Unit 3

Sunday, July 25, 2021 2:51 AM

6.2. $x = e^{rt} \Rightarrow mr^2 + br + K = 0$

$$r = \frac{-b \pm \sqrt{b^2 - 4mK}}{2m}$$

$$r = -s \pm i\omega_d$$

$$x = e^{rt} = e^{(-s \pm i\omega_d)t} = e^{-st}\left(e^{\pm i\omega_d t}\right)$$

$$e^{-st}\left(e^{i\omega_d t}\right) = e^{-st}(\cos\omega t + i\sin\omega t)$$

$$= e^{-st}\cos\omega t,\ e^{-st}\sin\omega t$$

$$(c+id)\,e^{-s+i\omega t} + (c-id)\,e^{-s-i\omega t}$$

Saved not Submitted.

$$y = C_1 e^{(-s+i\omega)t} + C_2 e^{(-s-i\omega)t}$$

$$\frac{\sqrt{b^2-4mK}}{2m} = i\frac{\sqrt{4mK-b^2}}{2m} = i\sqrt{\frac{K}{m} - \frac{b^2}{4m^2}} = i\omega_d$$

$a, b, \phi \in \mathbb{R}$

$A \geq 0 \in \mathbb{R}$

$$a\cos\theta + b\sin\theta = A\cos(\theta - \phi)$$

$$= \underbrace{A\cos\phi}_{a}\cos\theta - \underbrace{A\sin\phi}_{b}\sin\theta$$

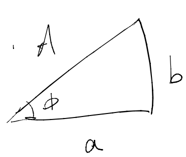
$A = \sqrt{a^2+b^2}$ ϕ : Angle between o&N and (a,b)

Rex is good. $\theta = \omega t$ for $\omega > 0$

$$-\cos(5t) - \sqrt{3}\sin(5t)$$

$$A = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2 \quad \phi =$$

$A = \sqrt{a^2+b^2}$ ✓



$$\phi = \operatorname{atan}\left(\frac{b}{a}\right)$$

Use the following Diagram for ϕ

$$\bar{c} = Ae^{i\phi} = a + bi \Rightarrow c = Ae^{-i\phi} = a - bi$$

$$\mathrm{Re}\left(ce^{i\omega t}\right) = \mathrm{Re}\left(Ae^{-i\phi}e^{i\omega t}\right)$$

$$= \mathrm{Re}\left(Ae^{i(\omega t - \phi)}\right) = A\cos(\omega t - \phi)$$

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3 ways of writing a Sinusoid function.

$A\cdot\cos(\omega t-\phi)$ $\quad A,\phi\in\mathbb{R},\ A\geq 0$

$\mathrm{Re}(ce^{i\omega t})$ $\quad c\in\mathbb{C}$

$a\cos\omega t + b\sin\omega t$

Assume $c = Ae^{-i\phi} = a - bi$

$\mathrm{Re}(ce^{i\omega t}) = \mathrm{Re}(Ae^{-i\phi}e^{i\omega t}) = A\cos(\omega t-\phi)$

$= \mathrm{Re}((a-bi)(\cos\omega t + i\sin\omega t))$

$= a\cos\omega t + b\sin\omega t$

$\cos(x-y) = \cos x\cos y + \sin x\sin y$

$A\cos(\omega t-\phi) = A(\cos\omega t\cos\phi + \sin\omega t\sin\phi)$

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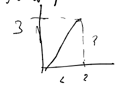
green (up) = orange (lower) + blue (lower)

Why have answers $\quad \phi = \mathrm{atan}$



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e

$$\frac{e^{i\omega t}}{2+3i} = \frac{e^{i\omega t}}{\sqrt{13}\,e^{i\,\mathrm{atan}(3/2)}} = \frac{1}{\sqrt{13}}\left(e^{i(\omega t-\mathrm{atan}(3/2))}\right)$$

$$\Rightarrow \frac{1}{\sqrt{13}}\left(\cos\left(\omega t-\mathrm{atan}\tfrac{3}{2}\right)\right) + i\sin\left(\omega t-\mathrm{atan}\left(\tfrac{3}{2}\right)\right)$$

$$\Rightarrow \mathrm{Re}\left(\frac{1}{\sqrt{13}}\cos(\omega t-\phi)\right) \quad \phi = \mathrm{atan}(3/2)$$

$$\frac{1}{\sqrt{13}}\cos(\omega t-\phi) \Rightarrow a\sin\theta + b\cos\theta$$

Rect Form: $\mathrm{Re}\left\{\frac{e^{i\omega t}}{2+3i}\right\} = \mathrm{Re}$

$$\frac{\cos\omega t + i\sin\omega t}{2+3i}\times\frac{2-3i}{2-3i} =$$

$$\frac{2\cos\omega t + 3\sin\omega t}{13}$$

Circular frequency of $\frac{e^{i\omega t}}{2+3i}$ ω labeled

ω?

Amplitude? $= \frac{1}{\sqrt{13}}$ Yes.

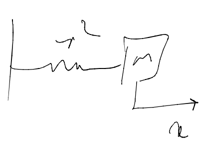
Phase lag $= \arctan(3/2) = \phi$

Sketch the diagram!

$$Re\left(\frac{1}{\sqrt{13}} e^{i(\omega t - \phi)}\right) = \frac{1}{\sqrt{13}} \cos(\omega t - \phi)$$

$\cos t + \pi/2$ $\quad \cos(\omega t) = \frac{\pi}{2\omega}, \frac{3\pi}{2\omega}$

Let's solve it with no initial velocity.



$K(0) = \pi^2 \times 1 = \pi^2$

$$F = ma = -\overset{\pi^2}{K}x = \overset{1}{m}\frac{d^2x}{dt^2}$$

$$-\pi^2 x = \frac{d^2x}{dt^2} \qquad x = e^{rt}$$

$\to -\pi^2 \cancel{e^{rt}} = r^2 \cancel{e^{rt}} \Rightarrow -\pi^2 = r^2$

$\Rightarrow r = \pi i$

$x = e^{i\pi t} = \cos \pi t + i \sin \pi t$

$x = C_1 \cos \pi t + C_2 \sin \pi t$

Pr for $x(0) = C_1 = -1$

$x' = -C_1 \pi \sin \pi t + \pi C_2 \cos \pi t$

$x'(0) = \pi C_2 \qquad x'(0) = \pi \Rightarrow C_2 = 1$

$\Rightarrow x = -\cos \pi t + \sin \pi t$

Walk through it one more time.

Hooke's law: $F = m\ddot{x} = -kx$

$k = \pi^2 \quad m = 1 \Rightarrow \ddot{x} = \pi^2 x$

$\Rightarrow x = a\cos(\pi t) + b \sin(\pi t)$

$x(0) = a = -1$

$x' = \pi a \sin(\pi t) + b\pi \cos(\pi t)$

$x'(0) = b\pi \qquad x'(0) = \pi$

$\longrightarrow$

$\Rightarrow b = 1$

$\Rightarrow \boxed{x = -\cos(\pi t) + \sin(\pi t)}$

$x = A\cos(\omega t - \phi)$

$A = \sqrt{a^2 + b^2} = \sqrt{2}$ where $\omega = \pi$

$a = A\cos\phi$ $b = A\sin\phi$

$a = -1 = \sqrt{2}\cos\phi$ $b = 1 = \sqrt{2}\sin\phi$

$\Rightarrow \frac{b}{a} = -1 = \tan\phi$ $a = -1$ $b = 1$

$\frac{3\pi}{4}$

$\Rightarrow \sqrt{2}\cos(\pi t - \frac{3\pi}{4})$

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t in secs. $\omega = \frac{1}{\text{sec}}$

Period: secs secs/cycle

fo: seconds

t_0 : Seconds

Freq $= \frac{1}{P} =$ cycles / sec $\quad$ Sec^{-1}

$\sqrt{2}\cos(\pi t - \frac{3\pi}{4}) = x(t)$

$P = \frac{2\pi}{\omega}$ since adding $\frac{2\pi}{\omega}$ to t

the $\omega t - \phi \to \omega(t + \frac{2\pi}{\omega}) - \phi$

$= \omega t + 2\pi - \phi$

$t_0 = \frac{\phi}{\omega}$ since $\omega t - \phi = 0$

$\omega(\frac{\phi}{\omega}) - \phi = 0$

$P = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

$t_0 = \frac{\phi}{\omega} = \frac{\frac{3\pi}{4}}{\pi} = \frac{3}{4}$

$\sqrt{2}\cos(\pi t - \frac{3\pi}{4})$ Sketch!

$t = (1 + \frac{3}{4}) = \frac{7}{4}$

Real eg $x(t) = \sin(t) + A\sin(\omega t)$

$f(t) \; eg: f(t) = \sin(t) + A\sin(\omega t)$

$\mathrm{Im}\left(R(t)\, e^{i\theta(t)}\right) = R(t)\sin(\theta(t))$

$$\frac{e^{it}-e^{-it}}{2i} + A \times \frac{e^{i\omega t}-e^{-i\omega t}}{2i} = \frac{1}{2i} \times$$

$$e^{it} - e^{-it} + A\left(e^{i\omega t} - e^{-i\omega t}\right)$$

Ex.

$\cos 2t + \sin 2t \qquad A = \sqrt{2}\cos\left(2t - \frac{\pi}{4}\right)$

$A = \sqrt{4} = 2 \qquad \omega = \pi \qquad \phi = \tan(-\sqrt{3})$

Ex.

$\sqrt{a^2+b^2} = 5 \Rightarrow a^2+b^2 = 25$

$\omega = 3, \quad \mathrm{atan}\left(\frac{b}{a}\right) = \frac{3\pi}{4}$??? Kvadrant

$\frac{b}{a} = -1 \qquad 2a^2 = 25 \Rightarrow a = \frac{5}{\sqrt{2}}$

$F = m\ddot{x} = -kx - b\dot{x} \qquad x = e^{rt}$

$\ddot{x} = -\frac{17}{16}x - \frac{1}{2}\dot{x} \qquad r^2 + \frac{1}{2}r + \frac{17}{16} = 0$

$r^2 = -\frac{17}{16} - \frac{1}{2}r \qquad \sqrt{\frac{4}{16} - \frac{4 \times 17}{16}} = 2i$

$$\frac{-\frac{1}{2} \pm 2i}{2} = -\frac{1}{4} \pm i = r$$

$$x = e^{\left(-\frac{1}{4}+i\right)t} = e^{-\frac{t}{4}}(\cos t + i\sin t)$$

$x = e^{-\frac{t}{4}}(C_1 \cos t + C_2 \sin t)$

$x(0) = 1 = C_1 \cos 0 \Rightarrow C_1 = 1$

$x'(0) = -\frac{1}{4} e^{-\frac{t}{4}}(C_1 \cos t + C_2 \sin t)$

$+ e^{-\frac{t}{4}}(-C_1 \sin t + C_2 \cos t)$

$t=0 \Rightarrow -\frac{1}{4}(1) + C_2 = \frac{3}{4}$

$A = \sqrt{2} \quad \Rightarrow C_2 = 1$

$x(t) = \sqrt{2}\, e^{-\frac{t}{4}} \cos(t - \frac{\pi}{4})$ No telling

$E = \sqrt{2}\, e^{-\frac{t}{4}} \qquad E(0) = \sqrt{2}$

$\frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\, e^{-\frac{t}{4}}} = 2 \Rightarrow \frac{1}{2} = e^{-\frac{t}{4}}$

$\Rightarrow \ln \frac{1}{2} = -\frac{t}{4}$

$\Rightarrow -\ln 2 = \frac{-t}{4} \Rightarrow 4 \ln 2$

$\cos(\omega t - \phi) \Rightarrow P = \frac{2\pi}{\omega}$

$\cos(\omega(t+P) - \phi) = \cos(\omega t + 2\pi - \phi)$

$= \cos(\omega t - \phi)$

$P = \frac{2\pi}{1} = 2\pi \quad f = \frac{1}{P} = \frac{\omega}{2\pi} = \frac{1}{2\pi}$

$\cos\left(t - \frac{\pi}{a}\right) \quad t - \frac{\pi}{a} \sim \quad \to \quad t \sim \frac{\pi}{a}$

$\cos\left(t - \frac{\pi}{a}\right) = 0 \qquad \frac{3\pi}{4}, \quad \frac{7\pi}{4}$

$\frac{\pi}{2} + \frac{\pi}{4}$

$\updownarrow \qquad\qquad \frac{11\pi}{4}, \quad \frac{15\pi}{4}$

$\frac{3\pi}{2} + \frac{\pi}{4}$

$\ddot{x} + 4x = 0 \qquad r^2 + 4 = 0$

$r = \pm 2i$

Cos $\quad \omega_0 =$ frequency $\qquad P = \frac{2\pi}{\omega} = \pi$

$m\ddot{x} + b\dot{x} + kx = 0 \qquad mr^2 + br + k = 0$

roots: $\frac{-b \pm \sqrt{b^2 - 4mk}}{2m} = \frac{-b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}$

$P = \frac{b}{2m}, \quad \omega_n \; \omega_0 = \sqrt{\frac{k}{m}} \Rightarrow$ Base / Natural frequency

Roots: $-P \pm \sqrt{P^2 - \omega_0^2} \implies$ Keep this in Mind

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Underdamped Case: $b^2 < 4mk \to$ Complex roots

$\omega_d = \frac{\sqrt{4mk - b^2}}{2m} = \sqrt{\omega_n^2 - P^2}$

$\Rightarrow$ Roots $= -p \pm i\omega_d$ $\quad e^{(-p+i\omega)t}, e^{(p-i\omega)t}$

$\Rightarrow A e^{-pt} \cos(\omega_d t - \phi)$

Notice that the damping has changed the angular frequency from ω_n to ω_d

The oscillation is not Harmonic anymore

$$\omega_d = \sqrt{\omega_n^2 - p^2} \qquad \text{if } b \to 0 \to \omega_d = \omega_n$$

$$\omega_d = \sqrt{\frac{K}{m} - \frac{b^2}{2m}}$$

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overdamped $\Rightarrow b^2 > 4mK$.

roots $= \dfrac{-b \pm \sqrt{b^2 - 4mK}}{2m}$ Both roots are real

and negative since $\sqrt{b^2 - 4mK} < b$

$\Rightarrow$ Calling the roots $-S_1$ and $-S_2$, the solutions

become $a e^{-S_1 t} + b e^{-S_2 t}$

becomes $ae^{-S_1 t} + be^{-S_2 t}$

the less negative root Controls the rate of return to equilibrium.

$x = ae^{-S_1 t} + be^{-S_2 t}$ $t \geq 0$

$\dot{x} = -S_1 a e^{-S_1 t} + -S_2 b e^{-S_2 t}$

$\dot{x}(0) = -S_1 a - S_2 b = 0 \Rightarrow S_1 a = -S_2 b$

$a = \frac{-S_2}{S_1} b$

for $t > 0$

$\Rightarrow x = \frac{-S_2}{S_1} b e^{-S_1 t} + b e^{-S_2 t} \neq 0$

$X_0 = \frac{-S_2}{S_1} e^{-S_1 t} + e^{-S_2 t}$

if $S_2 = S_1 + \varepsilon$

$\Rightarrow \frac{-S_1 - \varepsilon}{S_1} e^{-S_1 t} + e^{-(S_1 + \varepsilon)t}$

$= \left(-1 - \frac{\varepsilon}{S_1}\right) e^{-S_1 t} + e^{-S_1 t} e^{-\varepsilon t}$

$\Rightarrow e^{-S_1 t} \left(-1 - \frac{\varepsilon}{S_1} + e^{-\varepsilon t}\right)$

$$\Rightarrow \quad 1 + \frac{\varepsilon}{s_1} > e^{-\varepsilon t} \quad \text{for } t > 0$$

$$\text{if } \varepsilon > 0 \rightarrow \quad e^{-\varepsilon t} < 1 \quad \text{for } t > 0$$

$$X_0 = \frac{-s_2}{s_1} e^{-s_1 t} + e^{-s_2 t} \qquad \text{if } s_2 = s_1 - \varepsilon$$

$$\Rightarrow \frac{-s_1 + \varepsilon}{s_1} e^{-s_1 t} + e^{-(s_1 - \varepsilon) t}$$

$$= \left(-1 + \frac{\varepsilon}{s_1}\right) e^{-s_1 t} + e^{-s_1 t} e^{\varepsilon t}$$

$$\Rightarrow e^{-s_1 t}\left(-1 + \frac{\varepsilon}{s_1} + e^{\varepsilon t}\right) > 0$$

~~Oe~~ out the other way.

$$-1 + \frac{\varepsilon}{s_1} + e^{\varepsilon t} > 0$$

$$\frac{\varepsilon}{s_1} + e^{\varepsilon t} > 1$$

$$\text{if } \varepsilon \rightarrow 0^+ \rightarrow 1 + \varepsilon > 1$$

$$x = ae^{-s_1 t} + be^{-s_2 t}$$

Assume $x(t_1) = 0 \Rightarrow ae^{-s_1 t_1} + be^{-s_2 t_1} = 0$

Cross means non zero derivative $\quad -as_1 e^{-s_1 t_1} - bs_2 e^{-s_2 t_1} \neq 0$

t_1 is unique $\quad ae^{-s_1 t_2} + be^{-s_2 t_2} = 0$

$t_2 = t_1 + \varepsilon \quad -as_1 e^{-s_1 t_2} - bs_2 e^{-s_2 t_2} \neq 0$

$$ae^{-s_1 (t_1 + \varepsilon)} = -be^{-s_2 (t_1 + \varepsilon)}$$

$$-as_1 e^{-s_1 (t_1 + \varepsilon)} - bs_2 e^{-s_2 (t_1 + \varepsilon)} \neq 0$$

$$\left.\begin{aligned} ae^{-s_1 t_1} &= -be^{-s_2 t_1} \\ -as_1 e^{-s_1 t_1} - bs_2 e^{-s_2 t_1} &\neq 0 \end{aligned}\right\} \begin{aligned} &+s_1 b e^{-s_2 t_1} - bs_2 e^{-s_2 t_1} \neq 0 \\ &\rightarrow s_1 - s_2 \neq 0 \end{aligned}$$

assuming $b \neq 0$

$$x(t_1) = 0 = C_1 e^{-s_1 t_1} + C_2 e^{-s_2 t_2}$$

$$\Rightarrow C_1 e^{-s_1 t_1} = -C_2 e^{-s_2 t_1}$$

$$\Rightarrow \frac{-c_2}{c_1} = e^{(s_1-s_2)t_1}$$

$$\Rightarrow \frac{\ln\left(\frac{-c_2}{c_1}\right)}{s_1-s_2} = t$$

So if $\frac{-c_2}{c_1} > 0$, Since s_1 and s_2 are [illegible]

t is unique. for each in satisfied

Easier Solution. if $\dot{x}(0) = 0$ (assuming $\lambda_2 > \lambda_1 > 0$)

$$\Rightarrow \frac{c_2}{c_1} = \frac{\lambda_1}{\lambda_2} < 1$$

$\Rightarrow$ t becomes negative

$\Rightarrow$ at no time t does [illegible] cross [illegible]

Critically Damped: two equal and Real roots

But we know they must be negative

$\cap$ $\quad -at$

But we know ...

$\Rightarrow (r+a)^2 = 0 \Rightarrow y = e^{-at}$

But there's no second solution.

$\ddot{x} = Kx \Rightarrow$ Euler Method

$\ddot{x} = t \Rightarrow$ integrate

$\hookrightarrow \dot{x} = \frac{t^2}{2} + C_1$

$\rightarrow x = \frac{t^3}{6} + C_1 t + C_2$

$P = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$ $\quad m\ddot{x} + b\dot{x} + Kx = 0$

$b^2 = 4 \times m \times K = 4 \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{2}$

$\Rightarrow b = \frac{\sqrt{2}}{2}$ $\quad \frac{\sqrt{2}}{\sqrt{3}}$

$1 - 4 \times 3 =$

$\ddot{x} + 4\dot{x} + 3x$ $\quad \sqrt{16 - 12} = \sqrt{4}$

two real roots

-4

$\frac{-b \pm 2}{2} < \begin{matrix} -1 \\ -3 \end{matrix}$ $\qquad x(0) = 1$

$\dot{x}(0) = 0$

$x = C_1 e^{-t} + C_2 e^{-3t}$ $\qquad C_1 + C_2 = 1$

$x' = -C_1 e^{-t} - 3C_2 e^{-3t}$ $\qquad -C_1 - 3C_2 = 0$

$\Rightarrow C_2 = -\frac{1}{2}$ $\qquad \begin{array}{r} C_1 + 3C_2 = 0 \\ - \quad C_1 + C_2 = 1 \\ \hline 2C_2 = -1 \end{array}$

$C_1 = \frac{3}{2}$

$\frac{-b}{2a} = \frac{-4}{2} = -2$

$x(0) = 1$

$x = C_1 e^{-2t} + C_2 t e^{-2t}$ $\qquad \dot{x}(0) = 0$

$C_1 = 1$

$x = c_1 e \quad + c_2 t e$

$c_1 = \frac{1}{3}$

$x' = -2c_1 e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t}$

$c_2 = \frac{2}{3}$

$x'' = \cancel{\frac{4}{3} e^{-2t} - \frac{4}{3} e^{-2t}} - 2c_2 e^{-2t} + 4c_2 t e^{-2t}$

$x'' = -\frac{4}{3} e^{-2t} + \frac{8}{3} t e^{-2t}$

$x' = \cancel{-\frac{2}{3} e^{-2t} + \frac{2}{3} e^{-2t}} - \frac{4}{3} t e^{-2t}$

$x = \frac{1}{3} e^{-2t} + \frac{2}{3} t e^{-2t}$

$\Rightarrow -\frac{4}{3} e^{-2t} + \frac{8}{3} t e^{-2t} - \frac{16}{3} t e^{-2t}$

$+ \frac{4}{3} e^{-2t} + \frac{8}{3} t e^{-2t}$

$= 0$

$x = c_1 e^{-2t} + c_2 t e^{-2t}$

$x(0) = 1 \Rightarrow c_1 = 1$

$x'(0) = -2C_1 e^{-2t} + C_2 e^{-2t} =$

$-2C_1 + C_2 = 0$

~~$-2C_1$~~

$b = 0.6$

$b = 1 \quad t = 5.34$

$0.93 \quad 0.82 \quad 0.86$

0.4

$I\ddot{\theta} - K\theta = 0 \qquad \theta = e^{rt}$

$r^2 I e^{rt} - K e^{rt} = 0 \Rightarrow r^2 I - K = 0$

$\Rightarrow r = \pm\sqrt{\frac{K}{I}}$

$\theta = C_1 e^{-rt} + C_2 e^{+rt} \qquad \theta(0) =$

$C_1 + C_2 = 0$

$\dot{\theta} = -\sqrt{\frac{K}{I}} C_1 e^{-rt} + \sqrt{\frac{K}{I}} C_2 e^{+rt} \qquad C_1 = -C_2$

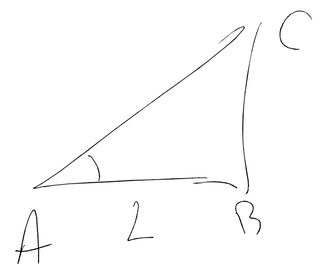
$\dot{\theta} = -\sqrt{\frac{K}{I}} C_1 e^{\dots} + \sqrt{\frac{K}{I}} C_2 e^{\dots}$

$\Rightarrow \sqrt{\frac{K}{I}}(-C_1 + C_2) = C$

$\Rightarrow 2\sqrt{\frac{K}{I}} C_2 = C \Rightarrow C_2 = \sqrt{\frac{I}{4K}}$

$C_1 = -\sqrt{\frac{I}{4K}}$

$P = \frac{2\pi}{\sqrt{K/I}} \simeq 2 \Rightarrow \pi$



$A = (0, 0)$

$B = (L, 0)$

$C = (L, y_b)$

$y_b = \tan\theta \times L$

$\frac{dy}{dt} = 3y \Rightarrow y = e^{3t} + C$

$u = r^2$

$u = r^2$

$r^4 + 4r^2 + 3 = 0$

$\sqrt{16-12} = \sqrt{4} = 2$

$u^2 + 4u + 3 = 0$

$r^2 = -1, -3$

$\dfrac{-4 \pm 2}{2} \begin{cases} -1 \\ -3 \end{cases}$

$i^4 + 4i^2 + 3$

$r = i, -i, \sqrt{3}i, -\sqrt{3}i$

$$x = c_1 e^{it} + c_2 e^{-it} + c_3 e^{-\sqrt{3}it} + c_4 e^{\sqrt{3}it}$$

$\Rightarrow x(0) = c_1 + c_2 + c_3 + c_4$

$\dot{x}(0) = ic_1 - ic_2 - \sqrt{3}ic_3 + \sqrt{3}ic_4$

$\ddot{x}(0) = -c_1 - c_2 - 3c_3 - 3c_4$

$\dddot{x}(0) = -ic_1 + ic_2 + 3\sqrt{3}ic_3 - 3\sqrt{3}ic_4$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ i & -i & -\sqrt{3}i & +\sqrt{3}i & 0 \\ -1 & -1 & -3 & -3 & 0 \\ -i & i & 3\sqrt{3}i & -3\sqrt{3}i & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & i & -i & 0 \\ 0 & 0 & -2 & 0 & 0 \\ -i & i & 3\sqrt{3}i & -3\sqrt{3}i & 0 \end{bmatrix}$$

$\Rightarrow C_3 = 0 \quad \Rightarrow C_4 = 0 \quad \Rightarrow$ All Zeros

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$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 2i & 0 & -\sqrt{3}i & +\sqrt{3}i & i \\ 0 & 0 & -2 & -2 & 0 \\ 0 & 0 & i & -i & 0 \end{bmatrix}$$

$C_3 \,\&\, C_4 = 0$

$C_1 \,\&\, C_2 = 1$

$2i C_1 = i \Rightarrow C_1 = \frac{1}{2}$

$\Rightarrow C_2 = \frac{1}{2}$

But $\quad i \quad 0$

$e^{i2t} = \cos 2t + i\sin 2t \quad e^{-i2t} = \cos -2t + i\sin -2t$

$\cos 2t \quad \sin 2t \quad = \cos 2t - i\sin 2t$

$\cos 2t + i\sin 2t$, $\cos 2t - i\sin 2t$

$r^3 - r = 0$ $\quad r^3 = r$

$1, -1, 0$

$x = c_1 e^{t} + c_2 e^{-t} + c_3$

$x(0) = c_1 + c_2 + c_3 = 0$

$x'(0) = c_1 - c_2 = 1$ $\qquad c_3 = -1$

$x''(0) = c_1 + c_2 = 1$

$2c_1 = 2 \Rightarrow c_2 = 0$

$x = e^{-2t} + 4t e^{-2t} - t^2 e^{-2t} + 4e^{-t}\cos(3t)$

3 t-powers of e^{-2t} $\qquad$ -2 was the root 3 times

$4e^{-t}\left(\frac{e^{3ti} + e^{-3ti}}{2}\right) = e^{(3i-1)t} + e^{(-3i-1)t}$

$$(r+2)^3 \times (r-3i+1) \times (r+3i+1)$$

$$= r^5 + 8r^4 + 34r^3 + 92r^2 + 136r + 8.$$

$$2i = 2e^{i\pi/2}$$

$$\sqrt{2i} = \sqrt{2} \times e^{i\frac{\pi}{4}} = \sqrt{2} \times \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

$$\sqrt{2} \times \left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 1 + i$$

$$-(1+i) \times -(1+i) = 1 \cancel{+} 2i$$

$$e^{at} \times e^{ibt} \times \left(\cos\cancel{\frac{\pi}{2}} - i\sin\frac{\pi}{2}\right)$$

$a = -3$, $b = 2$

$$(\cos 2t + i\sin 2t) \times (-i)$$

$e^{-4t} \alpha e^{5it}$

$Re(c\alpha(\cos\omega t + i\sin\omega t))$??

$c = A \times (\cos\theta + i\sin\theta)$

$\xrightarrow{Re} A \times (\cos\theta\cos\omega t - \sin\theta\sin\omega t)$

$= 3\cos\omega t - \sin\omega t$

$A\cos\theta = 3 \quad A\sin\theta = 1$

$\theta = \mathrm{Arctan}\left(\frac{1}{3}\right)$

$A^2\cos^2 + A^2\sin^2 = A^2 = 10 \Rightarrow A = \sqrt{10}$

$\omega = \frac{1}{14} \qquad A = 2 \qquad -\lambda$

$\omega \cdot T = 2\pi \Rightarrow \omega = \frac{2\pi}{T}$

Start at 1 ends at 7

Period 8

$\omega \cdot 8 = 2\pi \quad \omega = \frac{\pi}{4}$

$\ddot{x} + \pi^2 x = 0$ $\quad x = e^{rt}$,

$\Rightarrow r^2 + \pi^2 = 0 \Rightarrow r = \pm i\pi$

$x(0) = 1 \Rightarrow C_1 + C_2 = 1$

$x'(0) = C_1 i\pi e^{i\pi t} - i\pi C_2 e^{-i\pi t} = \pi\sqrt{3}$

$i(C_1 - C_2) = \sqrt{3}$

$$i(c_1 - c_2) = \sqrt{3}$$

$$i(c_1 + c_2) = i$$

$$\Rightarrow 2ic_1 = \sqrt{3} + i \Rightarrow c_1 = \frac{\sqrt{3}+i}{2i}$$

$$c_2 = \frac{-\sqrt{3}+i}{2i}$$

$i\sin\theta$ $\cos\theta$

$$\Rightarrow c_1 = -\frac{\sqrt{3}}{2}i + \frac{1}{2}, \quad c_2 = \frac{\sqrt{3}}{2}i + \frac{1}{2}$$

$$c_1 = e^{-i\pi/3} \qquad c_2 = e^{i\pi/3}$$

$$\Rightarrow e^{-i\pi/3}e^{i\pi t} + e^{i\pi/3}e^{-i\pi t} = x(t))$$

$$\Rightarrow x_1 = e^{i\pi t} = \cos\pi t + i\sin\pi t$$

$$x_2 = e^{-i\pi t} = \cos\pi t - i\sin\pi t$$

$$c_1 \cos\pi t + c_2 \sin\pi t = 1$$

$\Rightarrow x = c_1 \cos \pi t + c_2 \sin \pi t \quad = 1$

$x' = -\pi c_1 \sin \pi t + \pi c_2 \cos \pi t \quad = \pi \sqrt{3}$

$\Rightarrow \quad c_1 = 1$

$\Rightarrow \quad c_2 = \sqrt{3}$

$b^2 - 4mK < 0 \quad \rightarrow$ Underdamped.

$$\frac{-b \pm \sqrt{b^2 - 4mK}}{2m} t$$

e

$$\omega = \frac{\sqrt{b^2 - 4mK}}{2m} = 2$$

$$12 = \sqrt{b^2 - 12 \cdot \alpha 4 \alpha 3}$$ Valgurdu!

$$14\alpha = b^2 - 14\alpha_0$$ lol

$$r^4 + a_3 r^3 + a_2 r^2 + a_1 r + a_0 = 0$$

$$\Rightarrow (r-5)^2$$

$$e^{-\alpha t}\left(\frac{e^{i3t} - e^{-i3t}}{2i}\right)$$

$$e^{(-4+i3)t} \Rightarrow (r+4-3i)$$

$$(r-5)^2(r+4-3i)(r+4+3i)$$

$(r-5)($

$r^2+3r+5=0$ $\quad -3/2+i\sqrt{11}$

$\frac{-3\pm\sqrt{9-20}}{2}$ $\quad -3/2-i\sqrt{}$

$x = e^{-3/2\,t}\left(c_1\cos\sqrt{11}\,t + c_2\sin\sqrt{}\right.$

$3t^2+6-6t-10=-4$

other than

$r=$

$r^2-r+t^4=0$

$\ddot{y}+t^4\dot{y}=0$

t^4

$,0$

$\overline{}$

$\overline{u}$

$\overline{u\,t})$

$y + \,\, $

$r = t^4 \quad , 0$

$\ddot{y} - t^4 \dot{y} = 0 \qquad u = \dot{y}$

$u' - t^4 u = 0 \implies \frac{du}{dt} = t^4 u$

$\implies \ln u = \frac{t^5}{5} + C$

$\implies u = Ce^{\frac{t^5}{5}}$

$\frac{dy}{dt} = Ce^{t^5/5}$

$\implies y = \int Ce^{t^5/5} \, dt \quad dt.$

$\ddot{y} + t^2 y$ $\qquad$ $\ddot{y} = -t^2 y$

$\Rightarrow y(0) = 6 \Rightarrow \ddot{y}(0) = 0$

$y = e^{t^2}$ $\qquad$ $y' = 2t e^{t^2}$

$y' = 2e^{t^2} + 2t^2 e^{t^2}$

---

what? $\qquad$ $\downarrow$ $F = m\ddot{x} = mg - F_B$

volume of cylinder $= \pi r^2 h$

$(\pi r^2 h \frac{1}{2}) \ddot{x} = (\pi r^2 h \frac{1}{2}) x g - F_B$

$F_B = \pi r^2 h \dot{x} t$ $\qquad$?

Now!

$A\ddot{x} = Ag - 2A\dot{x}t$ New!

$\Rightarrow \ddot{x} = g - 2\dot{x} \Rightarrow$

$r^2 - g + 2r = 0 \Rightarrow$

$\Rightarrow V = \pi r^2 h \times 1$

$x(0) = 1$

$\dot{x}(0) = -1$

$$\frac{\ddot{x}}{2} + b\dot{x} + \frac{x}{2} = 0$$

$$-b \pm \sqrt{b^2 - 4}$$

$$\frac{\ddot{x}}{2} + b\dot{x} + \frac{x}{2} = 0$$

Assume $R = 1$

Penalty

$$\frac{r^2}{2} + br + \frac{1}{2} = 0$$

$x(0) = 1$

$\dot{x}(0) = -1$

$$r = -b \pm \sqrt{b^2 - 1}$$

if $b < 1$

$$x = e^{-b + \sqrt{b^2 - 1}\, t}$$

Based

$$x = e^{-bt}\left(c_1 \cos\sqrt{1-b^2}\,t + c_2 \sin\sqrt{1-b^2}\,t\right)$$

$$x(0) = 1 \Rightarrow c_1 = 1$$

$$\dot{x}(0) = -b e^{-bt}\left(c_1 \cdots + c_2 \cdots\right)$$

$$+ e^{-bt}\left(-c_1\sqrt{1-b^2}\,\sin\sqrt{1-b^2}\,t + c_2\sqrt{1-b^2}\,\cos\sqrt{1-b^2}\,t\right)$$

$$= -1 \Rightarrow -bc_1 + c_2\sqrt{1-b^2} = -1$$

$\Rightarrow \quad -b + c_2\sqrt{1-b^2} = -1$

$\Rightarrow \quad c_2 = \dfrac{b-1}{\sqrt{1-b^2}}$

$\dfrac{r^2}{2} + br + \dfrac{1}{2} = 0$ $\qquad x(0) = 1$

$x = c_1 e^{-bt} + c_2 t e^{-bt}$ $\qquad c_1 = 1$

$\Rightarrow -bc_1 e^{-bt} - c_2 e^{-bt} - bc_2 t e^{-bt}$

$-bc_1 - c_2 = -1$ $\qquad b + c_2 = 1$

$\Rightarrow -b - c_2 = -1$ $\qquad c_2 = 1 - b$

[illegible]

+ (1-b)**t*e^(-b*t)

$b > 1$ $\qquad \dfrac{r^2}{2} + br + \dfrac{1}{2} = 0$ $\qquad x(0) = 1$

$-b \pm \sqrt{b^2 - 1}$

$$-b \pm \sqrt{b^2-1}$$

$$x = e^{-bt}\left(c_1 e^{\sqrt{b^2-1}\,t} + c_2 e^{-\sqrt{b^2-1}\,t}\right)$$

$$c_1 + c_2 = 1$$

$$x'(0) = -b e^{-bt}\left(c_1 e^{\sqrt{b^2-1}\,t} + c_2 e^{-\sqrt{b^2-1}\,t}\right)$$

$$+ e^{-bt}\left(c_1\sqrt{b^2-1}\, e^{\sqrt{b^2-1}\,t} - \sqrt{b^2-1}\, c_2 e^{-\sqrt{b^2-1}\,t}\right)$$

$$x'(0) = -b(c_1+c_2)$$

$$+ \sqrt{b^2-1}\,(c_1-c_2) = -1$$

$$\sqrt{b^2-1}\,(c_1-c_2) = -1-b$$

$c_1 + c_2 = 1$ $\Rightarrow$ $c_1 - c_2 = \dfrac{-1-b}{\sqrt{b^2-1}}$

$c_2 = 1 - c_1$

$C_2 = 1 - C_1$

$$C_1 - (1 - C_1) \quad 2C_1 - 1 = \frac{1-b}{\sqrt{b^2-1}}$$

$$C_1 = \frac{1 - b + \sqrt{b^2-1}}{2\sqrt{b^2-1}}$$

$$C_2 = \frac{1 + b + \sqrt{b^2-1}}{2\sqrt{b^2-1}}$$

$$\frac{r^2}{2} + br + \frac{1}{2} = 0$$

$-b \pm A$

$$r = -b \pm \sqrt{b^2-1}$$

$$e^{-b \pm A} = e^{-b}(e^{A}) \quad e^{-b}(e^{-A}$$

$(-b+A)t$, $(-b-A)t$

$$x = C_1 e^{(-b+A)t} + C_2 e^{(-b-A)t}$$

$$x = e^{-bt}\left(C_1 e^{At} + C_2 e^{-At}\right)$$

$$x(0) = C_1 + C_2 = 1$$

$$x'(0) = -b e^{-bt}\left(C_1 + C_2\right) + e^{-bt}\left(AC_1 e^{At} - C_2 A e^{-At}\right) = -1$$

$$\Rightarrow -b(C_1 + C_2) + (AC_1 - AC_2) = -1$$

$$-b(C_1 + C_2) = -b$$

$$\Rightarrow -b + A(C_1 - C_2) = -1$$

$$\Rightarrow C_1 - C_2 = \frac{b-1}{A}$$

$$C_1 + C_2 = 1 \Rightarrow C_2 = (1 - C_1)$$

$$c_1 + c_2 = 1 \Rightarrow c_2 = (1-c_1)$$

$$c_1 - (1-c_1) = 2c_1 - 1 = \frac{b-1}{A}$$

$$\Rightarrow c_1 = \frac{b-1+A}{2A}$$

$$c_2 = 1 - c_1 = A-b+1$$

$$m\ddot{x} + b\dot{x} + Kx = 0$$

$$mr^2 + br + K = 0$$

$$\tfrac{1}{2}r^2 + b + \tfrac{1}{2} = 0$$

$$-b \pm \sqrt{b^2-1} \qquad \text{if } b > 1$$

$$x = c_1 e^{-bt} + c_2 t e^{bt} \qquad x(0) = 1$$

$$x = c_1 e^{t} + c_2 t e^{-t} \qquad x'(0) = -1$$

$$x'(0) = c_1 e^{t} + c_2 e^{-t}$$

$$c_1 = 1 \qquad c_1 + c_2 = -1$$

$$\Rightarrow c_2 = -2$$

$$x(t) = e^{t} - 2t e^{-t}$$

$$\frac{-b \pm \sqrt{b^2 - 1}}{A} \qquad \& \; b <$$

$$x = e^{-bt}\left(C_1 \cos(At) + C_2 \sin(At)\right)$$

$$x(0) = C_1 = 1$$

$$x'(0) = -b e^{-t}\left(C_1 \cos At + C_2 \sin At\right)$$

$$+ e^{-bt}\left(-C_1 A \sin At + A C_2 \cos At\right.$$

$$t = 0 \quad -b(C_1) + \left(A C_2 \cos At\right) = 1$$

$$e^{-t}\left(e^{0} + e^{0}\right) = 2e^{-t}$$

$$u = r^2$$

$$r^4 + 4r^2 + 3 = 0$$

$$\sqrt{\ldots} \quad 12 = \sqrt{4 \cdot 12}$$

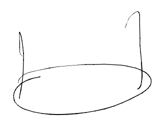
$r^2 + 4r + 3 = 0$

$\sqrt{16-12} = \sqrt{4} = 2$

$u^2 + 4u + 3 = 0$

$$\frac{-4 \pm 2}{2} < \begin{matrix} -3 = u^2 \\ -1 = u^2 \end{matrix}$$

$u = \sqrt{3}i, -\sqrt{3}i, i, -i$



$m\ddot{x} = mg - \pi r^2 \dot{x}$

$m = \pi r^2 h \frac{\rho}{2} = A$

$m\ddot{x} = mg - \frac{2A}{h}\dot{x}$ $\quad w = \frac{4}{16}$

$$\ddot{x} = -\frac{2g}{L}x \qquad r^2 = -\frac{2g}{L}$$

$$\text{Hence} \Rightarrow r = i\sqrt{\frac{2g}{L}} \qquad \sqrt{\frac{2g}{L}} = \omega$$

$$x = C_1 \cos\sqrt{\frac{2g}{L}}t + C_2 \sin\sqrt{\frac{2g}{L}}t + \frac{L}{2}$$

$$x(0) = 0 = C_1 + 1 \Rightarrow C_1 = -1$$

$$x'(0) = C_2\sqrt{g} = 0 \qquad C_2 = 0$$

$$x = \cos\sqrt{g} + \frac{L}{2}$$

$x = -\cos\sqrt{g}\,t + e_1$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{g}}$ $\quad A =$

$\sqrt{\;\;^2}$

$\cos\sqrt{g}\,t$

$= A\cos(\sqrt{g}\,t - \phi)$

$e^{2\pi i} +$ Jonas!

$m\ddot{x} = \left(\frac{1}{2}\pi r^2 L g\right) - (\pi r^2 \rho g)\,x$

$$m\ddot{x} = \left(\frac{1}{2}\lambda, \lambda y\right) - \ldots$$

$$\ddot{x} = g - \frac{2g}{h}x$$

$$v^2 = g - \frac{2g}{h}x$$

$$\sqrt{\frac{2g}{h}} = \frac{\pi}{10} \qquad \underset{(m)}{h} = \frac{10}{8}g$$

$$\Rightarrow$$

$$\frac{2g}{h} = \frac{8}{100} \qquad \frac{100}{8}g$$

$$\sqrt{20}\ 2\ \pi r^2 \frac{1}{2} h$$