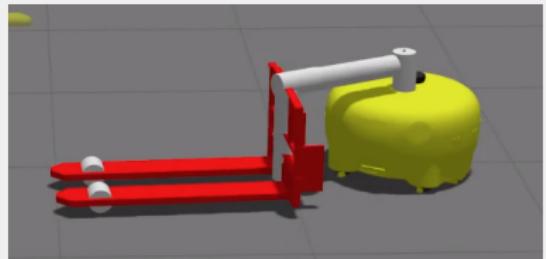


MOTION PLANNING FOR AUTONOMOUS VEHICLES

PATH PLANNING

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MARCH 21, 2023



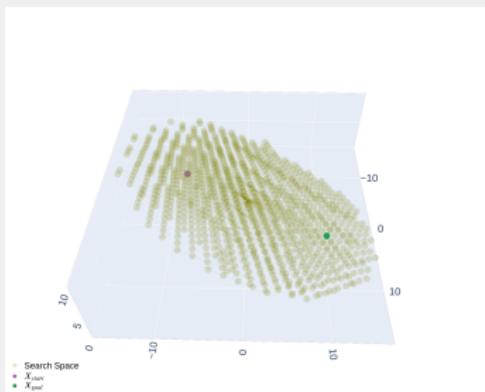
PATH PLANNING

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CONFIGURATION SPACE VS SEARCH SPACE FOR ROBOT

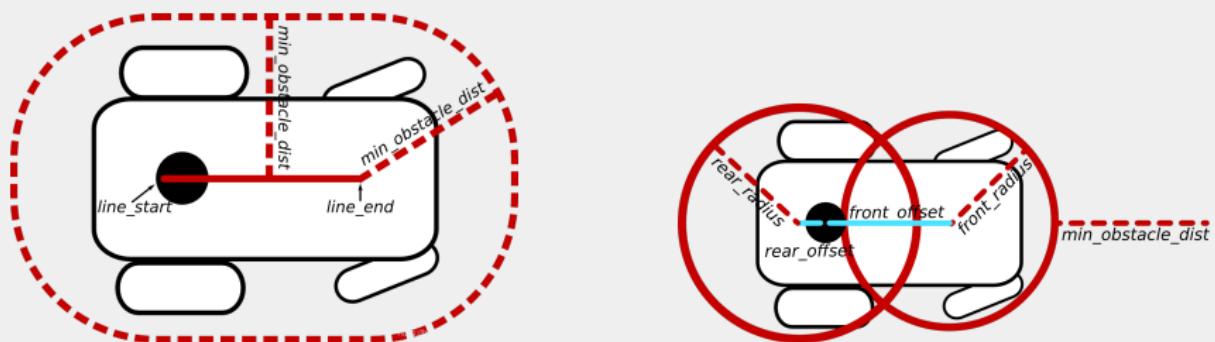
The **configuration space** is the space of all possible configurations robot can move which is a subset of the **search space (workspace)**



Robot configuration (robot footprint): specification of robot representation. **Robot configuration space**: possible all robot configuration with respect to robot degree of freedom (DOF)

CONFIGURATION SPACE VS SEARCH SPACE FOR ROBOT

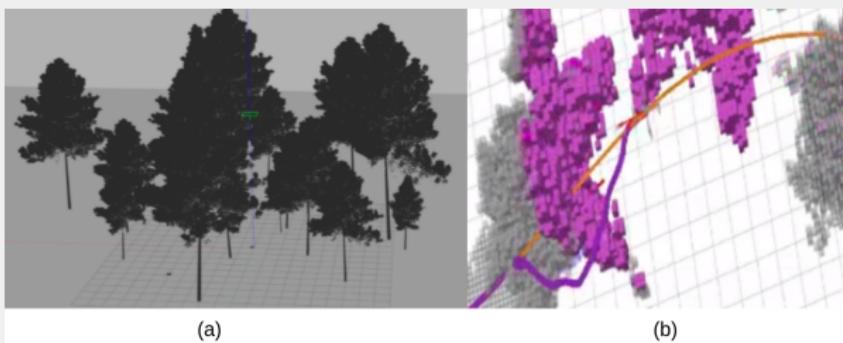
Robot configuration (robot footprint): specification of robot representation



<https://www.ncynl.com/archives/201809/2602.html>

CONFIGURATION SPACE FOR OBSTACLES

Planning in the workspace is computationally hard and time-consuming: both the robot and obstacles have different shapes

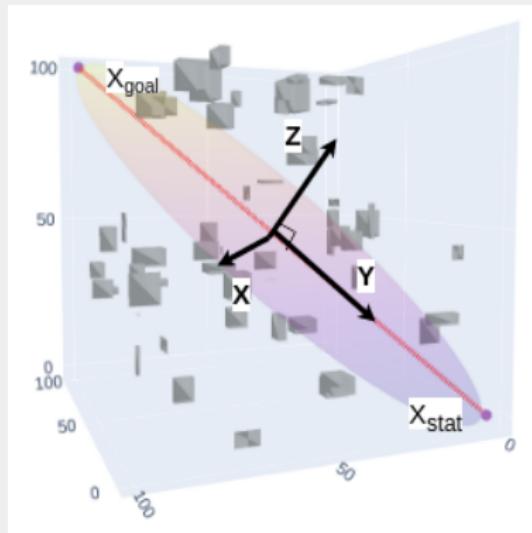


Obstacle representation in the **configuration space** is extremely complicated. Only **approximations** are applied with known pieces of information and computational capabilities

PATH PLANNING PROBLEM FORMULATION

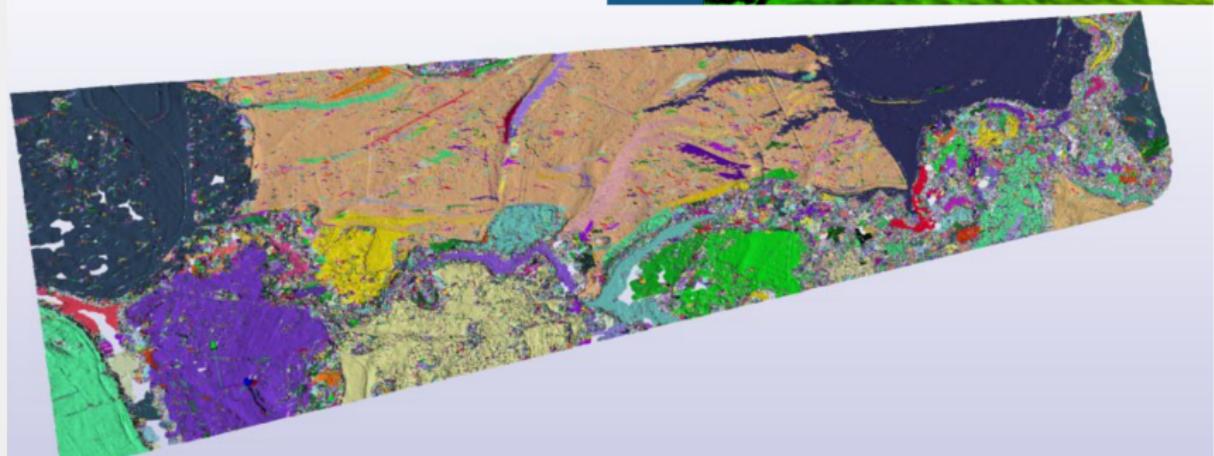
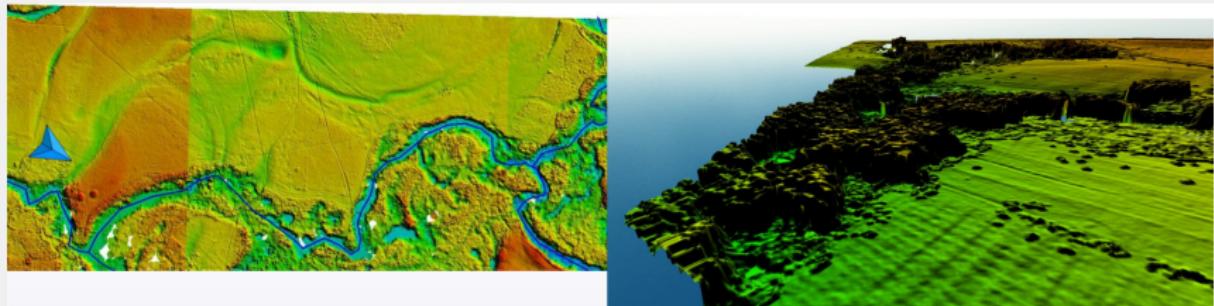
- $X = (0, 1)^d$ as the configuration space where $X \in \mathbb{R}^d$
- X_{obs} is the regions occupied by the obstacles
- $X \setminus X_{obs}$ becomes an open set where $cl(X \setminus X_{obs})$ can be defined as the traversable space (X_{free}) where $cl(\cdot)$ denotes the closure of set $X \setminus X_{obs}$
- X_{start} and X_{goal} are the start and target position for planning, where $X_{start} \in X_{free}$ and $X_{goal} \in X_{free}$
- $\sigma : [0, 1]^d \rightarrow \mathbb{R}^d$ denotes the waypoints of the planned path, provided that $\sigma(\tau) \in X_{free}$ for all $\tau \in [0, 1]$; This path is called as a obstacle free path

PATH PLANNING PROBLEM FORMULATION



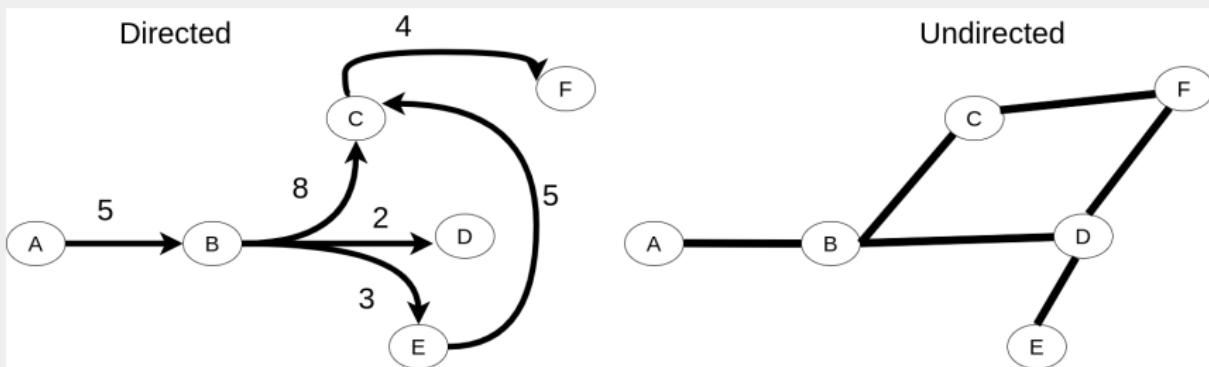
What kind of things do you have to consider for defining $cl(X \setminus X_{obs})$ in this workspace?

SEARCH-BASED PLANNING : MAPPING



SEARCH-BASED PLANNING : GRAPH

Graphs have edges and **nodes** whose **connectivity** is defined by **directed or undirected edges**. In motion planning, the mathematical representation of the search-based path planning algorithm is called **state space graph**



Search tree: searching the graph in a defined way produces a tree. Back-tracing a node in the search tree gives a path/path(s) from start to end node

GRAPH SEARCHING

- Need a **container** to keep all the nodes that are **needed to explore**
- Starts the container with starting node X_{start}
- Repeat
 - ▶ Visit **each node from the container** and **remove** it if it does not match with **defined constraints**, e.g., obstacle-free or obstacle-occupied
 - ▶ Check the desired constraint, if so terminate searching
 - ▶ Node whose constraints are satisfied, **obtain neighbours** of it
 - ▶ **Push back neighbours** back into the **container**

GRAPH SEARCHING

1. When to **terminate the search?** in the case of the **empty container** or **met the desired constraint** to terminate the search.
2. What if the **graph is cyclic?** when a **node is removed** from the container **it should never be considered**
3. **What are the ways to remove the right node** such that the search meets the **desired constraint** as **soon** as it can

GRAPH SEARCHING

SISTEMA
LIFO
LAST IN, FIRST OUT



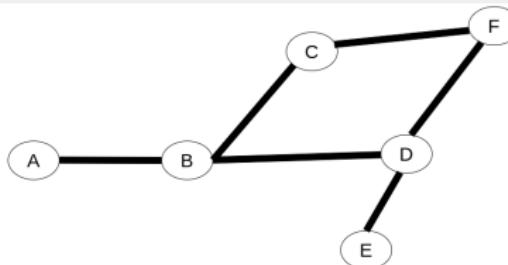
SISTEMA
FIFO
FIRST IN, FIRST OUT



<https://controlinventarios.files.wordpress.com/2021/11/image-10.png>

GRAPH SEARCHING: DEPTH FIRST SEARCH

The strategy is to expand the **deepest node** in the container



Algorithm 1 Depth first search

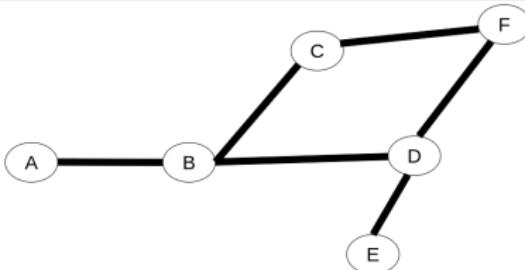
```
1: procedure DFS( $G, n$ ) ▷ The graph G, starting node n
2:    $n.visited \leftarrow true$ 
3:   for  $eache \in G.neighbours(n)$  do
4:     if  $e.visited == false$  then
5:       dfs( $G, v$ )
6:     end if
7:   end for
8: end procedure
```

| | | |
|--------|-----------|------------------|
| step 1 | visited | A |
| | container | B |
| step 2 | visited | A, B |
| | container | C, D |
| step 3 | visited | A, B, C |
| | container | F, D |
| step 4 | visited | A, B, C, F |
| | container | D |
| step 5 | visited | A, B, C, F, D |
| | container | E |
| step 6 | visited | A, B, C, F, D, E |
| | container | |

Uses Stack data structure

GRAPH SEARCHING: BREATH FIRST SEARCH

The strategy is to expand the **shallowest node** in the container



Algorithm 2 Breath first search

```
1: procedure BFS( $G, n$ )
2:    $container \leftarrow []$                                  $\triangleright$  The graph G, starting node n
3:    $n.visited \leftarrow true$ 
4:    $container.push(n)$ 
5:   while  $container.empty()$  do
6:      $v \leftarrow container.pop()$ 
7:     for each  $e \in G.neighbours(v)$  do
8:       if  $e.visited == false$  then
9:          $container.push(e)$ 
10:      end if
11:    end for
12:  end while
13: end procedure
```

| | | |
|--------|-----------|------------------|
| step 1 | visited | A |
| | container | B |
| step 2 | visited | A, B |
| | container | C, D |
| step 3 | visited | A, B, C |
| | container | D, F |
| step 4 | visited | A, B, C, D |
| | container | F, E |
| step 5 | visited | A, B, C, D, F |
| | container | E |
| step 6 | visited | A, B, C, D, F, E |
| | container | |

GRAPH SEARCHING: BREATH FIRST SEARCH

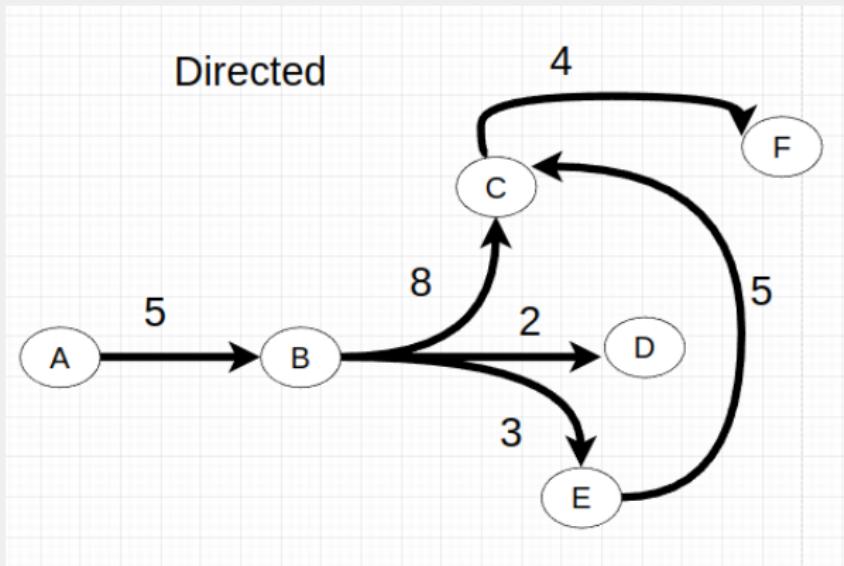
Breath first search can also be used for **flow field path-finding, distance map (or cost map)** generation, and much more.

Algorithm 3 Breath first search

```
1: procedure BFS( $G, n, g, dist$ )
2:    $openset \leftarrow []$   $\triangleright$  The graph  $G$ , starting node  $n$ , goal node  $g$ , minimum
   allowable residual  $dist$ 
3:    $n.id \leftarrow will\_be$ 
4:    $openset.insert(n)$ 
5:   while  $openset.empty()$  do
6:      $v \leftarrow container.pop()$ 
7:      $v.id \leftarrow was\_there$ 
8:     if  $dist < |v - g|$  then
9:        $terminate \leftarrow v$ 
10:      return
11:    end if
12:    for each  $e \in G.neighbours(v)$  do
13:      if  $e.id \neq was\_there$  then
14:         $e.parent \leftarrow v$ 
15:         $e.id \leftarrow will\_be$ 
16:         $openset.push(e)$ 
17:      end if
18:    end for
19:  end while
20: end procedure
```

GRAPH SEARCHING: COSTS CONSIDERATION

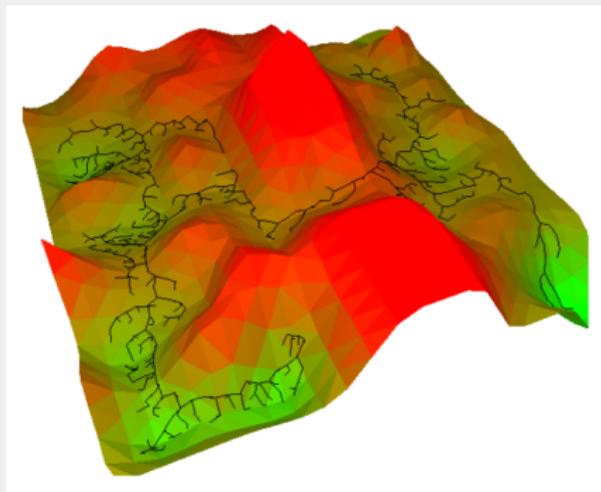
In real-world scenarios, different types of costs, e.g., length, time, and energy, are associated with nodes. If all **costs are equal**, the **breath first search** gives the optimal solution.



How can we find the **least-cost path** solution?

GRAPH SEARCHING: COSTS CONSIDERATION

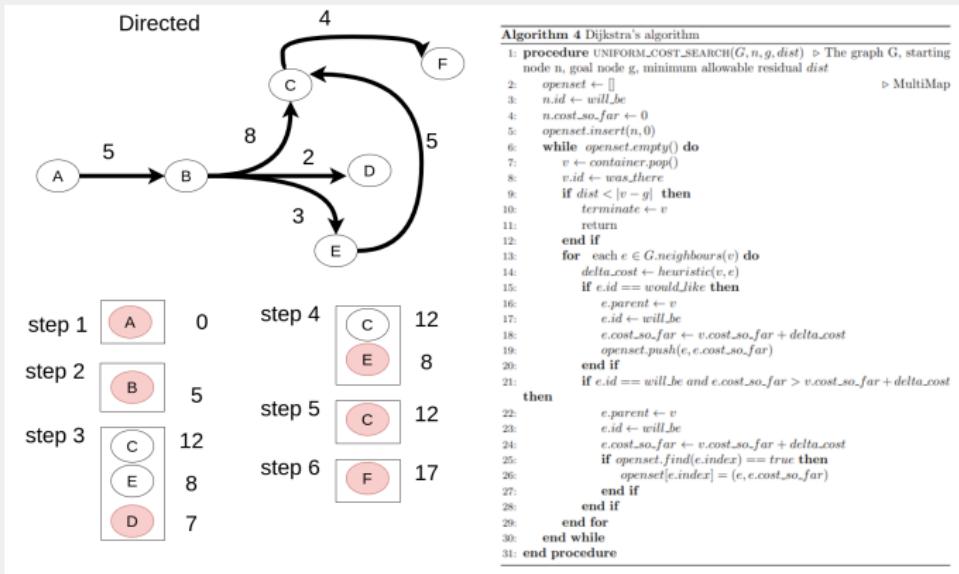
The strategy is to expand the node with **cheapest accumulated cost $g(v)$** , where $g(v)$ is the best-accumulated cost from the start node to node v . **Node** that has been **expanded** ensures having the **minimum cost from the start node**



Jaillet, L., Cortes, J., Simeon, T. (2008, September). Transition-based RRT for path planning in continuous cost spaces. In 2008 IEEE/RSJ International Conference on Intelligent Robots and Systems (pp. 2145-2150). IEEE.

GRAPH SEARCHING: DIJKSTRA'S ALGORITHM

The algorithm is **complete** and **optimal**



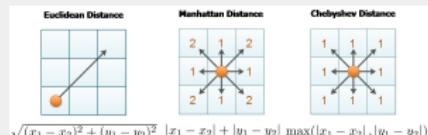
The algorithm incorporates **only the accumulated cost till the considered node**, and **every direction** has to incorporate to make the **next move**. **No information about the goal location**

GRAPH SEARCHING: GREEDY BEST FIRST SEARCH

The algorithm incorporates the **cost to the goal** $h(n)$ with heuristic cost estimation, e.g., Manhattan distance, Euclidean distance. Hence, start **exploring towards goal direction**

Algorithm 5 Greedy breath first search

```
1: procedure GBFS( $G, n, g, dist$ )  $\triangleright$  The graph G, starting node n, goal node  
g, minimum allowable residual dist  
2:    $openset \leftarrow []$   $\triangleright$  Priority queue  
3:    $n.id \leftarrow will\_be$   
4:    $n.cost\_to\_go \leftarrow 0$   
5:    $openset.insert(n, 0)$   
6:   while  $openset.empty()$  do  
7:      $v \leftarrow container.pop()$   
8:     if  $dist < |v - g|$  then  
9:       terminate  $\leftarrow v$   
10:      return  
11:    end if  
12:     $v.id \leftarrow was\_there$   
13:    for each  $e \in G.neighbours(v)$  do  
14:      if  $e.id \neq was\_there$  then  
15:         $e.parent \leftarrow v$   
16:         $e.id \leftarrow will\_be$   
17:         $e.cost\_to\_go \leftarrow heuristic(g, e)$   
18:         $openset.push(e, e.cost\_to\_go)$   
19:      end if  
20:    end for  
21:  end while  
22: end procedure
```

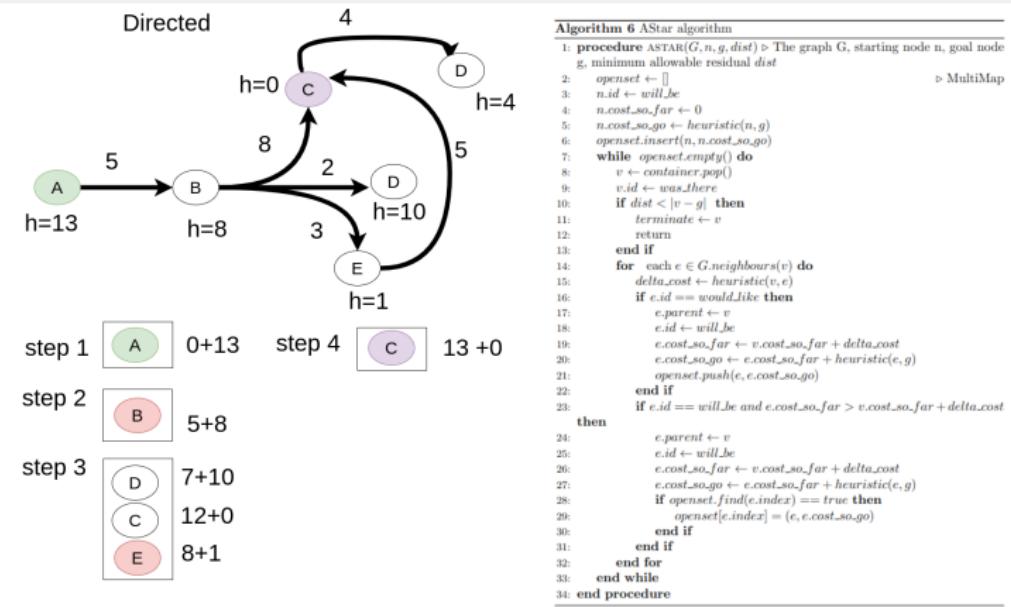


$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad |x_1 - x_2| + |y_1 - y_2| \quad \max(|x_1 - x_2|, |y_1 - y_2|)$$

<https://iq.opengenus.org/euclidean-vs-manhattan-vs-chebyshev-distance/>

A*: COMBINATION OF GREEDY BEST FIRST SEARCH AND DIJKSTRA'S ALGORITHM

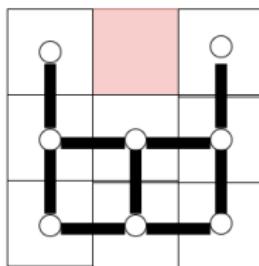
The strategy to expand the node with the **cheapest cost = $g(n) + h(n)$** , $g(n)$ **accumulated cost** from the start node to the node n and $h(n)$ estimated **heuristic cost** from node n to the goal node



A*: DESIGN CONSIDERATION

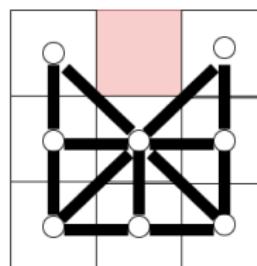
- The heuristic h is admissible if $f = h(n) \leq h^*(n)$ for all "n", where $h^*(n)$ is the least cost from node n to the goal node.
- The cost to go and the cost so far can be weighted in a defined way, e.g., closer to the goal $g(n) + \alpha h$, $\alpha > 1$. This strategy is called weighted A*.

$$\in \mathbb{R}^2$$



4 connections

$$\in \mathbb{R}^3$$



8 connections

A*: DESIGN CONSIDERATION

- Priority queue in c++ can be implemented in several ways
 - ▶ std::priority_queue
 - ▶ std::make_heap
 - ▶ std::multimap
- If more than one close-by node has the same f value, compare their h values and add a deterministic random value to the h, this idea is called **Tie Breaker**

$$\begin{aligned}\delta x_1 &= |n.x - g.x|, & \delta y_1 &= |n.y - g.y| \\ \delta x_2 &= |s.x - g.x|, & \delta y_2 &= |s.y - g.y|\end{aligned}\quad (1)$$

$$cross = |\delta x_1 \times \delta y_2 - \delta x_2 \times \delta y_1| / h = h + cross \times 0.001$$

there are many ways to modify these heuristics. One of the recently proposed approach: **Jump Point Search (JPS)[1]**

[1]. Harabor, D., Grastien, A. (2014, May). Improving jump point search. In Proceedings of the International Conference on Automated Planning and Scheduling (Vol. 24, pp. 128-135).

GRAPH-BASED SEARCH PROBLEM CLASSIFICATION

- **Search problem** can be seen in two different ways:
Uninformed or **Informed search problem**. In an uninformed search besides, the problem definition no further definition is provided (e.g., **breadth-first**, **depth-first**, etc), On the contrary, an informed search for future information pursues deterministic or heuristic way (**A***, **JPS**, **D***, etc)
- Performance of the search problem can be estimated in four different ways[1]
 - ▶ **Completeness**: does the algorithm find the solution when there is one?
 - ▶ **Optimality**: is the solution the best one of all possible solutions in terms of path cost?
 - ▶ **Time complexity**: how long does it take to find a solution?
 - ▶ **Space complexity**: how much memory is needed to perform the search?

[1]. <http://ais.informatik.uni-freiburg.de/teaching/ss11/robotics/slides/18-robot-motion-planning.pdf>

KINODYNAMIC A*:HEURISTICS

Consider KinoDynamic A*:Heuristics, where the objective function is expressed by:

$$J = \int_0^T g(x, u) dt = \int_0^T (1 + u^\top R u) dt = \int_0^T (1 + a_x^2 + a_y^2 + a_z^2) dt \quad (2)$$

where system state $x = (p_x, p_y, p_z, v_x, v_y, v_z)$, system input $u = a_x, a_y, a_z$, and motion model of the system $\dot{x} = f(x, u) = (v_x, v_y, v_z, a_x, a_y, a_z)$. Pontryagin's minimum principle can be used to solve this problem:

$$\dot{x}^*(t) = f(x^*(t), u^*(t)), \quad x^*(0) = x(0)$$

■ Define the Hamiltonian

$$\begin{aligned} H(x, u, \lambda) &:= g(x, u) + \lambda^\top f(x, u), \quad \lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6) \\ &= (1 + a_x^2 + a_y^2 + a_z^2) + \lambda^\top f(s, u) \\ &= (1 + a_x^2 + a_y^2 + a_z^2) + \lambda_1 v_x + \lambda_2 v_y + \lambda_3 v_z + \lambda_4 a_x \\ &\quad + \lambda_5 a_y + \lambda_6 a_z \end{aligned} \quad (3)$$

KINODYNAMIC A*:HEURISTICS

- Since x is a function of p and v ,

$$\begin{aligned}\dot{\lambda}^*(t) &= -\frac{H(\cdot)}{\partial x} = -\left(\frac{\partial f(\cdot)}{\partial x}\right)^\top \lambda^*(t) - \frac{\partial g(\cdot)}{\partial x} \\ \dot{\lambda} &= (0, 0, 0, -\lambda_1, -\lambda_2, -\lambda_3)^\top\end{aligned}\tag{4}$$

The costate equation is determined easily, for the latter convenience the solution is written in constants α, β

$$\lambda(t) = \begin{bmatrix} 2\alpha_1 \\ 2\alpha_2 \\ 2\alpha_3 \\ -2\alpha_1 t - 2\beta_1 \\ -2\alpha_2 t - 2\beta_2 \\ -2\alpha_3 t - 2\beta_3 \end{bmatrix}$$

KINODYNAMIC A*:HEURISTICS

- The optimal input is solved as:

$$\begin{aligned} o &= \frac{\mathcal{H}(\cdot)}{\partial u} = \left(\frac{\partial g(\cdot)}{\partial u} \right)^\top \lambda^*(t) + \frac{\partial f(\cdot)}{\partial u} \\ u^*(t) &= \begin{bmatrix} \alpha_1 t + \beta_1 \\ \alpha_2 t + \beta_2 \\ \alpha_3 t + \beta_3 \end{bmatrix} \end{aligned} \tag{5}$$

KINODYNAMIC A*:HEURISTICS

- From which, i.e., $u^*(t)$, the optimal state trajectory is solved by integration, i.e., an integral on u to get v and get p in the integral v :

$$x^*(t) = \begin{bmatrix} \frac{\alpha_1}{6}t^3 + \frac{\beta_1}{2}t^2 + v_{xo}t + p_{xo} \\ \frac{\alpha_2}{6}t^3 + \frac{\beta_2}{2}t^2 + v_{yo}t + p_{yo} \\ \frac{\alpha_3}{6}t^3 + \frac{\beta_3}{2}t^2 + v_{zo}t + p_{zo} \\ \frac{\alpha_1}{2}t^2 + \frac{\beta_1}{2}t + v_{xo} \\ \frac{\alpha_2}{2}t^2 + \frac{\beta_2}{2}t + v_{yo} \\ \frac{\alpha_3}{2}t^2 + \frac{\beta_3}{2}t + v_{zo} \end{bmatrix}, \quad (6)$$

where initial state $x(0) = (p_{xo}, p_{yo}, p_{zo}, v_{xo}, v_{yo}, v_{zo})$.

KINODYNAMIC A*:HEURISTICS

The remaining unknowns α, β are solved for as a function of the desired end transnational variable components as defined in eq. ??

$$\begin{bmatrix} \frac{T^3}{6} & 0 & 0 & \frac{T^2}{2} & 0 & 0 \\ 0 & \frac{T^3}{6} & 0 & 0 & \frac{T^2}{2} & 0 \\ 0 & 0 & \frac{T^3}{6} & 0 & 0 & \frac{T^2}{2} \\ \frac{T^2}{2} & 0 & 0 & T & 0 & 0 \\ 0 & \frac{T^2}{2} & 0 & 0 & T & 0 \\ 0 & 0 & \frac{T^2}{2} & 0 & 0 & T \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \Delta p_x \\ \Delta p_y \\ \Delta p_z \\ \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{bmatrix}}_{x(T) - x(0)} = \begin{bmatrix} p_{xf} - v_{xo}T - p_{xo} \\ p_{yf} - v_{yo}T - p_{yo} \\ p_{zf} - v_{zo}T - p_{zo} \\ v_{xf} - v_{xo} \\ v_{yf} - v_{yo} \\ v_{zf} - v_{zo} \end{bmatrix}, \quad (7)$$

where final state $x(T) = (p_{xf}, p_{yf}, p_{zf}, v_{xf}, v_{yf}, v_{zf})$.

KINODYNAMIC A*:HEURISTICS

Hence, solving for the unknown coefficients yields

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2} & 0 & 0 \\ 0 & -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2} & 0 \\ 0 & 0 & -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2} \\ \frac{6}{T^2} & 0 & 0 & -\frac{2}{T} & 0 & 0 \\ 0 & \frac{6}{T^2} & 0 & 0 & -\frac{2}{T} & 0 \\ 0 & 0 & \frac{6}{T^2} & 0 & 0 & -\frac{2}{T} \end{bmatrix} \begin{bmatrix} \Delta p_x \\ \Delta p_y \\ \Delta p_z \\ \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{bmatrix} \quad (8)$$

KINODYNAMIC A*:HEURISTICS

After finding these, $x^*(t)$ and cost function J can be determined.

$$\begin{aligned} J &= \frac{1}{T} \int_0^T (1 + a_x^2 + a_y^2 + a_z^2) dt \\ &= T + \left(\frac{\alpha_1^2 T^3}{3} + \alpha_1 \beta_1 T^2 + \beta_1^2 T \right) + \left(\frac{\alpha_2^2 T^3}{3} + \alpha_2 \beta_2 T^2 \right. \\ &\quad \left. + \beta_2^2 T \right) + \left(\frac{\alpha_3^2 T^3}{3} + \alpha_3 \beta_3 T^2 + \beta_3^2 T \right) \end{aligned} \quad (9)$$

The cost function is only a function of time, hence this kind of problem is called a minimum time problem.

KINODYNAMIC A*:HEURISTICS

$$\frac{dJ}{dT} = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 = 0$$

To find roots of a such polynomial, **matrix eigenvalue method** can be employed. The matrix eigenvalues are defined as

$$A\mathbf{y} = \lambda\mathbf{y}$$

If the roots of the polynomial are the eigenvalues of the matrix

$$A\mathbf{y} = x\mathbf{y}$$

That is to say if $\mathbf{y} = [x^3 \ x^2 \ x \ 1]^\top$, matrix A is constructed as

$$\underbrace{\begin{bmatrix} -\frac{c_3}{c_4} & -\frac{c_2}{c_4} & -\frac{c_1}{c_4} & -\frac{c_0}{c_4} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_A \begin{bmatrix} x^3 \\ x^2 \\ x \\ 1 \end{bmatrix} = x \begin{bmatrix} x^3 \\ x^2 \\ x \\ 1 \end{bmatrix}$$

HYBRID A*: MOTION MODEL

- What steering angles are possible? $[-\pi/2, \pi/2]$. It was assumed that the car moves in the direction that the rear wheels are pointing. When $\phi = \pi/2$, the front wheel is perpendicular to the rear wheels, then car has to rotate in place. In other words, $\dot{x} = \dot{y} = 0$ because the center of the rear axle does not translate. This behaviour is usually not possible because the front wheels would collide with the front axle when turned to $\phi = \pi/2$. Thus,

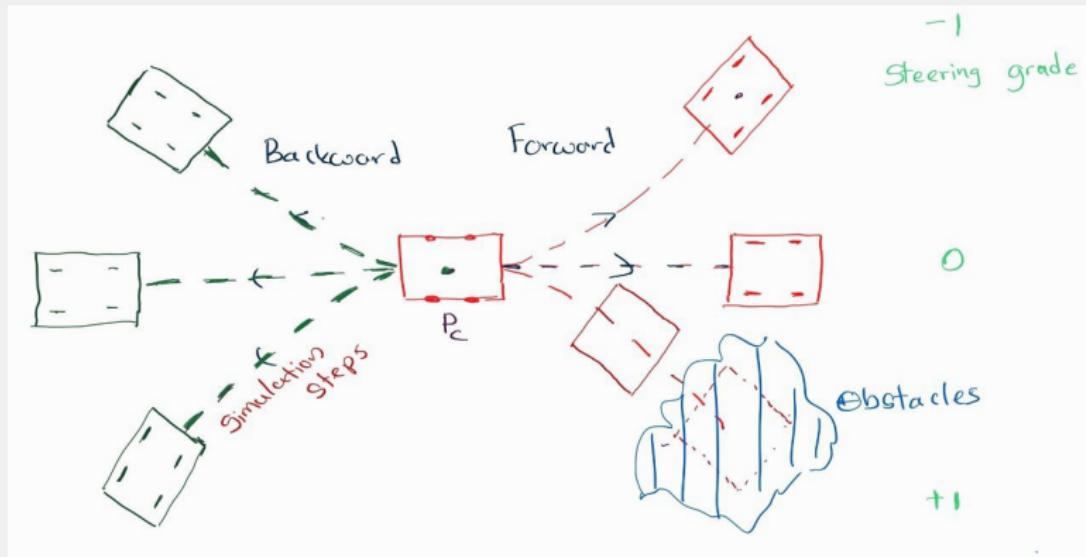
$$|\phi| \leq \phi_{max}$$

- A simple car is moving slowly to safely neglect dynamics, let's assume $|u_s| \leq 1$, where $u_s \in \{-1, 0, 1\}$. However, $0 < u_s < 1$, in this case, car can not drive in reverse

HYBRID A*: MOTION MODEL

- **Tricycle:** $U = [-1, 1] \times [-\pi/2, \pi/2]$, Assuming front-wheel drive, the vehicle can rotate in place if $u_\theta = \pi/2$. This kind of motion can be obtained using the unicycle model
- **Simple car:** $U = [-1, -1] \times [-\phi_{max}, \phi_{max}]$, by requiring that $|u_\phi| < \phi_{max} < \pi/2$, a car with minimum turning radius $\rho_{min} = \frac{L}{\tan \phi_{max}}$ is obtained
- **Reeds-Shepp Car:** Further restrict the speed of the car, i.e., $u_s = \{-1, 0, 1\}$ or a car with three gears: reverse, park, and forward.
- **Dubins Car:** After removing reverse speed $u_s = -1$ from a Reeds-Shepp car, $u_s = \{0, 1\}$ as the only possible speeds

HYBRID A*: FINDING NEIGHBORS



HYBRID A*: COST TO GO H

- When **close to the goal** location: use **appropriate kinematically feasible path**, e.g., Dubins or Reeds-Shepp.
- When **far from the goal** location: **euclidean** distance or **manhattan** distance

HYBRID A*: COST SO FAR G

Algorithm 13 Calculate cost so far g

```
1: procedure CALCULATEG( $p_c, p_n$ )            $\triangleright$  Current pos and neighbour pos
2:    $\lambda_s, \lambda_{sc}, \lambda_r$        $\triangleright$  steering penalty, steering change penalty, and reversing
   penalty
3:   seg.length                       $\triangleright$  number of simulation steps
4:   if  $p_n.direction == FORWARD$  then
5:     if  $p_n.steering\_grade != p_c.steering\_grade$  then
6:       if  $p_n.steering\_grade == 0$  then
7:          $g \leftarrow seg.length \cdot \lambda_{sc}$ 
8:       end if
9:       if  $p_n.steering\_grade != 0$  then
10:         $g \leftarrow seg.length \cdot \lambda_{sc} \cdot \lambda_s$ 
11:      end if
12:    end if
13:    if  $p_n.steering\_grade == p_c.steering\_grade$  then
14:      if  $p_n.steering\_grade == 0$  then
15:         $g \leftarrow seg.length$ 
16:      end if
17:      if  $p_n.steering\_grade != 0$  then
18:         $g \leftarrow seg.length \cdot \lambda_s$ 
19:      end if
20:    end if
21:  end if
22:  if  $p_n.direction != FORWARD$  then
23:    if  $p_n.steering\_grade != p_c.steering\_grade$  then
24:      if  $p_n.steering\_grade == 0$  then
25:         $g \leftarrow seg.length \cdot \lambda_r$ 
26:      end if
27:      if  $p_n.steering\_grade != 0$  then
28:         $g \leftarrow seg.length \cdot \lambda_{sc} \cdot \lambda_s \cdot \lambda_r$ 
29:      end if
30:    end if
31:    if  $p_n.steering\_grade == p_c.steering\_grade$  then
32:      if  $p_n.steering\_grade == 0$  then
33:         $g \leftarrow seg.length \cdot \lambda_r$ 
34:      end if
35:      if  $p_n.steering\_grade != 0$  then
36:         $g \leftarrow seg.length \cdot \lambda_s \cdot \lambda_r$ 
37:      end if
38:    end if
39:  end if
```

HYBRID A*

Algorithm 14 Hybrid A*

```
1: procedure HYBRIDASTAR( $p_e, p_n$ )  $\triangleright$  The graph G, starting pos, goal node  
pos  
2:    $openset \leftarrow []$   $\triangleright$  MultiMap  
3:    $shot\_distance$   $\triangleright$  Minimum allowable distance robot pose and the goal  
4:    $n.id \leftarrow will.be$   
5:    $n.steer.grade \leftarrow 0$   
6:    $n.cost.so\_far \leftarrow 0$   
7:    $n.cost.so\_go \leftarrow computeH(n, g)$   
8:    $openset.insert(n, n.cost.so\_go)$   
9:   while  $openset.empty()$  do  
10:     $v \leftarrow openset.pop()$   
11:     $v.id \leftarrow was.there$   
12:    if  $shot.distance < |v - g|$  then  
13:       $terminate \leftarrow EstimateAnalyticalExpansion$   
14:      return  
15:    end if  
16:    for each  $e \in G.neighbours(v)$  do  
17:       $delta\_cost \leftarrow computeG(v, e)$   
18:      if  $e.id == would.like$  then  
19:         $e.parent \leftarrow v$   
20:         $e.id \leftarrow will.be$   
21:         $e.cost.so\_far \leftarrow v.cost.so\_far + delta\_cost$   
22:         $e.cost.so\_go \leftarrow e.cost.so\_far + computeH(e, g)$   
23:         $openset.push(e, e.cost.so\_go)$   
24:      end if  
25:      if  $e.id == will.be$  and  $e.cost.so\_far > v.cost.so\_far + delta\_cost$   
        then  
26:         $e.parent \leftarrow v$   
27:         $e.id \leftarrow will.be$   
28:         $e.cost.so\_far \leftarrow v.cost.so\_far + delta\_cost$   
29:         $e.cost.so\_go \leftarrow e.cost.so\_far + computeH(e, g)$   
30:        if  $openset.find(e.index) == true$  then  
31:           $openset[e.index] = (e, e.cost.so\_go)$   
32:        end if  
33:      end if  
34:    end for  
35:  end while  
36: end procedure
```