Unit 5

Wednesday, August 18, 2021 12:35 AM

$P(x) y^{(n)} + \dots\ q(x) y + b(x) = 0$

$-\frac{x}{y} = 2 \Rightarrow y = \frac{-x}{2}$

$\frac{dy}{dx} = -\frac{x}{y} \Rightarrow y\,dy = -x\,dx$

$\Rightarrow \frac{y^2}{2} = \frac{-x^2}{2} + C_0$

$\rightarrow y = \sqrt{-x^2 + C}$

$\frac{dy}{dx} = xy^4 \Rightarrow y^{-4} dy = x\,dx$

$\Rightarrow \frac{y^{-3}}{-3} = \frac{x^2}{2} + C \Rightarrow \frac{1}{y^3} = \frac{-3}{2} x^2 + C$

$\Rightarrow y = \sqrt[3]{\frac{1}{\frac{-3}{2}x^2 + C}}$

$y(0) = 1 \Rightarrow C = 1 \qquad y = \sqrt[3]{\frac{1}{\frac{-3}{2}x^2 + 1}}$

$\frac{3}{2} x^2 = 1 \Rightarrow x = \pm\sqrt{\frac{2}{3}}$

outside of this can't satisfy $y(0) = 1$

$y = u^{-1} \rightarrow y' = -u' u^{-2}$

assuming $u^{-1} \neq 0$

$-u' u^{-2} + u^{-2} t - u^{-1} = 0$

$\rightarrow u' = \frac{-1 + u^{-1} t}{u^{-1}} = t - u$

$y' = \frac{1}{2} = x - 2y$

$y(0) = \frac{-1}{4}$

$\dot{y} = -y^2 + y = 1 \Rightarrow -y^2 + y + 1 = 0$

$\dot{y} = -y^2 + y = 1 \Rightarrow -y^2 + y + 1 = 0$

$$\frac{1 \pm \sqrt{5}}{2} < \begin{matrix} \frac{1+\sqrt{5}}{2} \\ \frac{1-\sqrt{5}}{2} \end{matrix}$$

$\frac{2}{100} \times 1.^{t} = 200$

$\dot{y} = ry - w \qquad y(0) < \frac{w}{r}$

$\dot{y} - ry = 0 \Rightarrow y = ue^{rt}$

$u'e^{rt} + \cancel{rue^{rt}} = \cancel{rue^{rt}} - w$

$\Rightarrow u'e^{rt} = -w \Rightarrow u' = -we^{-rt}$

$\Rightarrow u = \frac{w}{r}e^{-rt} + C$

$y = \frac{w}{r} + Ce^{rt} \qquad y(0) = \frac{w}{r} + C < \frac{w}{r}$

$C < 0$

$y(-\infty) = \frac{w}{r} - c$

$y' = 3y - y^2 = 0 \Rightarrow y = 0, \ y = 3$

$y(0) = 6 \rightarrow \frac{3}{1+C} = 6 \Rightarrow 1 + C = \frac{1}{2} \rightarrow C = -\frac{1}{2}$

$2 = e^{-3t} \rightarrow \ln 2 = -3t \Rightarrow t = \frac{\ln 2}{-3}$

$\frac{dy}{dt} = 3y - by^2 = 0 \qquad 3 = by$

$3 = 6 \times 10^3 b \Rightarrow b = \frac{1}{2} \times 10^{-3}$

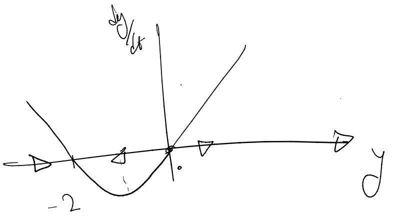
$\frac{dx}{dt} = 3y - \frac{1}{2}y^2$

$\dot{x} = -\frac{1}{100}x + C = 0 \qquad x = 700$

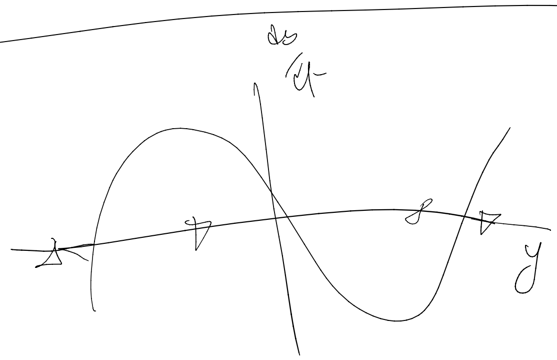
$x = -\frac{700}{100} + C \qquad C = 7$

x^2 Too

$y' = y^2 + 2y = 0 \Rightarrow y = 0 \quad y = -2$



$y' = 0$ at $-1, 1, 2$



$\dot{y} = f(y)$ Local Maxima for y?

non-Constant

$\dot{y} = f(y)$

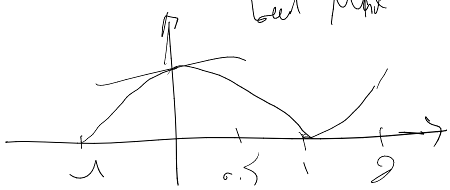
$\dot{y} = y(y-1)$

$y = -1 \to 2$???

Local Max

$y = 0 \to 0$

$y = 0.5 \to -\frac{1}{4}$



$y = 1 = 0$

$y = 2 = 2$

$\frac{dy}{dt} = y^2 - y \Rightarrow \int \frac{dy}{y^2 - y} = t$

$$\frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1} = \frac{Ay - A + By}{y(y-1)} = 1$$

$B = -A \quad A = -1 \Rightarrow$

$$\int \frac{-dy}{y} + \frac{dy}{y-1} = -\ln y + \ln(y-1) = t$$

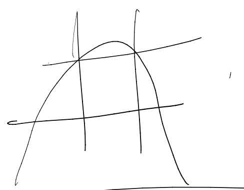
$$\Rightarrow e^{\ln(1-\frac{1}{y})} = e^t$$

$$\Rightarrow 1 - \frac{1}{y} = e^t$$

$$\Rightarrow 1 - e^t = \frac{1}{y} \Rightarrow$$

Not a local Maximum

$$y = \frac{1}{1-e^t}$$



$-y^2 + 3y - h = 0 \qquad 9 - 4h = 0$

$\frac{-b}{2a} = \frac{-3}{-2} = 1.5 \qquad -h = \frac{9}{4}$

$y(0) > y_L$

$-y^2 + 3y - 2 = 0$

$9 - 8 = 1 \qquad \frac{-3 \pm 1}{-2} < \begin{matrix} 2 \\ 1 \end{matrix}$ ✓

[illegible] Bifurcation [illegible] make it Unstable

Many Bifurcation a make it Unstable

15.

$-y^2+3y-h=0$ $\qquad \dfrac{-3\pm\sqrt{9-4h}}{-2}$

$= 1.5 \pm \sqrt{\frac{9}{4}-h}$

$F_{spring} = K(L-y)$ $\quad y \to$ Vertical Position

$F_{spring}(x) = K(L-y)\cos\theta$ $\quad y=\sqrt{h^2+x^2}$

$\theta = \arctan\left(\frac{h}{x}\right)$

$\dot{y} = -\frac{1}{4}+y-\frac{y^2}{2} = 0 \Rightarrow -2y^2+4y-1=0$

$\sqrt{16-8} = \sqrt{8} = 2\sqrt{2}$

$\dfrac{-4\pm 2\sqrt{2}}{-4} <$ $1-\frac{\sqrt{2}}{2}$, $1+\frac{\sqrt{2}}{2}$

$0 \to -\frac{1}{4}$ $\qquad 1 \to 1$

$2 \to 1$

$y' = -\frac{1}{4}+y-\frac{y^2}{a}=0$

$a=1 \to y=0$ $\qquad -y^2+y-\frac{1}{4}=0$

$y(0)=1$ $\qquad y'=x^2-y^2$ $\qquad y'(0)=-1$

Step-size: 0.1

$y(0.1) = 1 + -1 \times 0.1 = 0.9$ $\qquad y'(0.1) = -0.8$

$y(0.2) = 0.9 + 0.1 \times -0.8 = 0.82$ $\qquad y'(0.2) = -0.6324$

$y(0.1) = 1 + \ldots$

$y(0.2) = 0.9 + 0.1 \times -0.8 = 0.82 \qquad y'(0.2) = -0.6324$

$y(0.3) = 0.82 - 0.6324 \times 0.1 = 0.7876$

$y'' = 2y - 2 \qquad y''(-1,-1) = -4$

Convex

$y'' = 2yy' - 2x \qquad y'(0) = 1$

$y''(0) = 2 \times 1 \times 1 - 2 \times 0 = -2$

Convex

undershoot $\longrightarrow$ too large since negative

$\dfrac{1}{x+y} = c \Rightarrow x+y = \dfrac{1}{c} \Rightarrow y = -x + \dfrac{1}{c}$

$y' = e^x y \Rightarrow \dfrac{dy}{y} = e^x dx$

$\longrightarrow \ln y = e^x$

$\Rightarrow y = ce^{e^x} \qquad y(0) = ce = 2$

$\Rightarrow c = \dfrac{2}{e}$

$y' = y^3 \Rightarrow y^{-3} dy = dx \Rightarrow \dfrac{-1}{2y^2} = x + C$

$\Rightarrow y^2 = \dfrac{-1}{2x+c} \Rightarrow y = \sqrt{\dfrac{-1}{2x+c}}$

$y(0) = 2 \Rightarrow 2 = \sqrt{\dfrac{-1}{c}} \Rightarrow$

$\ldots c$

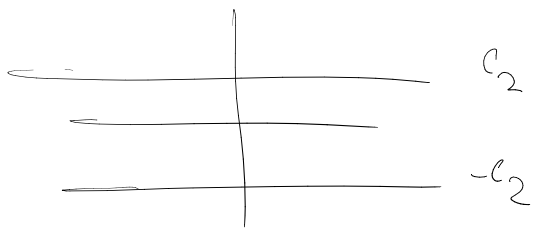
$\Rightarrow c = -\frac{1}{4}$

$\Rightarrow y = \sqrt{\frac{1}{2x - \frac{1}{4}}} \qquad x > \frac{1}{8}$

Done

$y^2 - 4 = e^c \Rightarrow y^2 = 4 + e^c = C_2$

$y = \sqrt{4 + e^c}$



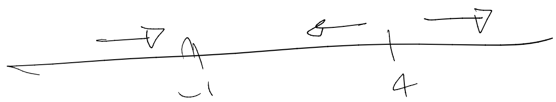
$\sqrt{9+16} = 5$

$\dot{x}(-2) > 0$

$\dot{x}(0) = -4$

$\dot{x}(1) > 0$

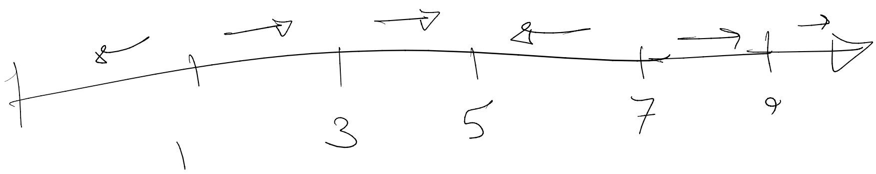
$\frac{+3 \pm 5}{2} < \begin{matrix} 4 \\ -1 \end{matrix}$



$\dot{x}(0) < 0 \qquad \dot{x}(2) > 0 \qquad \dot{x}(4) > 0$

$\dot{x}(6) < 0 \qquad \dot{x}(8) > 0$

$\dot{x}(10) > 0$



$y'' = 2y' - 2yy' \qquad y' = 2y - y^2$

$\Rightarrow y'' = 4y - 2y^2 - 4y^2 + 2y^3 = 0$

$14 > 0$

$\Rightarrow y'' = 4y - 2y'^2 - 4y' + 2y = 0$

$| y = 0$

$y'' = 4y - 6y^2 + 2y^3 = 0$

$36 - 16 \times 2 = 4$

$(2y^2 - 6y + 4)y = 0$

$\frac{+6 \pm 2}{4} < {2 \atop 1}$

$\frac{dy}{2y - y^2} = dx \rightarrow \frac{A}{y} + \frac{B}{2-y}$

$\Rightarrow \frac{2A - Ay + By}{2y - y^2} = \frac{1}{2y - y^2} \Rightarrow A = \frac{1}{2} \quad B = \frac{1}{2}$

$\Rightarrow \frac{1}{2}\int \frac{dy}{y} + \frac{1}{2}\int \frac{dy}{2-y} = x + C$

$= \frac{1}{2}(\ln y - \ln|2-y|) = x + C$

$\Rightarrow \ln \frac{y}{2-y} = 2x + C$

$\Rightarrow \frac{y}{2-y} = e^{(2x+C)} \qquad \frac{y}{2-y} = A$

$y = 2A - Ay \Rightarrow (A+1)y =$

2A

$$\Rightarrow \quad y = \frac{2A}{A+1} \Rightarrow y = \frac{2e^{(2x+C}}{1+e^{(2x+C)}}$$

$$y(0) = \frac{2e^{C}}{1+e^{C}} = \frac{1}{2} = \quad 4e^{C} = 1$$

$$\Rightarrow 3e^{C} =$$

$$\Rightarrow \quad e^{C} = \frac{1}{3} \Rightarrow C = \ln$$

$$y'' = 2y' - 2yy' = 0 \qquad y' = 2y - y^2$$

$$\Rightarrow \quad 4y - 2y^2 - 4y^2 + 2y^3 = 0$$

$$\Rightarrow y = 0, 1, 2 \qquad y'(y=0) = 0$$

$$y'(y=1) = 2 - 1 -$$

$$y'(y=2) = 0$$

)

$e\, e^{c}$

1

$\frac{1}{3}$

---

l

$y' = 2y - y^2 \qquad x = y - 2 \qquad y = x + 2$

$y' = -y(y-2) = -xy = -x^2 - 2x$

$\dot{x} = -x^2 - 2x \Rightarrow \dot{x} = -2x$

$\Rightarrow \dfrac{dx}{x} = -2dt \Rightarrow \ln x = -2t$

$\Rightarrow x = Ce^{-2t}$

$y = 2 + Ce^{-2t} \qquad y(0) = 2 + C =$

$\Rightarrow C = -2$

$16v^2 - 100 \times 16 = 0 \Rightarrow v = \pm 10$

$v'(-11) > 0 \qquad v'(0) < 0 \qquad v'(11) >$

$\dot v(-11) > 0$

$m = 100$
$k = 10$
$g = 10$

$$m\dot v = kv^2 - mg \Rightarrow \dot v = \frac{k}{m}v^2 - g$$

$$\dot v = \frac{v^2}{10} - 10$$

if $v(0) = 0 \longrightarrow \dot v = -10 + g$

if $\dot v = 0 \rightarrow$ or $g = 1g$

$$\dot v = \frac{400}{10} - 10 = 30 + g = 4g$$

$$m\dot v = k(t)v^2 - mg \qquad k(t) = \begin{cases} k_1 & 0 < t < D \\ k_2 & D < t \end{cases}$$

$\rightarrow v \downarrow$)

$$\dot v = \frac{k}{m}v^2 - g = \frac{k}{50}v^2 - 10$$

$$50k_1 - 10 = 0 \Rightarrow k_1 = \frac{1}{5}$$

$$\frac{k_2}{2} - 10 = 0 \Rightarrow k_2 = 20$$

z c

Next y : (assuming $V(0)=0$)

After the Parachute opens :

$\frac{2}{5} \times 50^2$

$\frac{\cancel{8}\, 10 \times 10}{\cancel{5}}$

$y^2 - x^2 = 2$

$\Rightarrow y = \pm\sqrt{2 + x^2}$

$y^2 - x^2 = 0$ $\quad y^2 =$

$y = x \quad y = -x$

$y^2 - x^2 = 2 \Rightarrow y =$

$\dot{v} = (a - v^2)\, v$

$v = 0$

$v = \pm\sqrt{a}$

$\dot{v}^2 > 0$

$-\sqrt{a} \quad 0 \quad \sqrt{a}$

~~= lo log~~

= 2 loo 2 log

$= x^2$

log

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x

x

β_{or} $\quad v > 0$ $\quad \alpha < 0$ $\quad (-) \times (+$

β_{or} $\quad v < 0$ $\quad \alpha < 0$ $\quad (-) \times (-$

$e) = \overline{\phi}$

$) = t$