# Tutorial for the simulation of wall-bounded turbulent channel flow

Zheng Gong (Department of Hydraulic Engineering, Tsinghua University, Beijing, China)

## 1. Introduction

This tutorial illustrates the step by step usage of Fourth-order **Channel3d** for wall-bounded turbulent channel flow at $Re_\tau \approx 180$. Here, periodic boundary conditions are used in streamwise (x) and spanwise (z) directions, while no-slip conditions are adopted for top and bottom walls in normal direction (y). The computational sketch is shown is Fig.1.

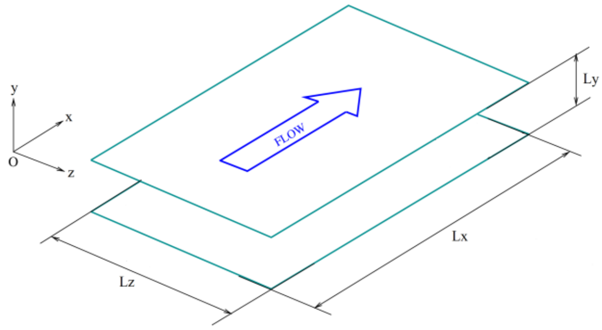


**Fig.1.** Sketch of computational domain for wall-bounded turbulent flows

The simulation has been initiated with a laminar parabolic Poiseuille velocity profile, plus a divergence-free large-amplitude sinusoidal perturbation:

$$u_0 = r_d u_b \bar{y} \exp\left(-\frac{9}{2}\bar{y}^2\right)\cos\left(\frac{\pi z}{100h}\mathrm{Re}_\tau\right) + 3u_b\left(\bar{y} - \frac{1}{2}\bar{y}^2\right) \tag{1a}$$

$$w_0 = r_d u_b \bar{y} \exp\left(-\frac{9}{2}\bar{y}^2\right)\sin\left(\frac{\pi x}{250h}\mathrm{Re}_\tau\right) \tag{1b}$$

$$v_0 = 0 \tag{1c}$$

where $\bar{y}$ is the dimensionless y coordinate defined as $\bar{y} = \min(y, 2h - y)/h$, and h denotes half channel height. $r_d$ is a random number, ranging from 0.9 to 1.1, to further disturb the initial flow. The flow is driven by a uniform pressure gradient, which varies in time to maintain a constant mass flow rate. After several time unit, the laminar profiles transit to turbulent ones, and the pressure gradient begins to fluctuate near an almost constant value.

## 2. Preparation

The friction Reynolds number is defined as $Re_\tau = u_\tau h/\nu$, where $\nu$ is the fluid kinematic viscosity, and $u_\tau$ is friction velocity: $u_\tau = \sqrt{\left(\nu \partial\bar{u}/\partial y\right)_{wall}} = \sqrt{\left(-dp/dx\right)h}$ . $Re_b$ denotes the bulk Reynolds number: $Re_b = u_b h/\nu$, where $u_b$ is the bulk velocity. For wall-bounded turbulence, $Re_b$ and $Re_\tau$ have the following empirical relationship:

$$\mathrm{Re}_\tau = 0.1538\,\mathrm{Re}_b^{0.8877} \tag{2}$$

Eq.(2) is regressed from the data provided by Lee & Moser [1], and the regression result is also shown in Fig.2. If we want to simulate the case $Re_\tau \approx 180$, we can use Eq.(2) to get an approximate value for $Re_b$, i.e. $Re_b \approx 2860$. If we further set $u_b = 2/3$ and $\mathrm{h} = 1$, we can get the corresponding viscosity: $\nu = (u_b h)/\mathrm{Re}_b = 2.3310\times10^{-4}$ .

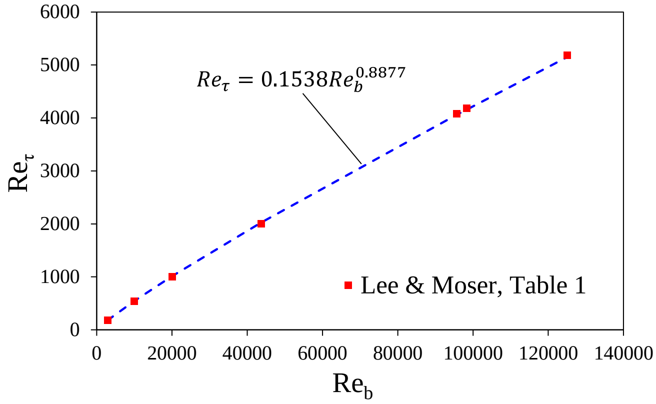


**Fig.2.** Relationship between bulk and friction Reynolds number

## 3. Parameter setting

The computational domain sizes are set to 4πh, 2h, 2πh in x-, y- and z-directions respectively. Details about the flow parameters and grid mesh are listed in Table 1. The input parameter text can be found in file ./doc/TurbulentChannel_4th_direct.prm. The illustrations for every input parameters can be found in file ./doc/channel4th_prm.md.

**Table 1**

List of parameters for turbulent channel flow cases. Here $N_x$, $N_y$, and $N_z$ are the number of grid points in streamwise, wall-normal and spanwise directions, respectively. $\Delta x^+$ and $\Delta z^+$ denote grid spacings in wall-parallel directions. $\Delta y_w^+$ and $\Delta y_c^+$ represent the grid spacing at the wall and the centerline in wall-normal direction respectively. $Tu_\tau/h$ is the time averaging window without transition.

| Flow Case | $Re_\tau$ | $Re_b$ | $N_x$ | $N_y$ | $N_z$ | $\Delta x^+$ | $\Delta y_w^+$ | $\Delta y_c^+$ | $\Delta z^+$ | $Tu_\tau/h$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Cha180 | 182.3 | 2860 | 384 | 192 | 256 | 5.97 | 0.0244 | 2.98 | 4.48 | 20.40 |

## 4. Running the code

If the executable file ./channel4th and ./Tool/interpolateField/interpolateField has not been compiled, you should compile it firstly, following the guidance written in ./README.md, Installation part. The simulation results and restart files will be stored in the folder ./CFD/Results/ and ./CFD/Restart, respectively.

Now, you can run this test case directly by typing:

```
chmod a+x ./channel4th
mpirun –n 8 ./channel4th ./Input/TurbulentChannel_4th_direct.prm
```

Alternatively, we can run the case on a coarse grid firstly, and then interpolate the resulting flow field to the fine mesh. If so, we can do as follows.

### 4.1 Running on the coarse mesh

Firstly, we run on a coarse grid where $N_x \times N_y \times N_z = 180 \times 132 \times 120$. Here we choose an arbitrary coarse mesh configuration. The evolution of $Re_\tau$ for coarse mesh is shown in Fig.3.

```
mpirun –n 8 ./channel4th ./Input/TurbulentChannel_4th_1.prm
```

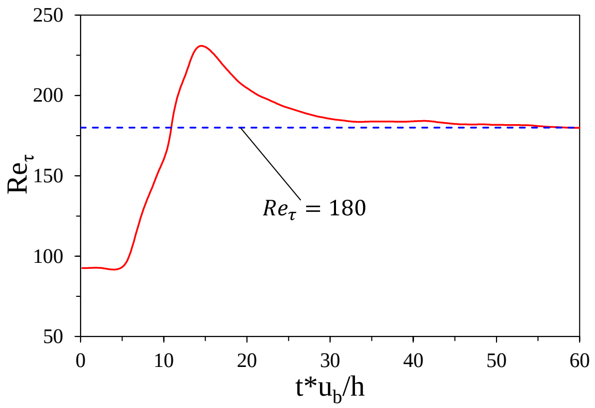


**Fig.3.** Time evolution of the friction Reynolds number $Re_\tau$ for coarse mesh

## 4.2 Interpolating flow field from coarse mesh to fine mesh

Now, we need to interpolate flow field from coarse mesh to fine mesh, using the executable file ./Tool/interpolateField/interpolateField:

```
1    cd ./Tool/interpolateField/
2    chmod a+x interpolateField
3    mpirun –n 8 interpolateField interp1.prm
4    cd ../../
```

After that, New restart file (./CFD/RestartForCha180_2_0000003000) will be generated.

## 4.3 Running on the fine mesh

Lastly, we run on the fine mesh where $\mathrm{N_x} \times \mathrm{N_y} \times \mathrm{N_z} = 384 \times 192 \times 256$. The evolution of $Re_\tau$ for fine mesh is shown in Fig.4.

```
1    mpirun –n 8 ./channel4th ./Input/TurbulentChannel_4th_2.prm
```

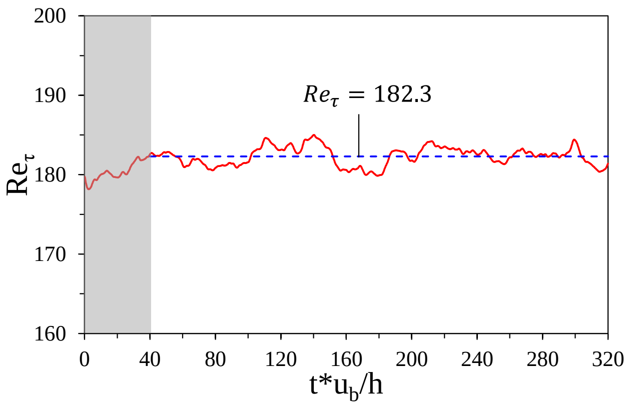


**Fig.4.** Time evolution of the friction Reynolds number $Re_\tau$ for fine mesh. Gray area indicates the adjustment stage from coarse mesh to fine mesh, and the corresponding time interval will be excluded from the statistic calculation.

# 5. Visualization

One can use Paraview for visualization. An interface file ended with the suffix .xmf is included in the folder ./CFD/Results/ (./CFD/Results/VisuForCha180_2_.xmf here), and this file can be read by Paraview directly.

# 6. Post-processing

Post-processing might be the most interesting part for wall-bounded turbulence. Here I will illustrate how to get the desired statistic data using **Channel3d**.

## 6.1 Data storage

The canonical statistic data include mean velocity profile, turbulent intensity, Reynolds stress, vorticity correlation, skewness, flatness, turbulent budget, energy spectra and so on.

If *a*, *b*, *c*, and *d* denote four variables. The second-order correlation can be defined as:

$$\langle a'b' \rangle = \langle [a - \langle a \rangle][b - \langle b \rangle] \rangle \tag{3}$$

While Eq.(3) can not be used directly, due to the fact that the exact $\langle a \rangle$ and $\langle b \rangle$ are not available until end of the simulation. So alternately, we can use the following expression instead:

$$\langle a'b' \rangle = \langle ab \rangle - \langle a \rangle \langle b \rangle \tag{4}$$

Similarly, the third-order and fourth-order correlations read:

$$\langle a'b'c' \rangle = \langle abc \rangle - \langle a \rangle \langle bc \rangle - \langle b \rangle \langle ac \rangle - \langle c \rangle \langle ab \rangle + 2 \langle a \rangle \langle b \rangle \langle c \rangle \tag{5}$$

$$\begin{aligned} \langle a'b'c'd' \rangle = {} & \langle abcd \rangle - 3 \langle a \rangle \langle b \rangle \langle c \rangle \langle d \rangle \\ & + \langle a \rangle \langle b \rangle \langle cd \rangle + \langle b \rangle \langle c \rangle \langle da \rangle + \langle c \rangle \langle d \rangle \langle ab \rangle \\ & + \langle d \rangle \langle a \rangle \langle bc \rangle + \langle a \rangle \langle c \rangle \langle bd \rangle + \langle b \rangle \langle d \rangle \langle ac \rangle \\ & - \langle a \rangle \langle bcd \rangle - \langle b \rangle \langle cda \rangle - \langle c \rangle \langle dab \rangle - \langle d \rangle \langle abc \rangle \end{aligned} \tag{6}$$

For instance, if we want to determine pressure strain term $\pi_{1,2} = \langle p' (\partial u'/\partial y + \partial v'/\partial x) \rangle$, we can further expand it as follows:

$$\pi_{1,2} = \left\langle p \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right\rangle + \langle p \rangle \left[ \frac{\partial \langle u \rangle}{\partial y} + \frac{\partial \langle v \rangle}{\partial x} \right] = \left\langle p \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right\rangle + \langle p \rangle \frac{\partial \langle u \rangle}{\partial y} \tag{7}$$

So we need to calculate and store $\langle p (\partial u/\partial y + \partial v/\partial x) \rangle$, $\langle p \rangle$, and $\langle u \rangle$ in advance. Table 2 lists the raw averaged date contained in file ./CFD/Results/stats*. It is also worth nothing that due to the use of fully staggered grids, so averaged variables (e.g. <u>) are placed in the y-center layer, while others (e.g. <v>) are stored in the y-node layer. The definition of turbulent budget terms are listed in Appendix A.

The raw averaged 1D energy spectra data are also stored in the file ./CFD/Results/Spec*.

## 6.2 Statistic results calculation

The final statistic results can be calculated by the post-processing Matlab code, ./Tool/readStat_CH.m. If Matlab is not available, one can use Octave instead:

```
cd ./Tool/
octave readStat_CH.m
cd ../
```

After running this Matlab code, the final statistic results will be generated in the folder ./StatOut/.

**Table 2**

List of the raw averaged date contained in file `./CFD/Results/stats*`. Here 'location' column indicates where the variable is placed. 'C' means 'Center', and 'N' stands for 'Node'.

| id | variable name | location | id | variable name | location |
|---|---|---|---|---|---|
| 1 | <u> | C | 19 | <uuu> | C |
| 2 | <v> | N | 20 | <www> | C |
| 3 | <w> | C | 21 | <uuuu> | C |
| 4 | <p> | C | 22 | <vvvv> | N |
| 5 | <uu> | C | 23 | <wwww> | C |
| 6 | <vv> | N | 24 | <p $\partial u/\partial x$> | C |
| 7 | <ww> | C | 25 | <p $\partial v/\partial y$> | C |
| 8 | <pp> | C | 26 | <p $\partial w/\partial z$> | C |
| 9 | <uv> | C | 27 | <p$(\partial u/\partial y + \partial v/\partial x)$> | N |
| 10 | <vw> | C | 28 | $\langle u\nabla^2 u\rangle$ | C |
| 11 | <uw> | C | 29 | $\langle v\nabla^2 v\rangle$ | N |
| 12 | <up> | C | 30 | $\langle w\nabla^2 w\rangle$ | C |
| 13 | <vp> | C | 31 | $\langle \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y} \rangle$ | C |
| 14 | <wp> | C | | | |
| 15 | <uuv> | C | 32 | $\langle (\partial u/\partial z)(\partial v/\partial z) \rangle$ | N |
| 16 | <vvv> | N | 33 | $\langle \omega_x\omega_x \rangle$ | N |
| 17 | <wwv> | C | 34 | $\langle \omega_y\omega_y \rangle$ | C |
| 18 | <uvv> | C | 35 | $\langle \omega_z\omega_z \rangle$ | N |

# 7. Statistic results validation

This section compare the statistic results with Lee & Moser [1], and Vreman & Kuerten [2] to for validation.

## 7.1 Basic profiles

Fig.5 shows some turbulent statistic profiles, i.e. mean velocity, velocity fluctuations, Reynolds stress, vorticity fluctuations, mean pressure, pressure fluctuation.

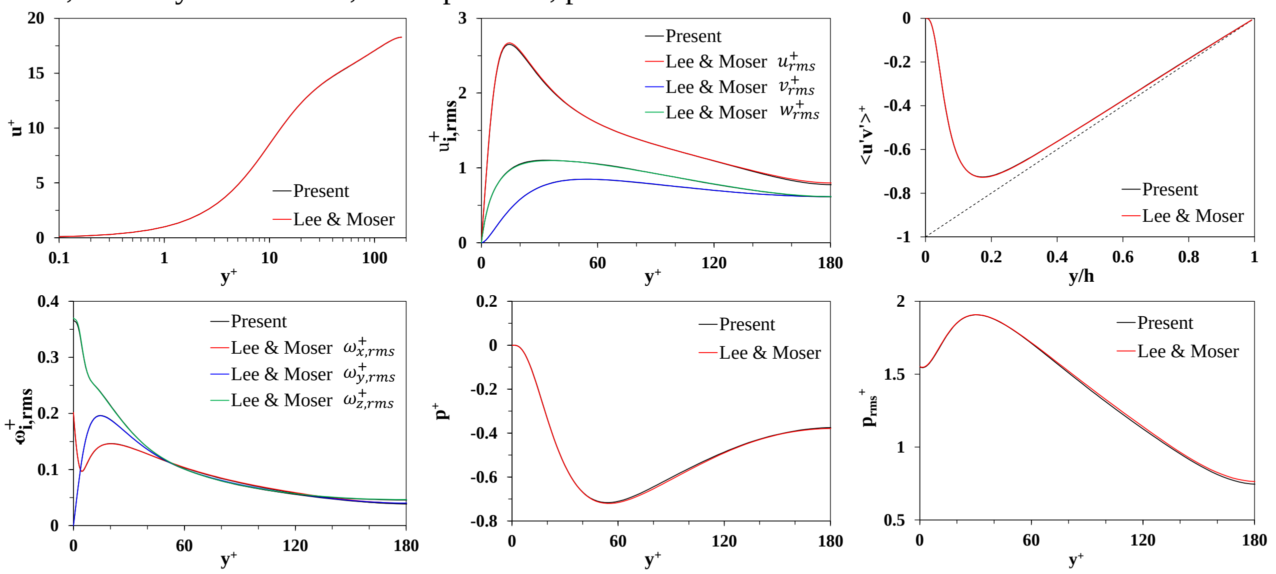


**Fig.5.** Basic turbulent statistic profiles

## 7.2 Budget profiles for TKE

Fig.6 shows the budget profiles for TKE.

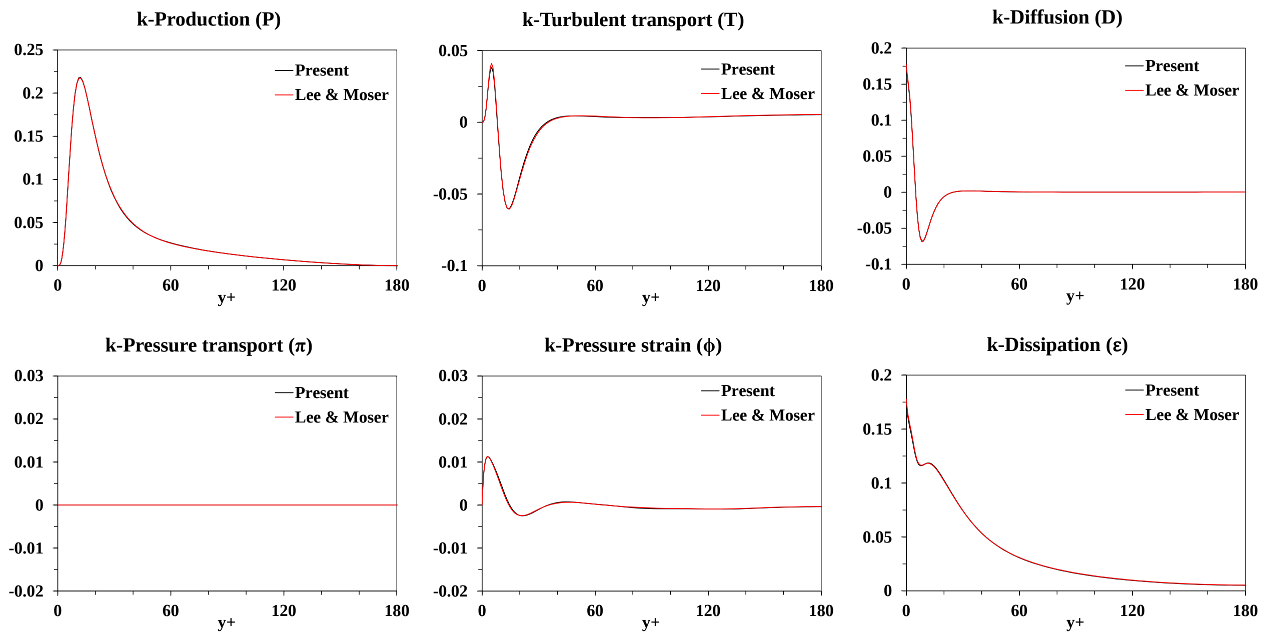


**Fig.6.** Budget profiles for TKE

## 7.3 Budget profiles for $< u'u' >$

Fig.7 shows the budget profiles for $< u'u' >$.

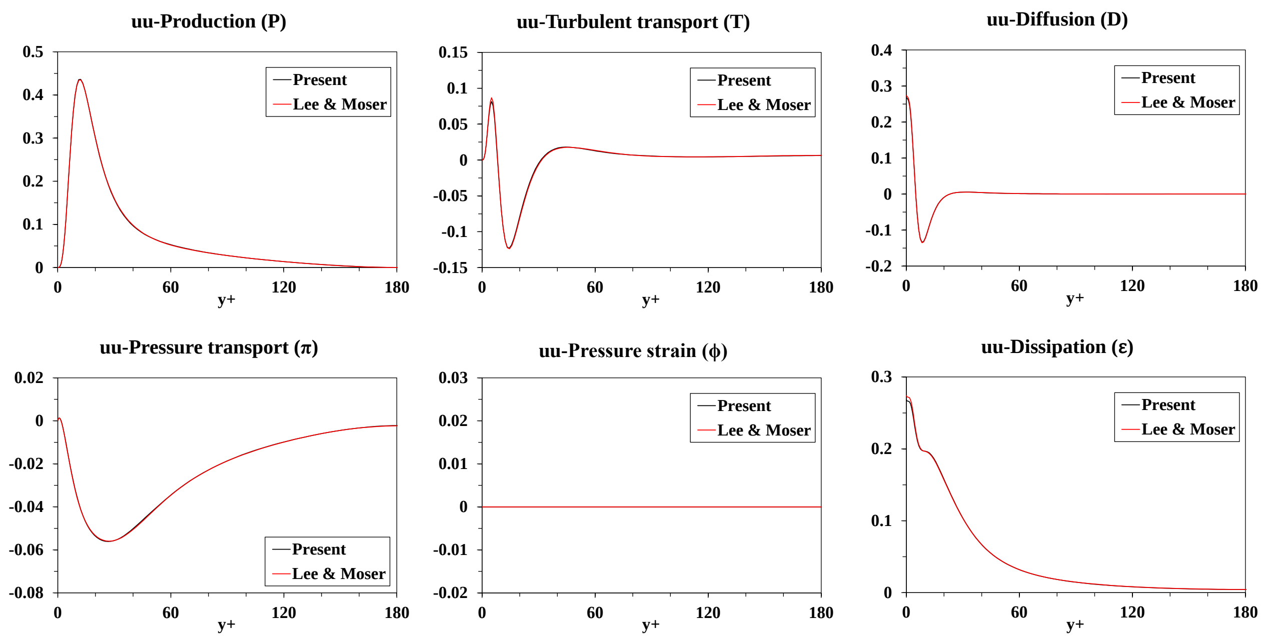


**Fig.7.** Budget profiles for $< u'u' >$

## 7.4 Budget profiles for $<v'v'>$

Fig.8 shows the budget profiles for $<v'v'>$.

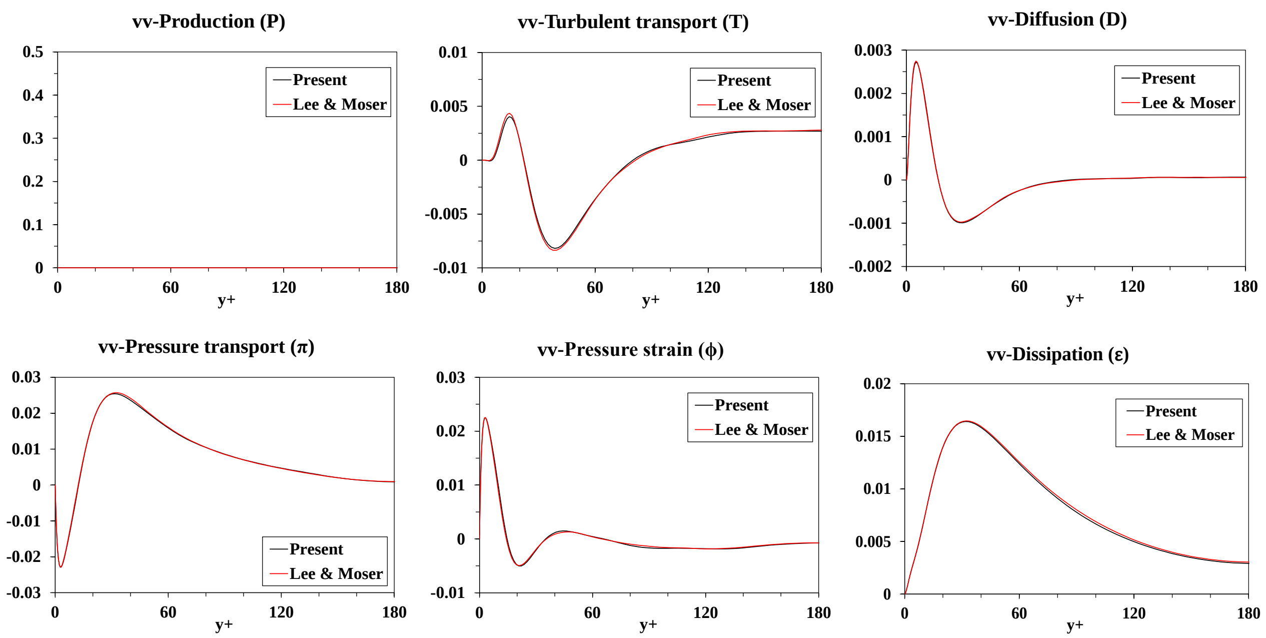


**Fig.8.** Budget profiles for $<v'v'>$

## 7.5 Budget profiles for $<w'w'>$

Fig.9 shows the budget profiles for $<w'w'>$.

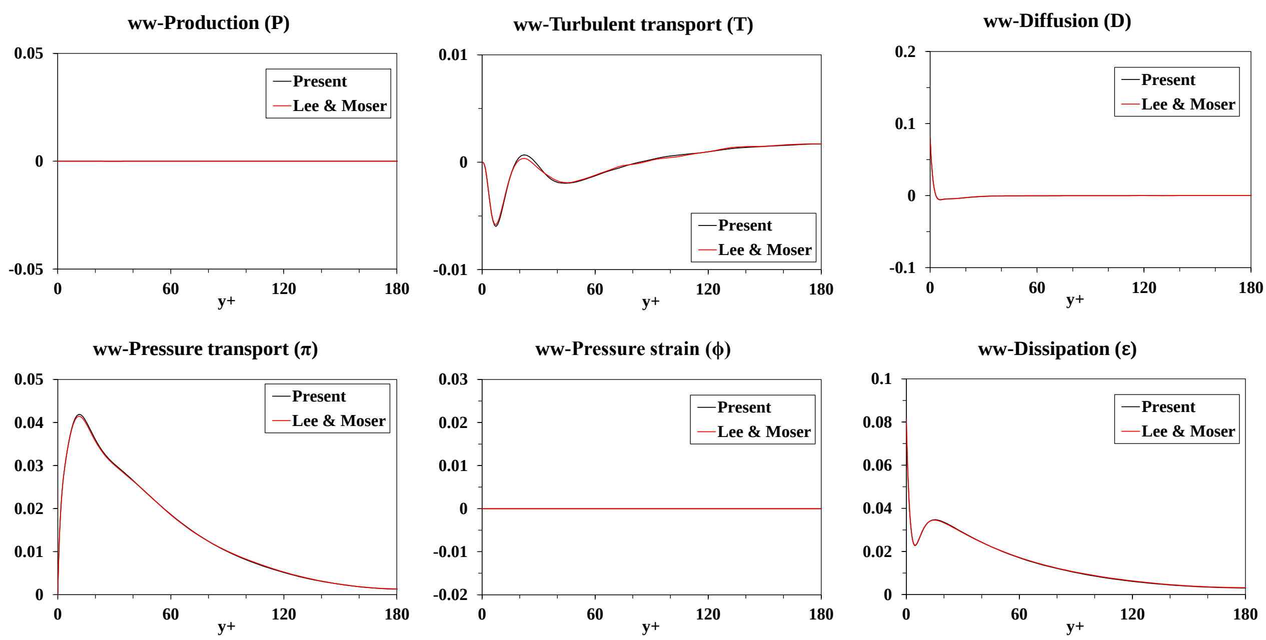


**Fig.9.** Budget profiles for $<w'w'>$

## 7.6 Budget profiles for $< u'v' >$

Fig.10 shows the budget profiles for $< u'v' >$.

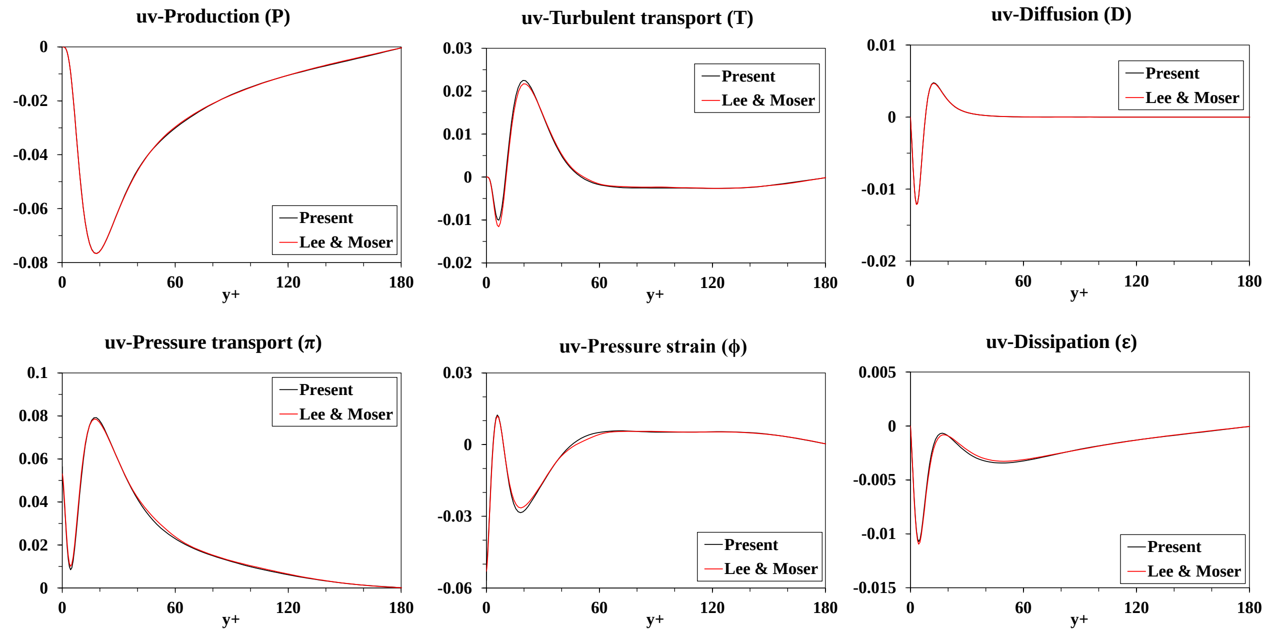


**Fig.10.** Budget profiles for $< u'v' >$

## 7.7 Energy spectra profiles

Fig.11 shows the energy spectra profiles at $y^+ \approx 36.5$.

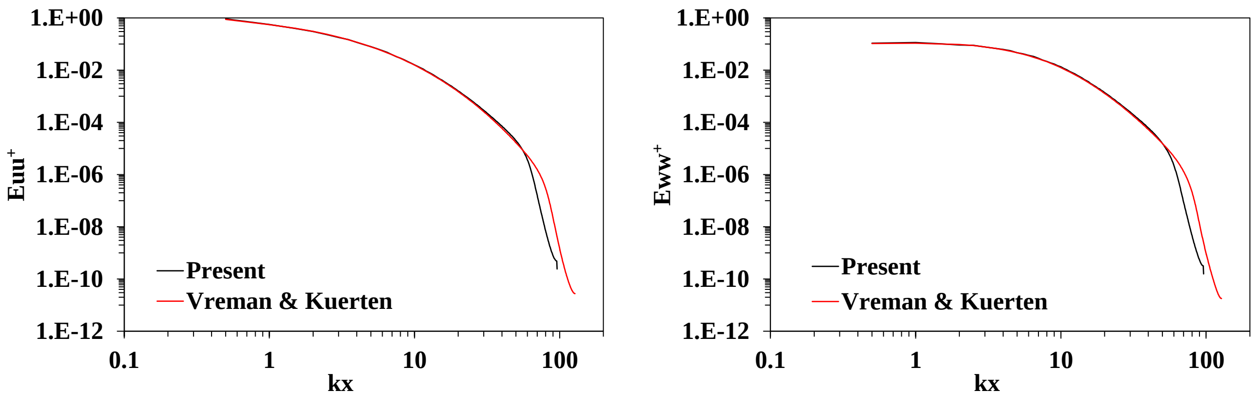


**Fig.11.** Energy spectra profiles at $y^+ \approx 36.5$

## Reference:

[1] Lee M, Moser RD. Direct numerical simulation of turbulent channel flow up to Re τ ≈5200. J Fluid Mech.2015;774:395-415. doi:10.1017/jfm.2015.268.

[2] Vreman AW, Kuerten JGM. Comparison of direct numerical simulation databases of turbulent channel flow at Reτ = 180. Phys Fluids. 2014;26(1):1-21. doi:10.1063/1.4861064.

# Appendix A. Definition of turbulent budget terms for wall-bounded turbulence

The general Reynolds stress transport equation reads:

$$\frac{\partial\langle u_i'u_j'\rangle}{\partial t}+\langle u_k\rangle\frac{\partial\langle u_i'u_j'\rangle}{\partial x_k}=P_{i,j}+T_{i,j}+\phi_{i,j}+\pi_{i,j}+D_{i,j}-\varepsilon_{i,j} \tag{A.1}$$

where $P_{i,j}$, $T_{i,j}$, $\phi_{i,j}$, $\pi_{i,j}$, $D_{i,j}$, and $\varepsilon_{i,j}$ stand for production term, turbulent transport term, pressure transport term, pressure strain term, viscous transport (diffusion) term, and viscous dissipation term respectively:

$$P_{i,j}=-\langle u_i'u_k'\rangle\frac{\partial\langle u_j\rangle}{\partial x_k}-\langle u_j'u_k'\rangle\frac{\partial\langle u_i\rangle}{\partial x_k} \tag{A.2a}$$

$$T_{i,j}=-\frac{\partial\langle u_i'u_j'u_k'\rangle}{\partial x_k} \tag{A.2b}$$

$$\phi_{i,j}=-\frac{\partial\langle p'u_i'\rangle}{\partial x_j}-\frac{\partial\langle p'u_j'\rangle}{\partial x_i} \tag{A.2c}$$

$$\pi_{i,j}=\left\langle p'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)\right\rangle \tag{A.2d}$$

$$D_{i,j}=\nu\frac{\partial^2\langle u_i'u_j'\rangle}{\partial x_k\partial x_k} \tag{A.2e}$$

$$\varepsilon_{i,j}=2\nu\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right\rangle \tag{A.2f}$$

Specifically, for wall-bounded turbulent flow, where periodic boundary conditions are applied in streamwise and spanwise directions. So Eq.(A.2) can be further simplified as follows:

$$\begin{aligned}
&P_{i,j}=-\langle u_i'v'\rangle\frac{\partial\langle u_j\rangle}{\partial y}-\langle u_j'v'\rangle\frac{\partial\langle u_i\rangle}{\partial y}, && T_{i,j}=-\frac{\partial\langle u_i'u_j'v'\rangle}{\partial y}\\
&\phi_{i,j}=-\frac{\partial\langle p'u_i'\rangle}{\partial x_j}-\frac{\partial\langle p'u_j'\rangle}{\partial x_i}, && \pi_{i,j}=\left\langle p'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)\right\rangle\\
&D_{i,j}=\nu\frac{\partial^2\langle u_i'u_j'\rangle}{\partial y^2}, && \varepsilon_{i,j}=2\nu\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right\rangle
\end{aligned} \tag{A.3}$$

And for $\langle u'u'\rangle$:

$$\begin{aligned}
&P_{1,1}=-2\langle u'v'\rangle\frac{\partial\langle u\rangle}{\partial y},\quad T_{1,1}=-\frac{\partial\langle u'u'v'\rangle}{\partial y},\phi_{1,1}=0\\
&\pi_{1,1}=2\left\langle p'\frac{\partial u'}{\partial x}\right\rangle,\quad D_{1,1}=\nu\frac{\partial^2\langle u'u'\rangle}{\partial y^2},\quad \varepsilon_{1,1}=2\nu\left\langle\frac{\partial u'}{\partial x_k}\frac{\partial u'}{\partial x_k}\right\rangle
\end{aligned} \tag{A.4}$$

For $\langle v'v' \rangle$:

$$\begin{aligned} &P_{2,2} = -2\langle v'v' \rangle \frac{\partial \langle v \rangle}{\partial y} = 0, \quad T_{2,2} = -\frac{\partial \langle v'v'v' \rangle}{\partial y}, \quad \phi_{2,2} = -2\frac{\partial \langle p'v' \rangle}{\partial y} \\ &\pi_{2,2} = 2\left\langle p' \frac{\partial v'}{\partial y} \right\rangle, \quad D_{2,2} = \nu \frac{\partial^2 \langle v'v' \rangle}{\partial y^2}, \quad \varepsilon_{2,2} = 2\nu \left\langle \frac{\partial v'}{\partial x_k} \frac{\partial v'}{\partial x_k} \right\rangle \end{aligned} \tag{A.5}$$

For $\langle w'w' \rangle$:

$$\begin{aligned} &P_{3,3} = -2\langle v'w' \rangle \frac{\partial \langle w \rangle}{\partial y}, \quad T_{3,3} = -\frac{\partial \langle w'w'v' \rangle}{\partial y}, \quad \phi_{3,3} = 0 \\ &\pi_{3,3} = 2\left\langle p' \frac{\partial w'}{\partial z} \right\rangle, \quad D_{3,3} = \nu \frac{\partial^2 \langle w'w' \rangle}{\partial y^2}, \quad \varepsilon_{3,3} = 2\nu \left\langle \frac{\partial w'}{\partial x_k} \frac{\partial w'}{\partial x_k} \right\rangle \end{aligned} \tag{A.6}$$

For $\langle u'v' \rangle$:

$$\begin{aligned} &P_{1,2} = -\langle v'v' \rangle \frac{\partial \langle u \rangle}{\partial y}, \quad T_{1,2} = -\frac{\partial \langle u'v'v' \rangle}{\partial y}, \quad \phi_{1,2} = -\frac{\partial \langle p'u' \rangle}{\partial y} \\ &\pi_{1,2} = \left\langle p' \left( \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right) \right\rangle, \quad D_{1,2} = \nu \frac{\partial^2 \langle u'v' \rangle}{\partial y^2}, \quad \varepsilon_{1,2} = 2\nu \left\langle \frac{\partial u'}{\partial x_k} \frac{\partial v'}{\partial x_k} \right\rangle \end{aligned} \tag{A.7}$$