Tutorial for the simulation of wall-bounded turbulent channel flow

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1. Introduction

This tutorial illustrates the step by step usage of Fourth-order **Channel3d** for wall-bounded turbulent channel flow at $Re_\tau \approx 180$. Here, periodic boundary conditions are used in streamwise (x) and spanwise (z) directions, while no-slip conditions are adopted for top and bottom walls in normal direction (y). The computational sketch is shown is Fig.1.

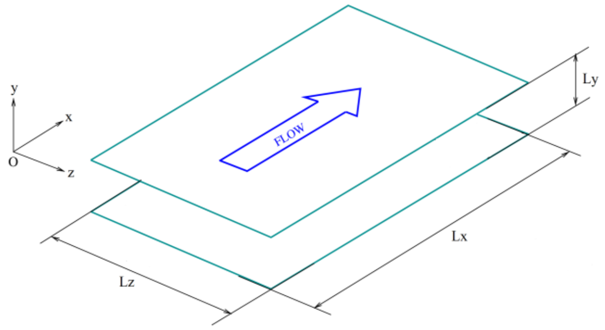

Fig.1. Sketch of computational domain for wall-bounded turbulent flows

The simulation has been initiated with a laminar parabolic Poiseuille velocity profile, plus a divergence-free large-amplitude sinusoidal perturbation:

$$u_0 = r_d u_b \bar{y} \exp\left(-\frac{9}{2}\bar{y}^2\right)\cos\left(\frac{\pi z}{100h}\mathrm{Re}_\tau\right) + 3u_b\left(\bar{y} - \frac{1}{2}\bar{y}^2\right) \tag{1a}$$

$$w_0 = r_d u_b \bar{y} \exp\left(-\frac{9}{2}\bar{y}^2\right)\sin\left(\frac{\pi x}{250h}\mathrm{Re}_\tau\right) \tag{1b}$$

$$v_0 = 0 \tag{1c}$$

where $\bar{y}$ is the dimensionless y coordinate defined as $\bar{y} = \min(y, 2h - y)/h$, and h denotes half channel height. r_d is a random number, ranging from 0.9 to 1.1, to further disturb the initial flow. The flow is driven by a uniform pressure gradient, which varies in time to maintain a constant mass flow rate. After several time unit, the laminar profiles transit to turbulent ones, and the pressure gradient begins to fluctuate near an almost constant value.

2. Preparation

The friction Reynolds number is defined as $Re_\tau = u_\tau h/\nu$, where ν is the fluid kinematic viscosity, and u_τ is friction velocity: $u_\tau = \sqrt{(\nu \partial \bar{u}/\partial y)_{wall}} = \sqrt{(-dp/dx)h}$. Re_b denotes the bulk Reynolds number: $Re_b = u_b h/\nu$, where u_b is the bulk velocity. For wall-bounded turbulence, Re_b and Re_τ have the following empirical relationship:

$$\mathrm{Re}_\tau = 0.1538\,\mathrm{Re}_b^{0.8877} \tag{2}$$

Eq.(2) is regressed from the data provided by Lee & Moser [1], and the regression result is also shown in Fig.2. If we want to simulate the case $Re_\tau \approx 180$, we can use Eq.(2) to get an approximate value for Re_b, i.e. $Re_b \approx 2860$. If we further set $u_b = 2/3$ and $\mathrm{h} = 1$, we can get the corresponding viscosity: $\nu = (u_b h)/\mathrm{Re}_b = 2.3310 \times 10^{-4}$.

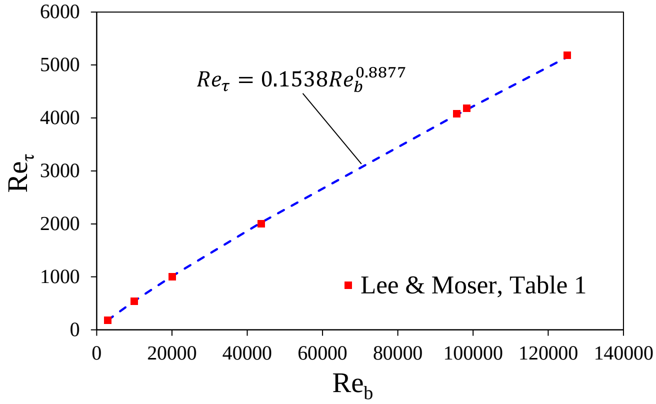

Fig.2. Relationship between bulk and friction Reynolds number

3. Parameter setting

The computational domain sizes are set to 4πh, 2h, 2πh in x-, y- and z-directions respectively. Details about the flow parameters and grid mesh are listed in Table 1. The input parameter text can be found in file ./doc/TurbulentChannel_4th_direct.prm. The illustrations for every input parameters can be found in file ./doc/channel4th_prm.md.

Table 1

List of parameters for turbulent channel flow cases. Here N_x, N_y, and N_z are the number of grid points in streamwise, wall-normal and spanwise directions, respectively. Δx^+ and Δz^+ denote grid spacings in wall-parallel directions. Δy_w^+ and Δy_c^+ represent the grid spacing at the wall and the centerline in wall-normal direction respectively. Tu_τ/h is the time averaging window without transition.

Flow Case	Re_τ	Re_b	N_x	N_y	N_z	Δx^+	Δy_w^+	Δy_c^+	Δz^+	Tu_τ/h
Cha180	182.3	2860	384	192	256	5.97	0.0244	2.98	4.48	20.40

4. Running the code

If the executable file ./channel4th and ./Tool/interpolateField/interpolateField has not been compiled, you should compile it firstly, following the guidance written in ./README.md, Installation part. The simulation results and restart files will be stored in the folder ./CFD/Results/ and ./CFD/Restart, respectively.

Now, you can run this test case directly by typing:

```
chmod a+x ./channel4th
mpirun –n 8 ./channel4th ./Input/TurbulentChannel_4th_direct.prm
```

Alternatively, we can run the case on a coarse grid firstly, and then interpolate the resulting flow field to the fine mesh. If so, we can do as follows.

4.1 Running on the coarse mesh

Firstly, we run on a coarse grid where $N_x \times N_y \times N_z = 180 \times 132 \times 120$. Here we choose an arbitrary coarse mesh configuration. The evolution of Re_τ for coarse mesh is shown in Fig.3.

```
mpirun –n 8 ./channel4th ./Input/TurbulentChannel_4th_1.prm
```

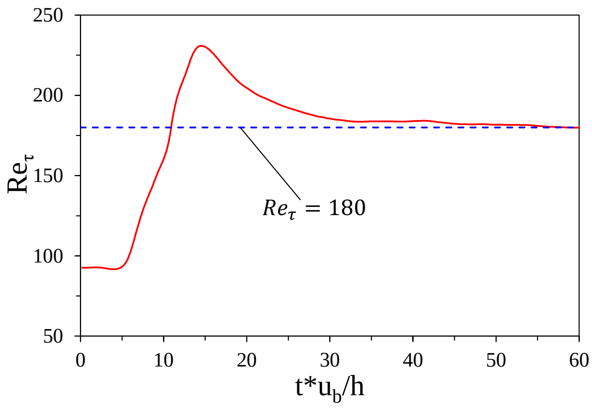

Fig.3. Time evolution of the friction Reynolds number Re_τ for coarse mesh

4.2 Interpolating flow field from coarse mesh to fine mesh

Now, we need to interpolate flow field from coarse mesh to fine mesh, using the executable file ./Tool/interpolateField/interpolateField:

```
cd ./Tool/interpolateField/
chmod a+x interpolateField
mpirun –n 8 interpolateField interp1.prm
cd ../../
```

After that, New restart file (./CFD/RestartForCha180_2_0000003000) will be generated.

4.3 Running on the fine mesh

Lastly, we run on the fine mesh where $N_x \times N_y \times N_z = 384 \times 192 \times 256$. The evolution of Re_τ for fine mesh is shown in Fig.4.

```
mpirun –n 8 ./channel4th ./Input/TurbulentChannel_4th_2.prm
```

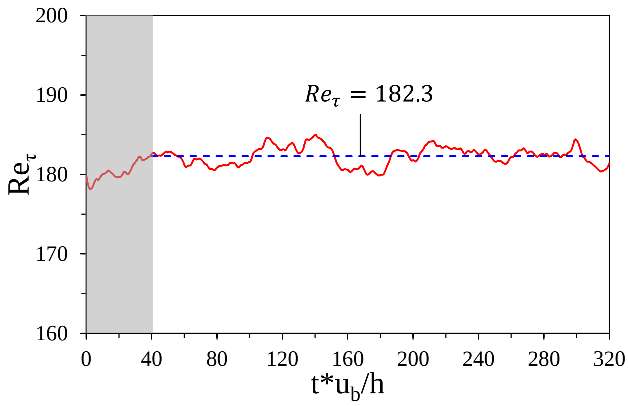

Fig.4. Time evolution of the friction Reynolds number Re_τ for fine mesh. Gray area indicates the adjustment stage from coarse mesh to fine mesh, and the corresponding time interval will be excluded from the statistic calculation.

5. Visualization

One can use Paraview for visualization. An interface file ended with the suffix .xmf is included in the folder ./CFD/Results/ (./CFD/Results/VisuForCha180_2_.xmf here), and this file can be read by Paraview directly.

6. Post-processing

Post-processing might be the most interesting part for wall-bounded turbulence. Here I will illustrate how to get the desired statistic data using **Channel3d**.

6.1 Data storage

The canonical statistic data include mean velocity profile, turbulent intensity, Reynolds stress, vorticity correlation, skewness, flatness, turbulent budget, energy spectra and so on.

If *a*, *b*, *c*, and *d* denote four variables. The second-order correlation can be defined as:

$$\langle a'b' \rangle = \langle [a - \langle a \rangle][b - \langle b \rangle] \rangle \tag{3}$$

While Eq.(3) can not be used directly, due to the fact that the exact $\langle a \rangle$ and $\langle b \rangle$ are not available until end of the simulation. So alternately, we can use the following expression instead:

$$\langle a'b' \rangle = \langle ab \rangle - \langle a \rangle \langle b \rangle \tag{4}$$

Similarly, the third-order and fourth-order correlations read:

$$\langle a'b'c' \rangle = \langle abc \rangle - \langle a \rangle \langle bc \rangle - \langle b \rangle \langle ac \rangle - \langle c \rangle \langle ab \rangle + 2 \langle a \rangle \langle b \rangle \langle c \rangle \tag{5}$$

$$\begin{aligned} \langle a'b'c'd' \rangle = &\langle abcd \rangle - 3 \langle a \rangle \langle b \rangle \langle c \rangle \langle d \rangle \\ &+ \langle a \rangle \langle b \rangle \langle cd \rangle + \langle b \rangle \langle c \rangle \langle da \rangle + \langle c \rangle \langle d \rangle \langle ab \rangle \\ &+ \langle d \rangle \langle a \rangle \langle bc \rangle + \langle a \rangle \langle c \rangle \langle bd \rangle + \langle b \rangle \langle d \rangle \langle ac \rangle \\ &- \langle a \rangle \langle bcd \rangle - \langle b \rangle \langle cda \rangle - \langle c \rangle \langle dab \rangle - \langle d \rangle \langle abc \rangle \end{aligned} \tag{6}$$

For instance, if we want to determine pressure strain term $\pi_{1,2} = \langle p' (\partial u'/\partial y + \partial v'/\partial x) \rangle$, we can further expand it as follows:

$$\pi_{1,2} = \left\langle p \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right\rangle + \langle p \rangle \left[\frac{\partial \langle u \rangle}{\partial y} + \frac{\partial \langle v \rangle}{\partial x} \right] = \left\langle p \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right\rangle + \langle p \rangle \frac{\partial \langle u \rangle}{\partial y} \tag{7}$$

So we need to calculate and store $\langle p(\partial u/\partial y + \partial v/\partial x) \rangle$, $\langle p \rangle$, and $\langle u \rangle$ in advance. Table 2 lists the raw averaged date contained in file ./CFD/Results/stats*. It is also worth nothing that due to the use of fully staggered grids, so averaged variables (e.g. <u>) are placed in the y-center layer, while others (e.g. <v>) are stored in the y-node layer. The definition of turbulent budget terms are listed in Appendix A.

The raw averaged 1D energy spectra data are also stored in the file ./CFD/Results/Spec*.

6.2 Statistic results calculation

The final statistic results can be calculated by the post-processing Matlab code, ./Tool/readStat_CH.m. If Matlab is not available, one can use Octave instead:

```
cd ./Tool/
octave readStat_CH.m
cd ../
```

After running this Matlab code, the final statistic results will be generated in the folder ./StatOut/.

Table 2

List of the raw averaged date contained in file ./CFD/Results/stats*. Here 'location' column indicates where the variable is placed. 'C' means 'Center', and 'N' stands for 'Node'.

id	variable name	location	id	variable name	location
1	<u>	C	19	<uuu>	C
2	<v>	N	20	<www>	C
3	<w>	C	21	<uuuu>	C
4	<p>	C	22	<vvvv>	N
5	<uu>	C	23	<wwww>	C
6	<vv>	N	24	<p $\partial u/\partial x$>	C
7	<ww>	C	25	<p $\partial v/\partial y$>	C
8	<pp>	C	26	<p $\partial w/\partial z$>	C
9	<uv>	C	27	<p$(\partial u/\partial y + \partial v/\partial x)$>	N
10	<vw>	C	28	<$u\nabla^2 u$>	C
11	<uw>	C	29	<$v\nabla^2 v$>	N
12	<up>	C	30	<$w\nabla^2 w$>	C
13	<vp>	C	31	<$\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}$>	C
14	<wp>	C			
15	<uuv>	C	32	<$(\partial u/\partial z)(\partial v/\partial z)$>	N
16	<vvv>	N	33	<$\omega_x\omega_x$>	N
17	<wwv>	C	34	<$\omega_y\omega_y$>	C
18	<uvv>	C	35	<$\omega_z\omega_z$>	N

7. Statistic results validation

This section compare the statistic results with Lee & Moser [1], and Vreman & Kuerten [2] to for validation.

7.1 Basic profiles

Fig.5 shows some turbulent statistic profiles, i.e. mean velocity, velocity fluctuations, Reynolds stress, vorticity fluctuations, mean pressure, pressure fluctuation.

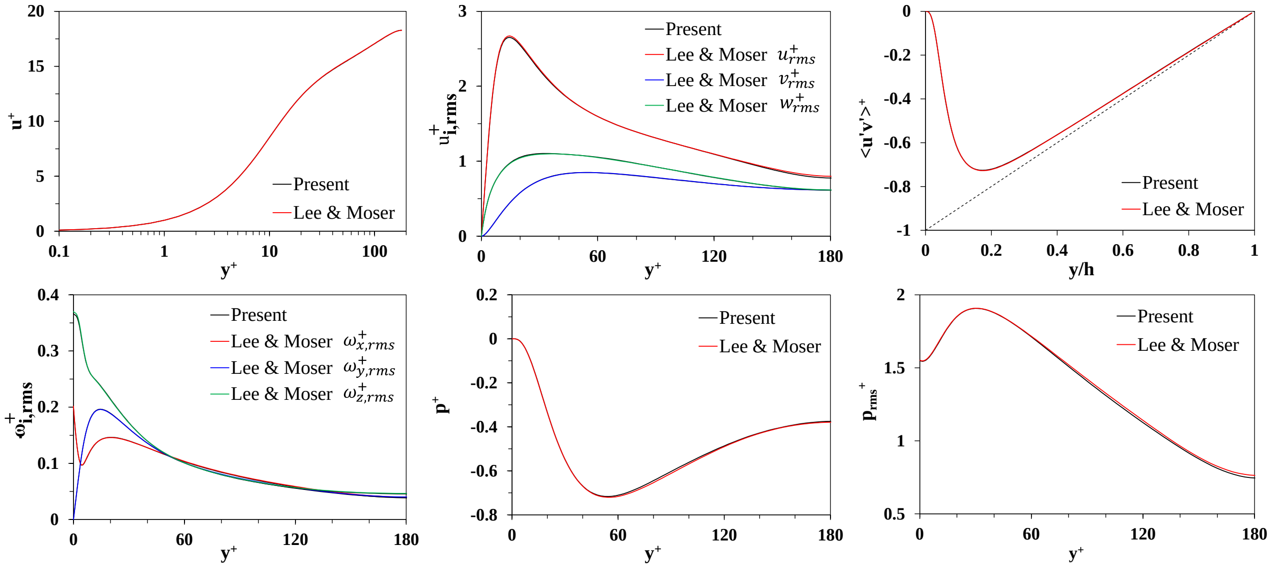

Fig.5. Basic turbulent statistic profiles

7.2 Budget profiles for TKE

Fig.6 shows the budget profiles for TKE.

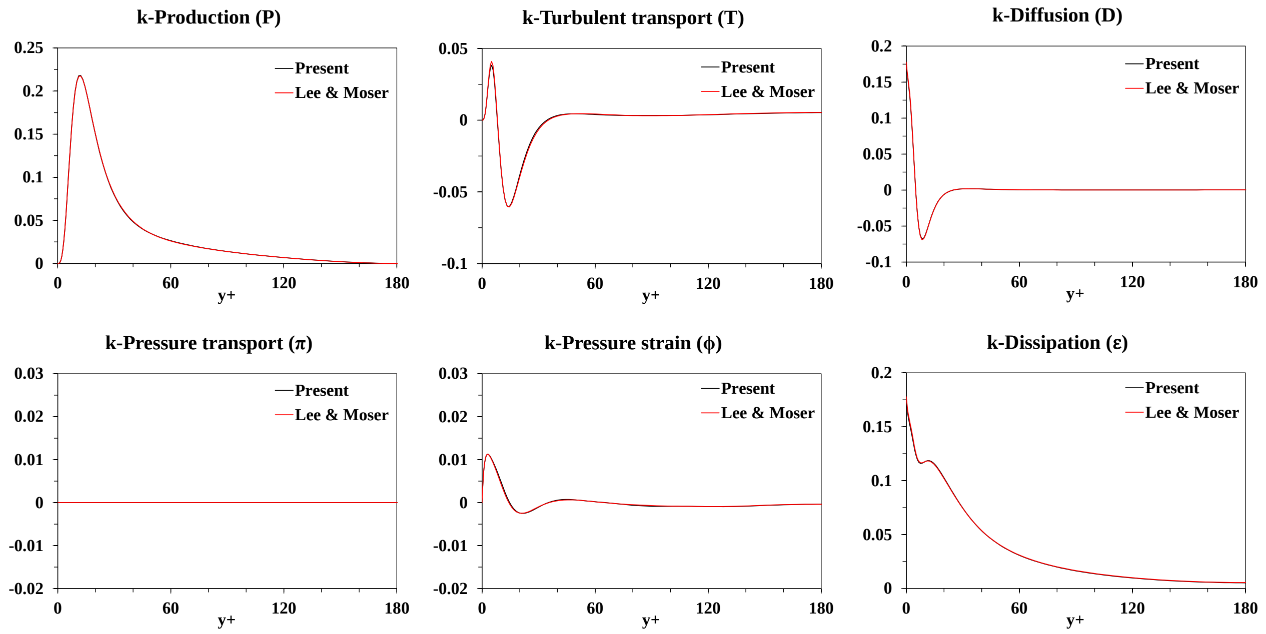

Fig.6. Budget profiles for TKE

7.3 Budget profiles for $< u'u' >$

Fig.7 shows the budget profiles for $< u'u' >$.

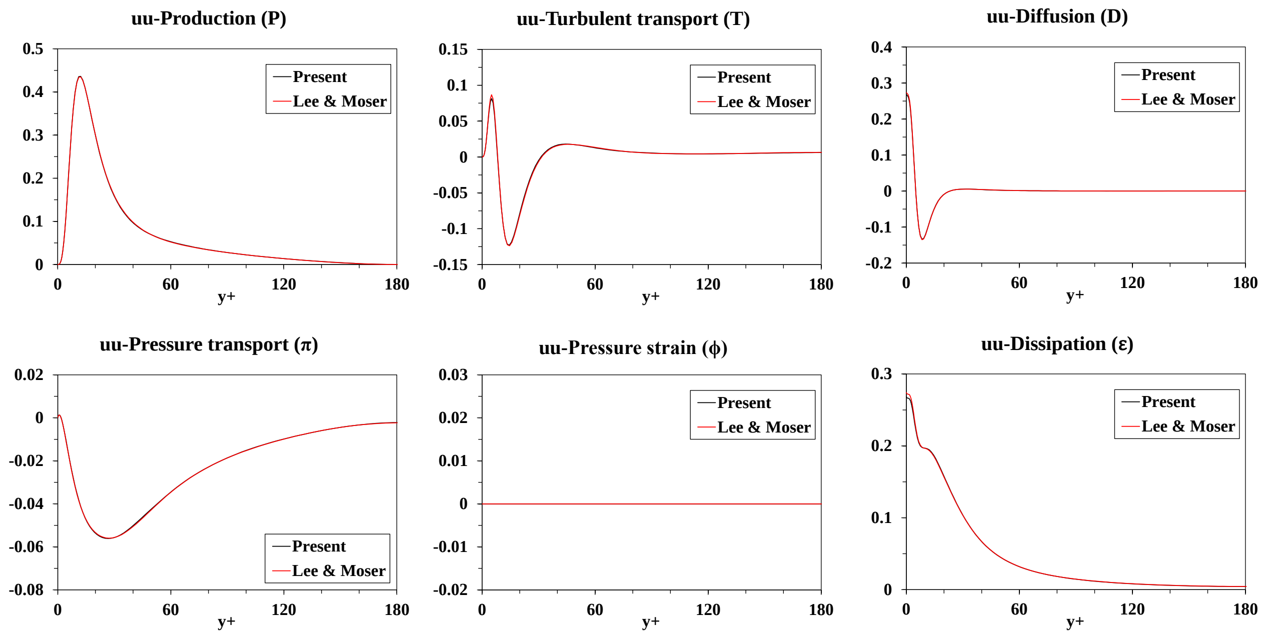

Fig.7. Budget profiles for $< u'u' >$

7.4 Budget profiles for $< v'v' >$

Fig.8 shows the budget profiles for $< v'v' >$.

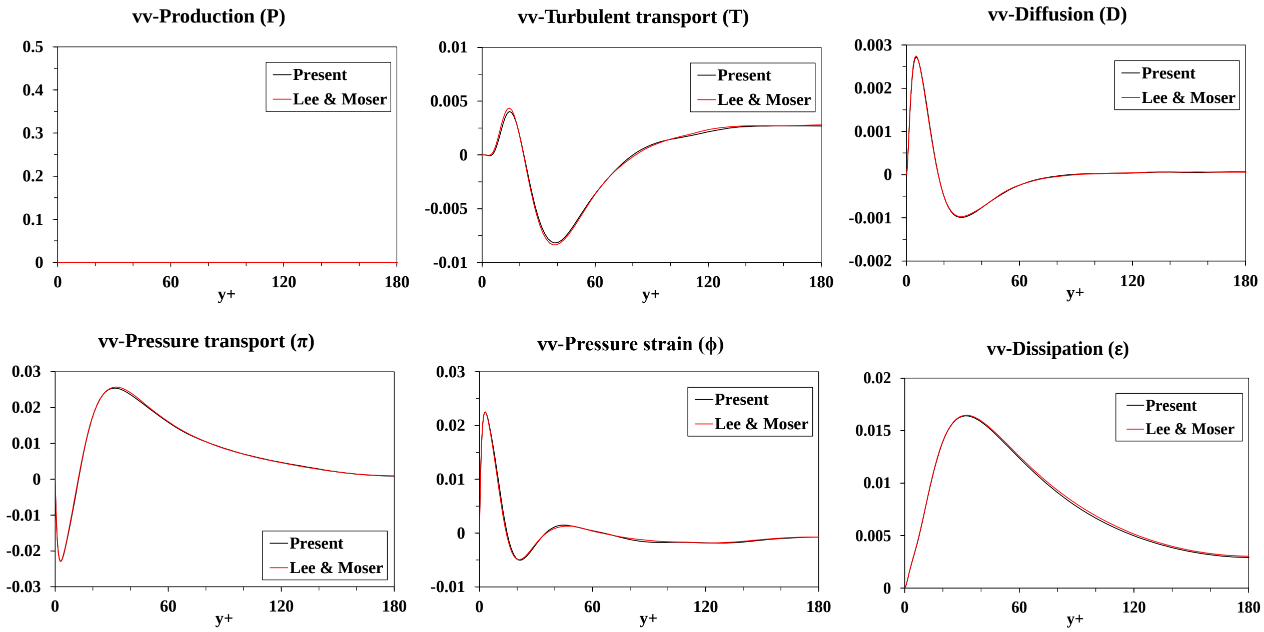

Fig.8. Budget profiles for $< v'v' >$

7.5 Budget profiles for $< w'w' >$

Fig.9 shows the budget profiles for $< w'w' >$.

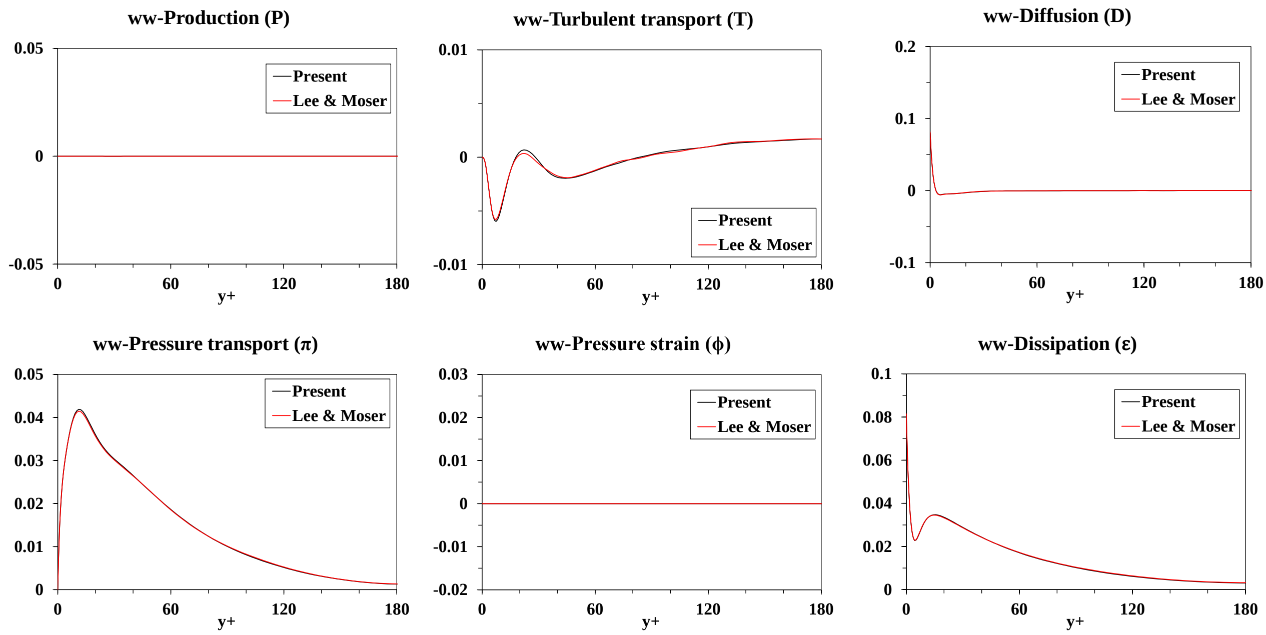

Fig.9. Budget profiles for $< w'w' >$

7.6 Budget profiles for $< u'v' >$

Fig.10 shows the budget profiles for $< u'v' >$.

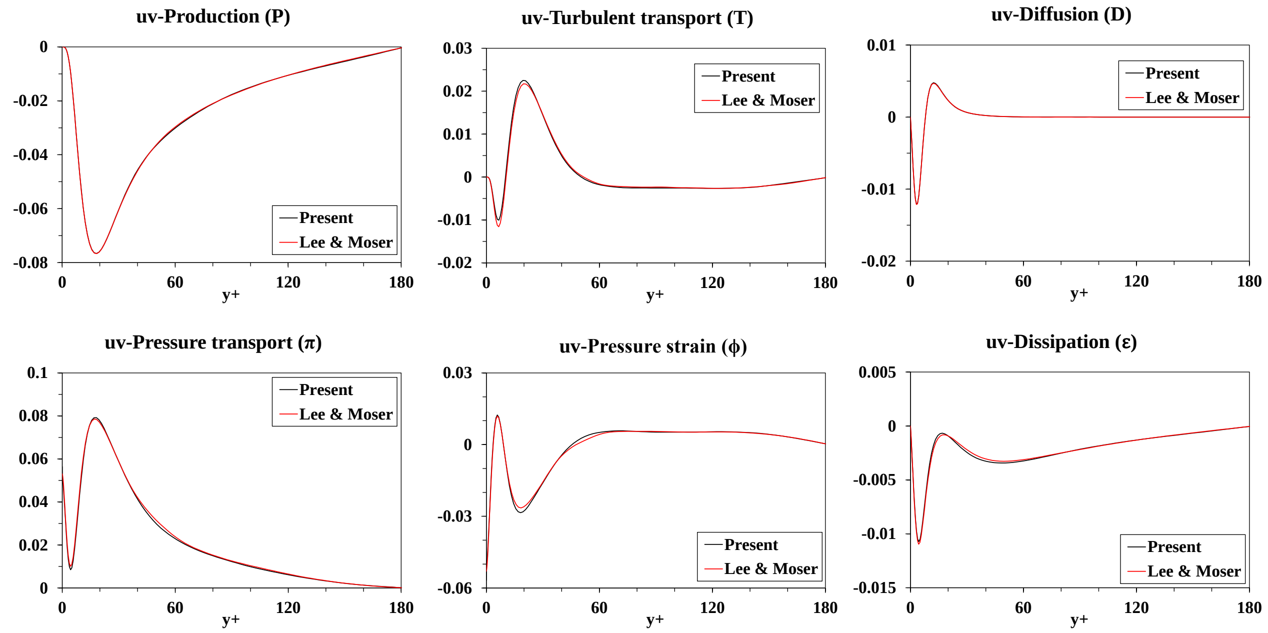

Fig.10. Budget profiles for $< u'v' >$

7.7 Energy spectra profiles

Fig.11 shows the energy spectra profiles at $y^+ \approx 36.5$.

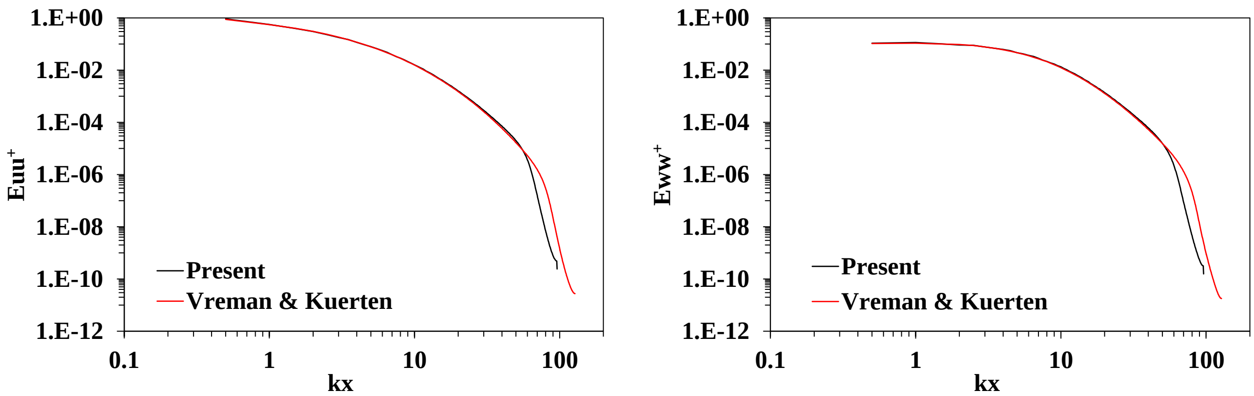

Fig.11. Energy spectra profiles at $y^+ \approx 36.5$

Reference:

[1] Lee M, Moser RD. Direct numerical simulation of turbulent channel flow up to Re τ ≈5200. J Fluid Mech.2015;774:395-415. doi:10.1017/jfm.2015.268.

[2] Vreman AW, Kuerten JGM. Comparison of direct numerical simulation databases of turbulent channel flow at Reτ = 180. Phys Fluids. 2014;26(1):1-21. doi:10.1063/1.4861064.

Appendix A. Definition of turbulent budget terms for wall-bounded turbulence

The general Reynolds stress transport equation reads:

$$\frac{\partial\left\langle u_i'u_j'\right\rangle}{\partial t}+\left\langle u_k\right\rangle\frac{\partial\left\langle u_i'u_j'\right\rangle}{\partial x_k}=P_{i,j}+T_{i,j}+\phi_{i,j}+\pi_{i,j}+D_{i,j}-\varepsilon_{i,j} \tag{A.1}$$

where $P_{i,j}$, $T_{i,j}$, $\phi_{i,j}$, $\pi_{i,j}$, $D_{i,j}$, and $\varepsilon_{i,j}$ stand for production term, turbulent transport term, pressure transport term, pressure strain term, viscous transport (diffusion) term, and viscous dissipation term respectively:

$$P_{i,j}=-\left\langle u_i'u_k'\right\rangle\frac{\partial\left\langle u_j\right\rangle}{\partial x_k}-\left\langle u_j'u_k'\right\rangle\frac{\partial\left\langle u_i\right\rangle}{\partial x_k} \tag{A.2a}$$

$$T_{i,j}=-\frac{\partial\left\langle u_i'u_j'u_k'\right\rangle}{\partial x_k} \tag{A.2b}$$

$$\phi_{i,j}=-\frac{\partial\left\langle p'u_i'\right\rangle}{\partial x_j}-\frac{\partial\left\langle p'u_j'\right\rangle}{\partial x_i} \tag{A.2c}$$

$$\pi_{i,j}=\left\langle p'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)\right\rangle \tag{A.2d}$$

$$D_{i,j}=\nu\frac{\partial^2\left\langle u_i'u_j'\right\rangle}{\partial x_k\partial x_k} \tag{A.2e}$$

$$\varepsilon_{i,j}=2\nu\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right\rangle \tag{A.2f}$$

Specifically, for wall-bounded turbulent flow, where periodic boundary conditions are applied in streamwise and spanwise directions. So Eq.(A.2) can be further simplified as follows:

$$\begin{aligned}
&P_{i,j}=-\left\langle u_i'v'\right\rangle\frac{\partial\left\langle u_j\right\rangle}{\partial y}-\left\langle u_j'v'\right\rangle\frac{\partial\left\langle u_i\right\rangle}{\partial y}, && T_{i,j}=-\frac{\partial\left\langle u_i'u_j'v'\right\rangle}{\partial y}\\
&\phi_{i,j}=-\frac{\partial\left\langle p'u_i'\right\rangle}{\partial x_j}-\frac{\partial\left\langle p'u_j'\right\rangle}{\partial x_i}, && \pi_{i,j}=\left\langle p'\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)\right\rangle\\
&D_{i,j}=\nu\frac{\partial^2\left\langle u_i'u_j'\right\rangle}{\partial y^2}, && \varepsilon_{i,j}=2\nu\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right\rangle
\end{aligned} \tag{A.3}$$

And for $\left\langle u'u'\right\rangle$:

$$\begin{aligned}
&P_{1,1}=-2\left\langle u'v'\right\rangle\frac{\partial\left\langle u\right\rangle}{\partial y},\quad T_{1,1}=-\frac{\partial\left\langle u'u'v'\right\rangle}{\partial y},\phi_{1,1}=0\\
&\pi_{1,1}=2\left\langle p'\frac{\partial u'}{\partial x}\right\rangle,\quad D_{1,1}=\nu\frac{\partial^2\left\langle u'u'\right\rangle}{\partial y^2},\quad \varepsilon_{1,1}=2\nu\left\langle\frac{\partial u'}{\partial x_k}\frac{\partial u'}{\partial x_k}\right\rangle
\end{aligned} \tag{A.4}$$

For $\langle v'v' \rangle$:

$$
\begin{aligned}
&P_{2,2} = -2\langle v'v' \rangle \frac{\partial \langle v \rangle}{\partial y} = 0, \quad T_{2,2} = -\frac{\partial \langle v'v'v' \rangle}{\partial y}, \quad \phi_{2,2} = -2\frac{\partial \langle p'v' \rangle}{\partial y} \\
&\pi_{2,2} = 2\left\langle p' \frac{\partial v'}{\partial y} \right\rangle, \quad D_{2,2} = \nu \frac{\partial^2 \langle v'v' \rangle}{\partial y^2}, \quad \varepsilon_{2,2} = 2\nu \left\langle \frac{\partial v'}{\partial x_k} \frac{\partial v'}{\partial x_k} \right\rangle
\end{aligned} \tag{A.5}
$$

For $\langle w'w' \rangle$:

$$
\begin{aligned}
&P_{3,3} = -2\langle v'w' \rangle \frac{\partial \langle w \rangle}{\partial y}, \quad T_{3,3} = -\frac{\partial \langle w'w'v' \rangle}{\partial y}, \quad \phi_{3,3} = 0 \\
&\pi_{3,3} = 2\left\langle p' \frac{\partial w'}{\partial z} \right\rangle, \quad D_{3,3} = \nu \frac{\partial^2 \langle w'w' \rangle}{\partial y^2}, \quad \varepsilon_{3,3} = 2\nu \left\langle \frac{\partial w'}{\partial x_k} \frac{\partial w'}{\partial x_k} \right\rangle
\end{aligned} \tag{A.6}
$$

For $\langle u'v' \rangle$:

$$
\begin{aligned}
&P_{1,2} = -\langle v'v' \rangle \frac{\partial \langle u \rangle}{\partial y}, \quad T_{1,2} = -\frac{\partial \langle u'v'v' \rangle}{\partial y}, \quad \phi_{1,2} = -\frac{\partial \langle p'u' \rangle}{\partial y} \\
&\pi_{1,2} = \left\langle p' \left(\frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right) \right\rangle, \quad D_{1,2} = \nu \frac{\partial^2 \langle u'v' \rangle}{\partial y^2}, \quad \varepsilon_{1,2} = 2\nu \left\langle \frac{\partial u'}{\partial x_k} \frac{\partial v'}{\partial x_k} \right\rangle
\end{aligned} \tag{A.7}
$$