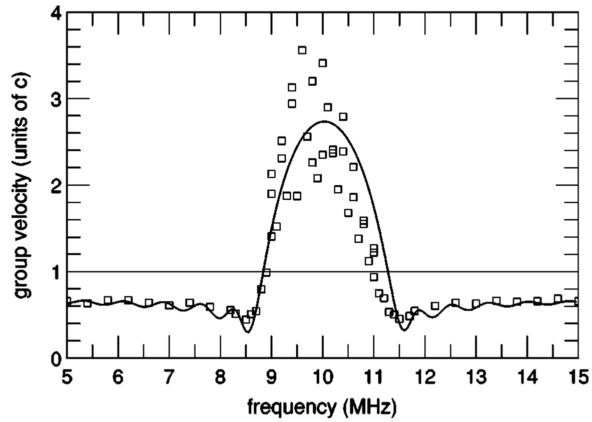

Fig. 6. Measured (squares) and calculated (solid curve) group velocity of a pulse in 120 m of coaxial crystal.

shift between the model and the data because the boundary conditions in each case are different. In the simulation, the first mass is forced to oscillate with fixed amplitude whereas, in reality, a node appears there, as it should at the ends of each unit cell. As a result, the minima and maxima in the simulation and the data appear out of phase. A partial solution to this problem would be to take a snapshot of the wave at a time when the first mass is at zero, but although the curve would be offset to match the data, it would not exactly represent the real spatial amplitude of the wave.

Other configurations can be used to observe more field distributions. For example, by connecting the last and first segments, a loop is obtained in which the wave from the generator splits into two equal counter-propagating components. The result is a symmetric field distribution on each side of the emitter, as shown in Fig. 5 (the signal is at 10 MHz). Here, an additional segment of 5 m of 50 Ω was added at the end to make the loop perfectly symmetrical and 125 m in length. Interestingly, closing the loop imposes a boundary condition similar to the Born–von Karman periodic boundary condition used in solid-state physics which leads to the discretization of the wave vector.[17] Similarly, it can be shown experimentally that in this configuration the smallest increment of k is $4\pi n/L$, where L is the crystal length and n is the effective index.

Figure 6 shows the measured group velocity for a 4 μs Gaussian pulse envelope containing various carrier frequencies. In agreement with the theory, superluminal velocities are observed inside the band gap, where dispersion is anomalous, whereas values close to the phase velocity are seen elsewhere. Group delays of the order of 150 ns were measured at 10 MHz, giving group velocities near $3c_0$. To limit the effect of noise, the time delay between the input and output pulses was measured not from the peak of each, but from the "center of mass," according to

$$t_{\mathrm{cm}}=\frac{\int t|E(t)|dt}{\int|E(t)|dt}, \tag{18}$$

where $|E|$ is the amplitude of the electric signal. This method of measuring delays is especially useful at frequencies in the band gap where the transmitted signal has a lower signal-to-noise ratio. The theoretical group velocity was calculated with Eq. (10) and from the effective index of refraction. This superluminal effect is similar to quantum tunneling of a wave packet through a barrier, in which case the velocity of the leaked wave function also is known to be superluminal.[18]

V. CONCLUSIONS

We have demonstrated that a coaxial model of a photonic crystal is a useful system for exploring experimentally many effects generally encountered in periodic media. A band gap at radio frequencies was observed and the dispersion relations were measured and found to be similar to ones observed in optical photonic crystals and crystalline solids. The field distribution measured inside the cable structure is in good agreement with a simple mass-spring model. The group velocity of electrical pulses also was measured and found to exceed the speed of light in vacuum for a frequency band centered at the band gap in accordance with theory.

The model coaxial photonic crystal is an educational tool with the advantages of being easily modifiable and only requiring equipment that is widely used in undergraduate laboratories. It is rich in possibilities as components can be added (for example, segments of various impedance or length), allowing students to make and verify their own theoretical predictions. The model coaxial photonic crystal also is a unique medium for demonstrating superluminal effects with electrical signals, as no equivalent system as simple as this one exists.

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