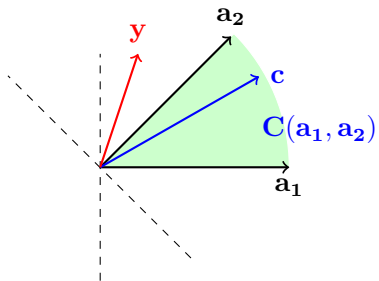
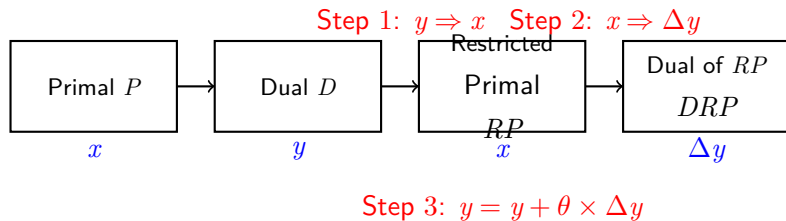


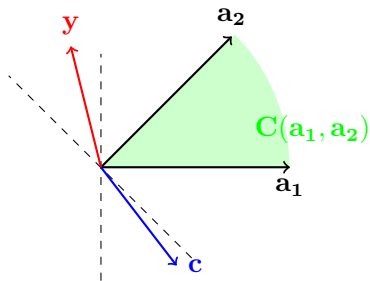
bbbb;

Farkas lemma

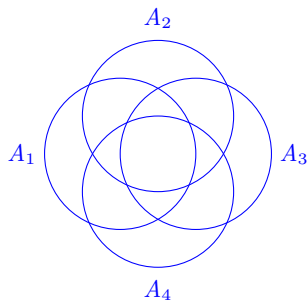




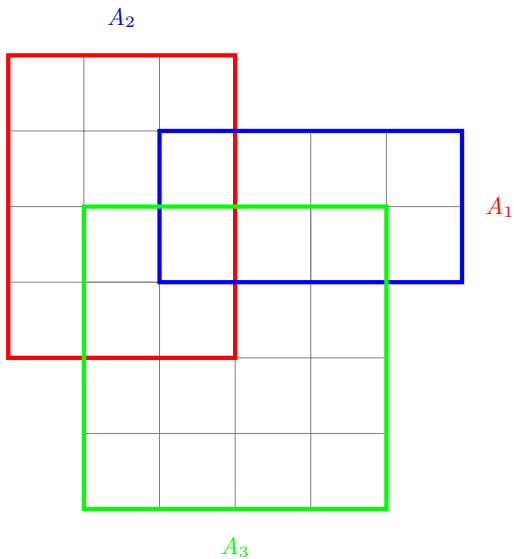
Farkas lemma



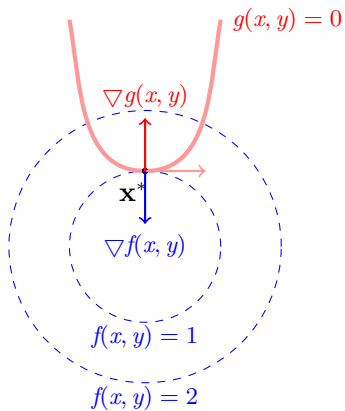
Max Coverage Problem



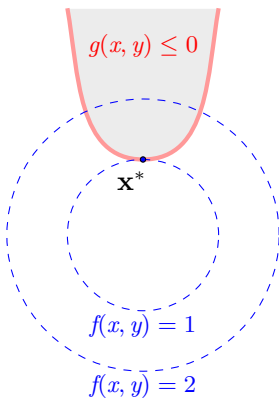
Max Coverage Problem2



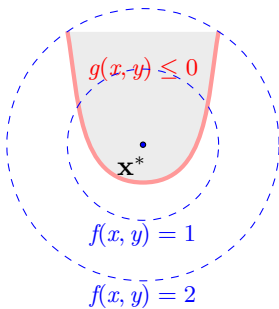
Lagrangian

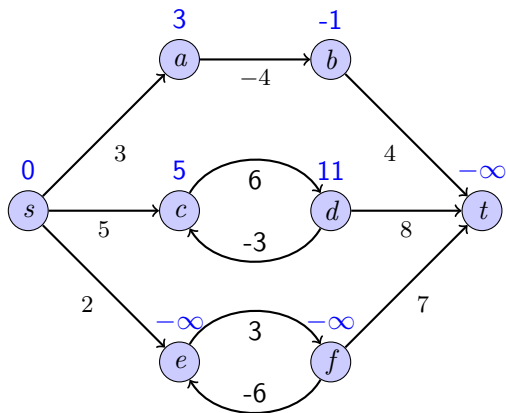


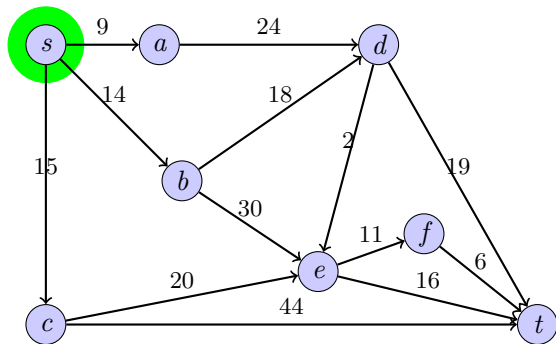
Lagrangian

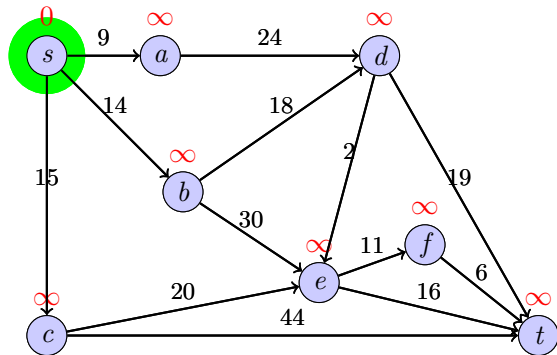


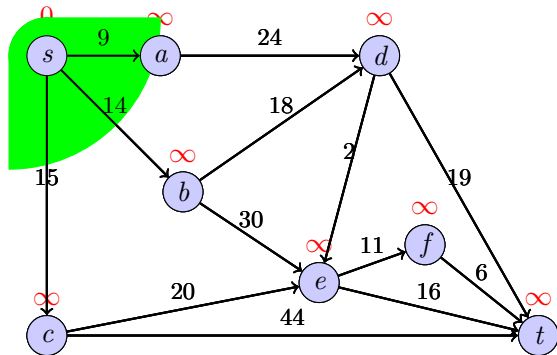
Lagrangian

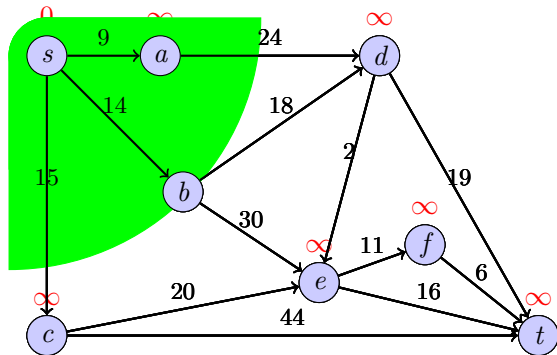


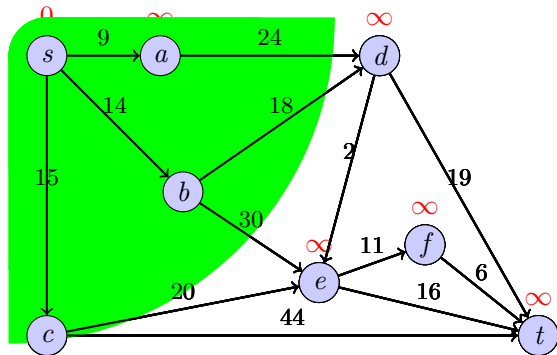


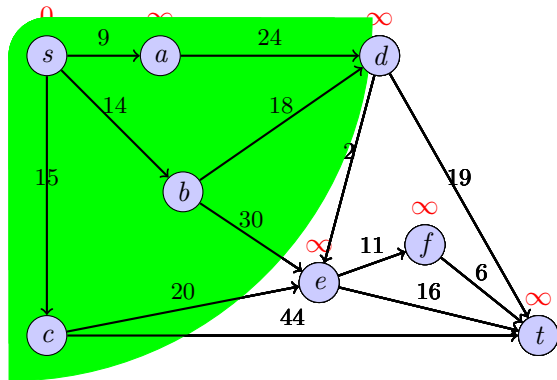


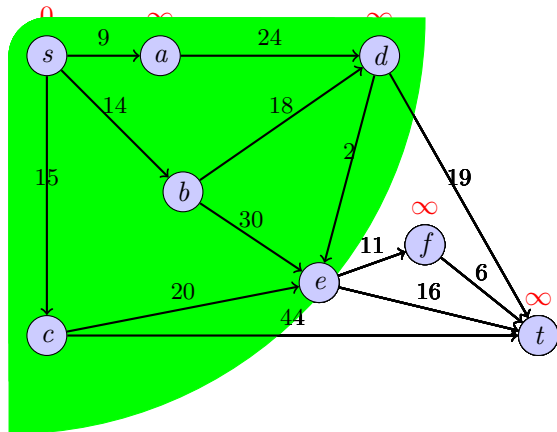


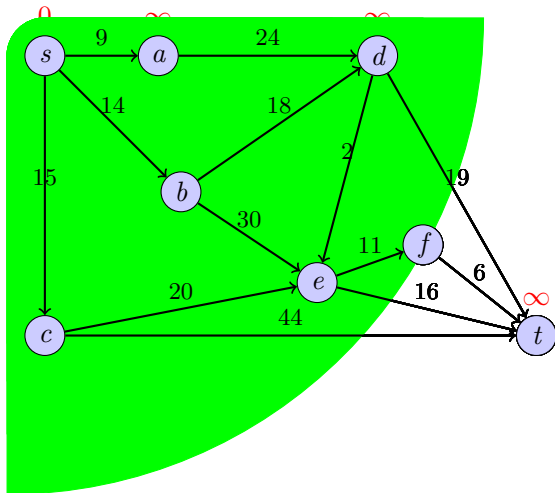


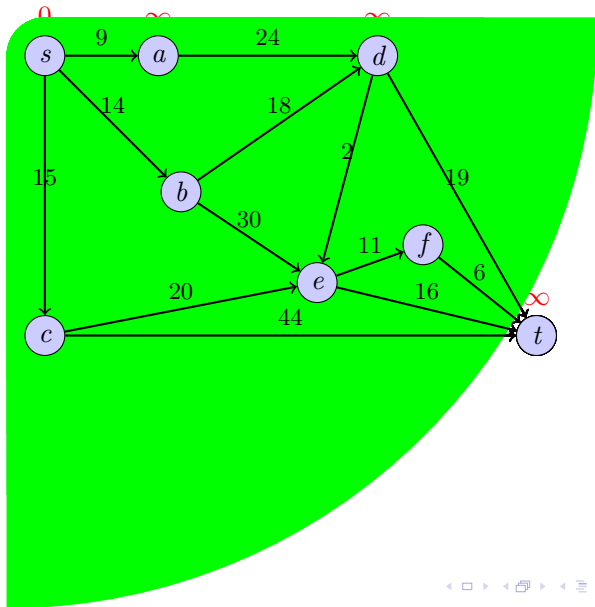


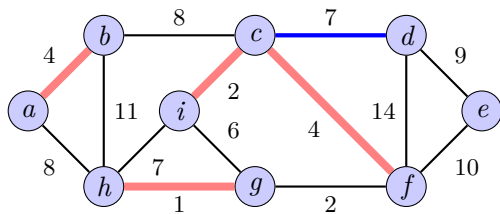




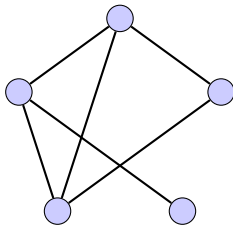


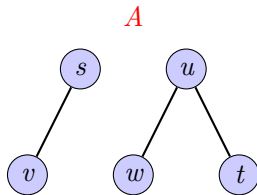
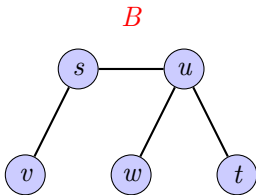
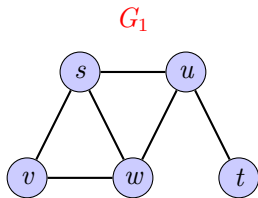


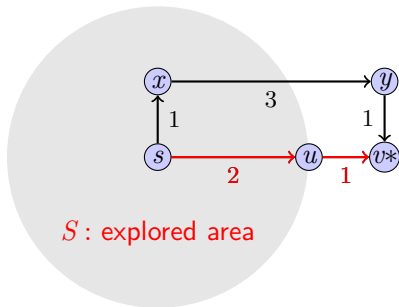


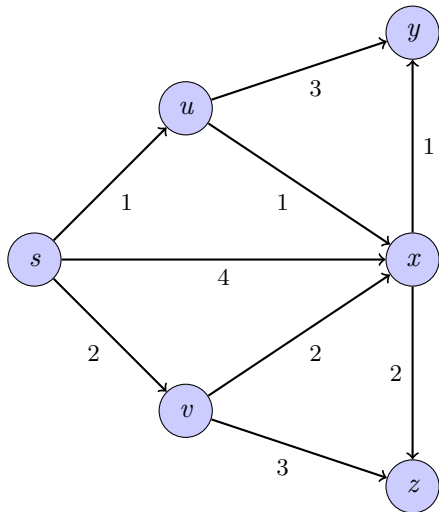


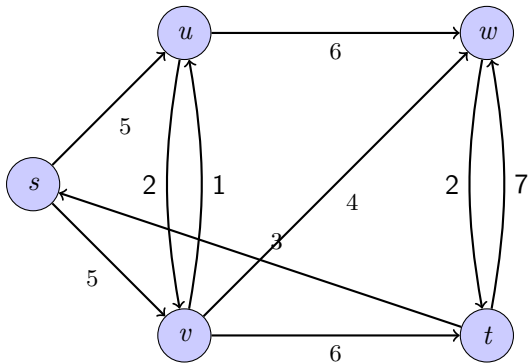
Pentagon











Lec6 Alignment matrix Suffix FULL

4	0	-4	-10	-12	-16	-18	-22	-26	-30	E
5	3	-1	-7	-9	-13	-15	-19	-23	-27	C
3	6	2	-4	-6	-10	-12	-16	-20	-24	N
-1	2	5	-1	-3	-7	-9	-13	-17	-21	E
-5	-2	1	4	0	-4	-6	-10	-14	-18	R
-9	-6	-3	0	3	-1	-3	-7	-11	-15	R
-13	-10	-7	-4	-1	2	0	-4	-8	-12	U
-15	-12	-9	-6	-3	0	3	-1	-5	-9	C
-19	-16	-13	-10	-7	-4	-1	2	-2	-6	C
-23	-20	-17	-14	-11	-8	-5	-2	1	-3	O
-27	-24	-21	-18	-15	-12	-9	-6	-3	0	'
										'
										T
E	C	N	A	R	R	U	C	O	'	S

Lec6 Alignment matrix Prefix FULL

S: ' ' O C U R R A N C E

T: ' ' O C U R R A N C E

0	-3	-6	-9	-12	-15	-18	-21	-24	-27
-3	1	-2	-5	-8	-11	-14	-17	-20	-23
-6	-2	2	-1	-4	-7	-10	-13	-16	-19
-9	-5	-1	1	-2	-5	-8	-11	-12	-15
-12	-8	-4	0	0	-3	-6	-9	-12	13
-15	-11	-7	-3	1	1	-2	-5	-8	-11
-18	-14	-10	-6	-2	2	-	-3	-6	-9
-21	-17	-13	-9	-5	-1	1	-1	-4	-5
-24	-20	-16	-12	-8	-4	-2	2	-1	-4
-27	-23	-19	-15	-11	-7	-5	-1	3	0
-30	-26	-22	-18	-14	-10	-8	-4	0	4

Lec7 Shortest Path example FULL

	k=0	1	2	3	4	5
S	0	0	0	0	0	0
U	—	1	1	1	1	1
V	—	2	2	2	2	2
X	—	4	2	2	2	2
Y	—	—	4	3	3	3
Z	—	—	5	4	4	4

Lec6 Alignment matrix Prefix

S:	'	'	O	C	U	R	R	A	N	C	E
T:	0	-3	-6	-9	-12	-15	-18	-21	-24	-27	
O	-3										
C	-6										
C	-9										
U	-12										
R	-15										
R	-18										
E	-21										
N	-24										
C	-27										
E	-30										

Lec6 Step1

OPT

1	2	3	4	
0	6			1
	0	24		2
		0	60	3
			0	4

SPLITTER

1	2	3	4	
	1			1
		2		2
			3	3
				4

Lec6 Step2

OPT

1	2	3	4	
0	6	18		1
	0	24	64	2
		0	60	3
			0	4

SPLITTER

1	2	3	4	
	1	2		1
		2	3	2
			3	3
				4

Lec6 Step3

OPT

1	2	3	4	
0	6	18	38	1
	0	24	64	2
		0	60	3
			0	4

SPLITTER

1	2	3	4	
	1	2	3	1
		2	3	2
			3	3
				4

$$\frac{n}{2}$$

S: OCUR RANCE

T: OCCUR RENCE

$$1 \leq q \leq n$$

Pair

S: OCURRANC E
T: OCCURRENC E

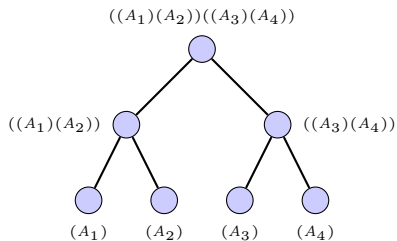
Insertion

S: OCURRANC E
T: OCCURRENC E -

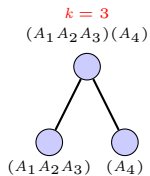
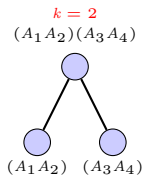
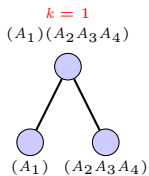
Deletion

S: OCURRANCE -
T: OCCURRENC E

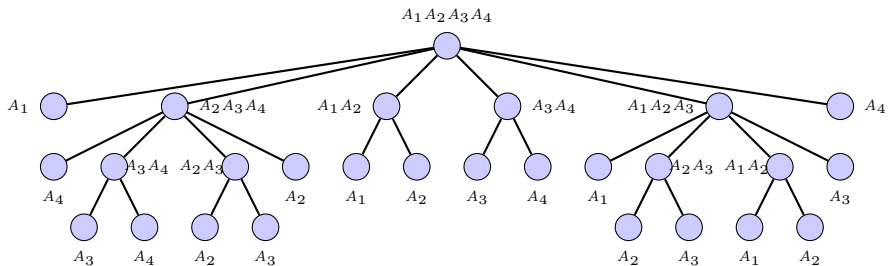
Lec6 DP 1



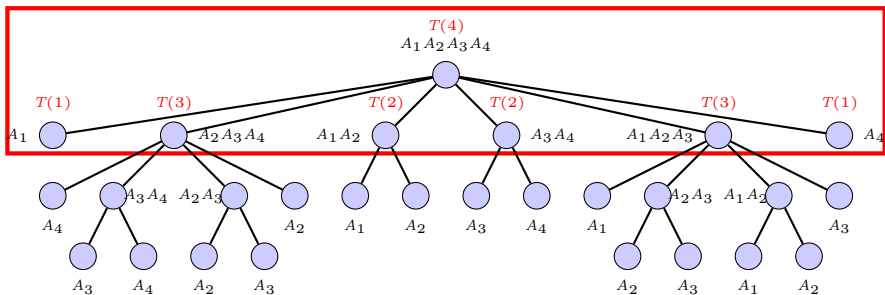
Lec6 DP 2

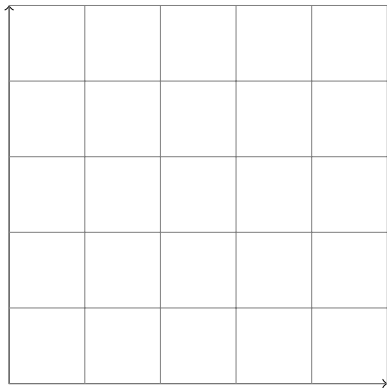


Lec6 DP 3

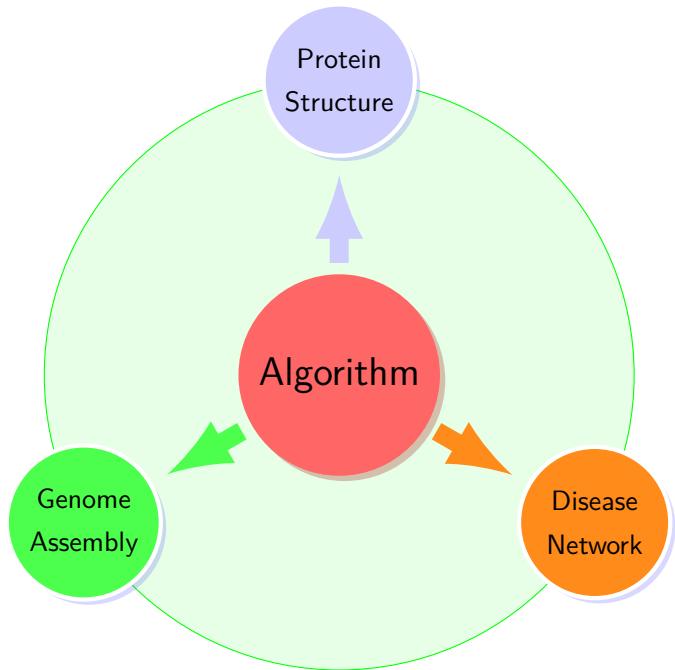


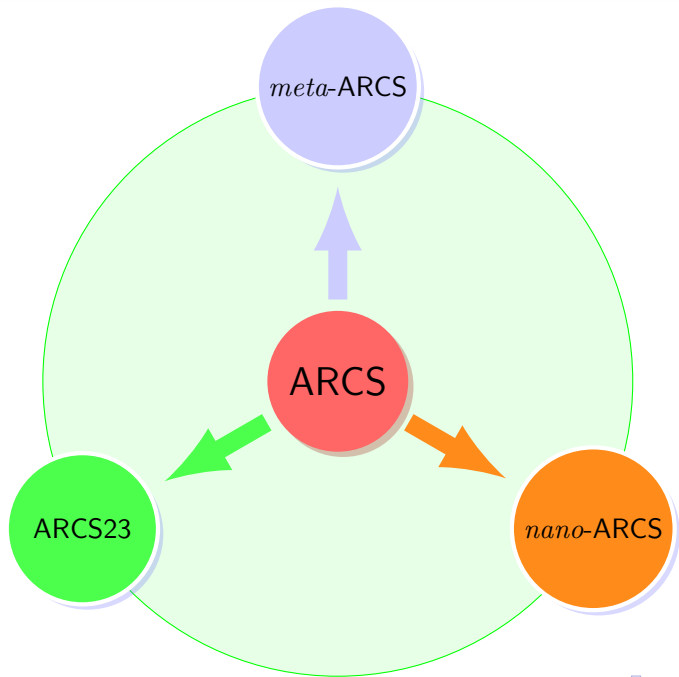
Lec6 DP4

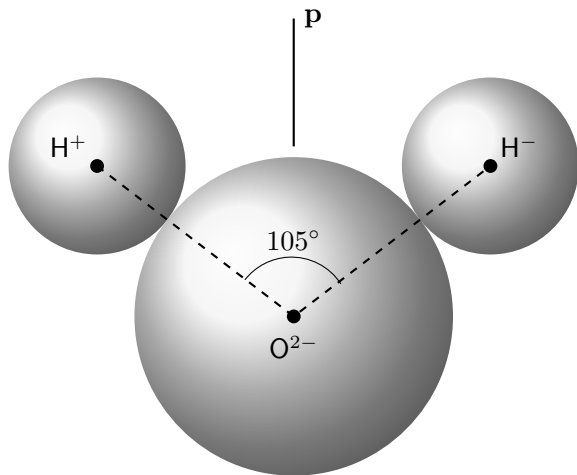


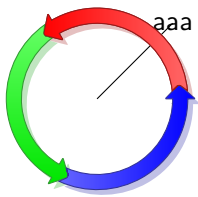


$$\begin{array}{ccccccc}
 & & 1 \times 2 & & 2 \times 3 & & 3 \times 4 & & 4 \times 5 \\
 A_1 = \begin{bmatrix} & 1 & & 2 & & \end{bmatrix} & A_2 = \begin{bmatrix} & 1 & & 2 & & 3 & & \end{bmatrix} & A_3 = \begin{bmatrix} & 1 & & 2 & & 3 & & 4 & & \end{bmatrix} & A_4 = \begin{bmatrix} & 1 & & 2 & & 3 & & 4 & & 5 & & \end{bmatrix}
 \end{array}$$

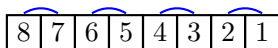




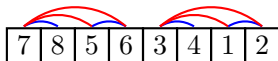




Lec5. Where did we save? Merge sort



MERGESORT step 1: 4 ops

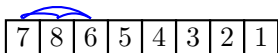
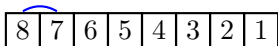


MERGESORT step 2: 4 ops, save: 4 ops

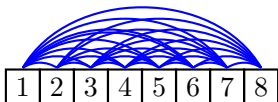


MERGESORT step 3: 4 ops, save: 12 ops

Lec5. Where did we save?

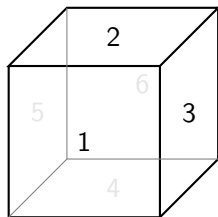


⋮

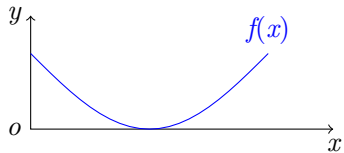
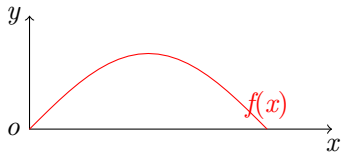


INSERTSORT: 28 ops

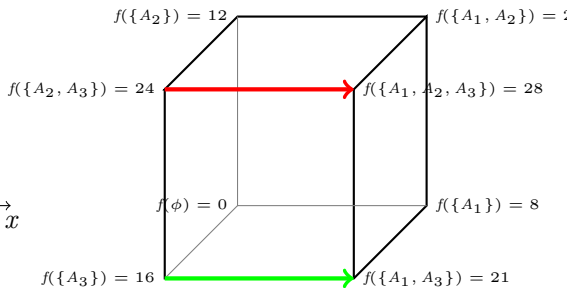
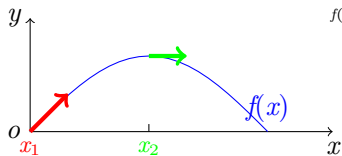
cube



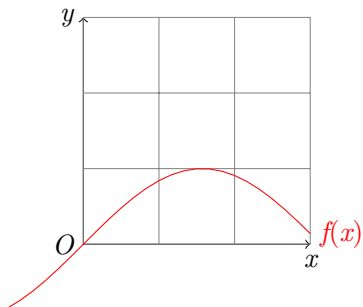
$f(x)$ convex



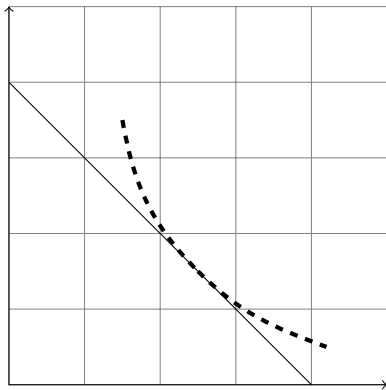
$f(x)$ convex



$$f(x)$$

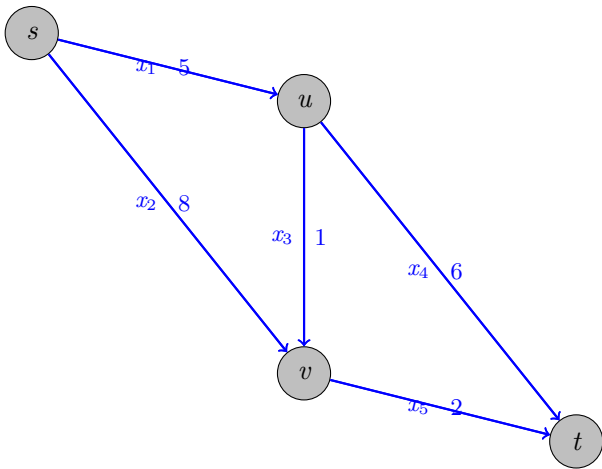


$f(x)$

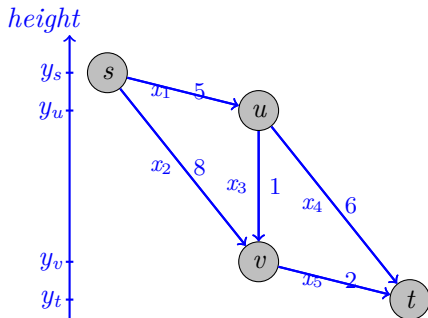


Lec5 Closest Pair n points



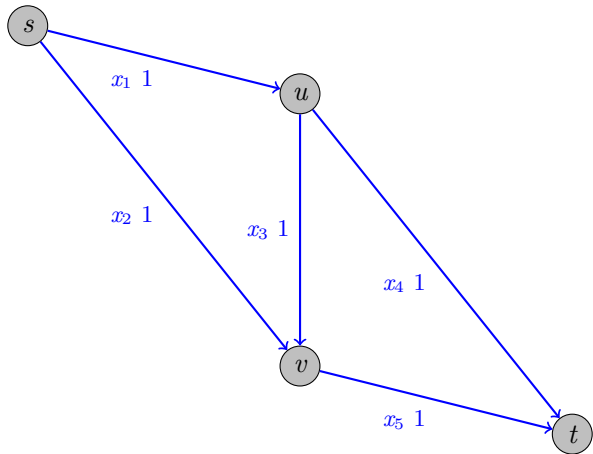


Shortest path

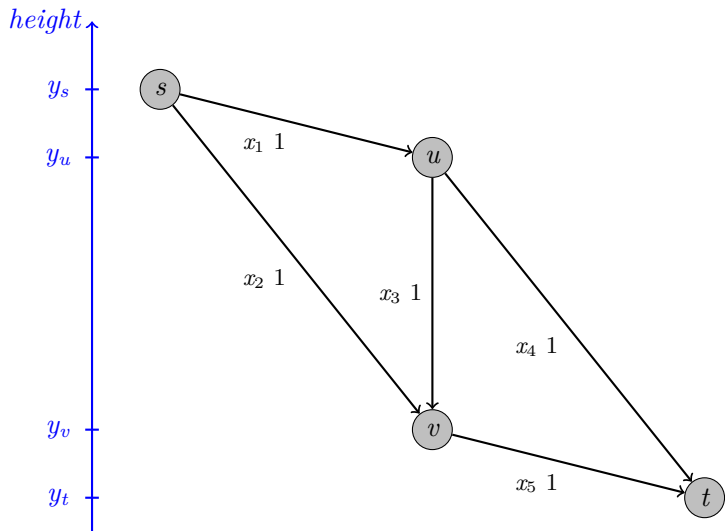


Dual

$$\begin{array}{cccccc}
 & & & & & x_n = 0 \\
 & & & & & \uparrow \\
 & & & & & \uparrow \\
 c_1 & c_2 & \dots & c_n & & \\
 \parallel & \parallel & & \vee & \max & \\
 y_1 a_{11} & y_1 a_{12} & \dots & y_1 a_{1n} & y_1 b_1 & \\
 + & + & & + & + & \\
 y_2 a_{21} & y_2 a_{22} & \dots & y_2 a_{2n} & y_2 b_2 & \\
 + & + & & + & + & \\
 \vdots & \vdots & \dots & \vdots & \vdots & \\
 + & + & & + & + & \\
 y_m a_{m1} & y_m a_{m2} & \dots & y_m a_{mn} & y_m b_m &
 \end{array}$$

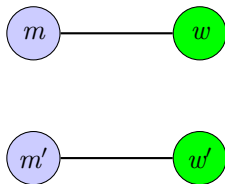


dual of shortest path

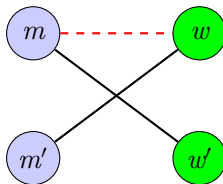


Stable 1

Match 1

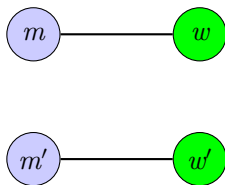


Match 2

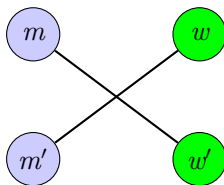


Stable 2

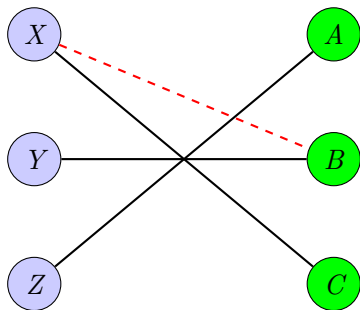
Match 1



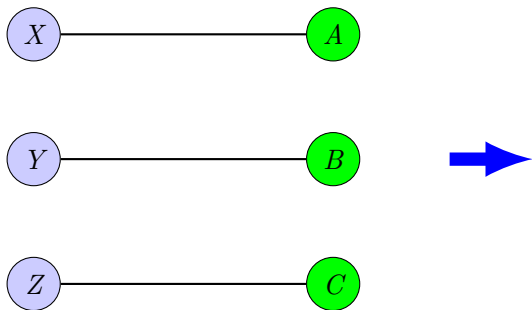
Match 2



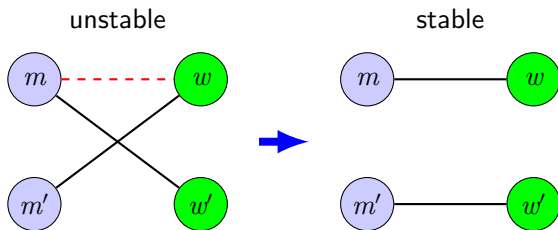
Stable 4



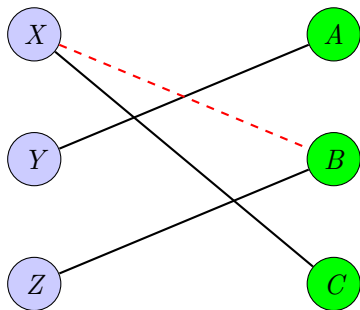
Stable 5



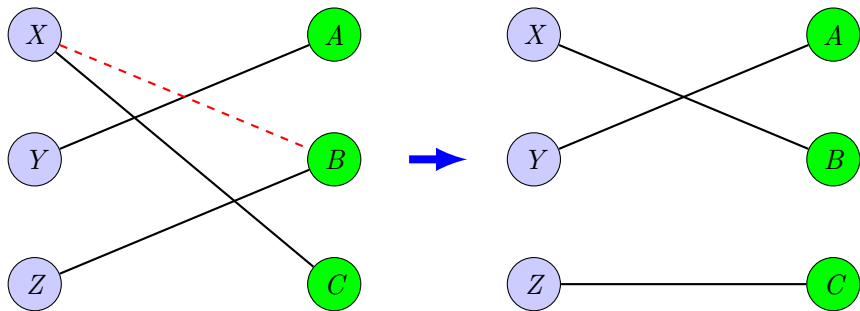
Stable 6



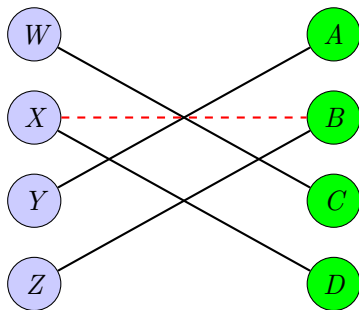
Stable 7



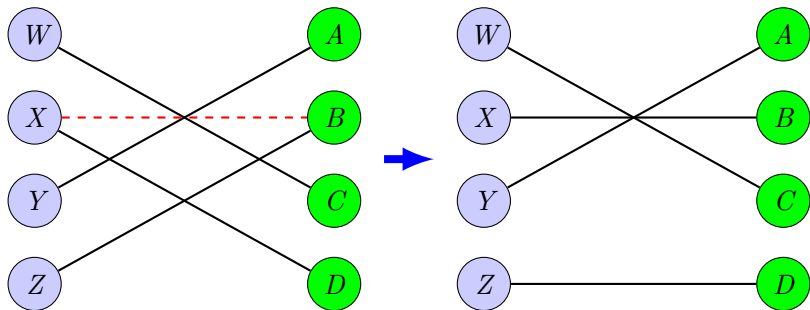
Stable 8



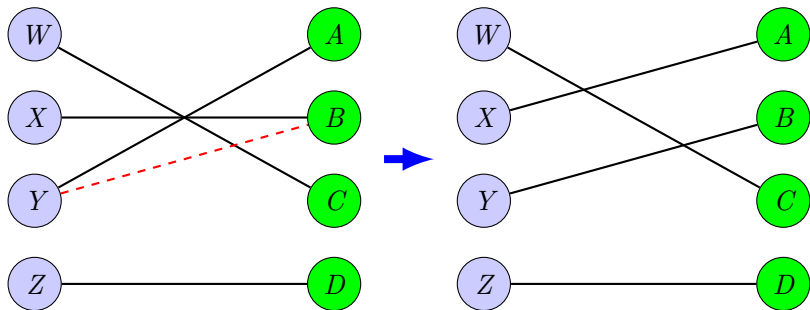
Stable 9



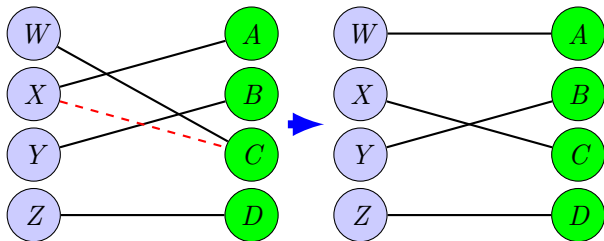
Stable 10



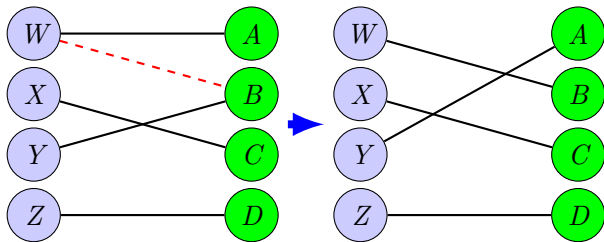
Stable 11



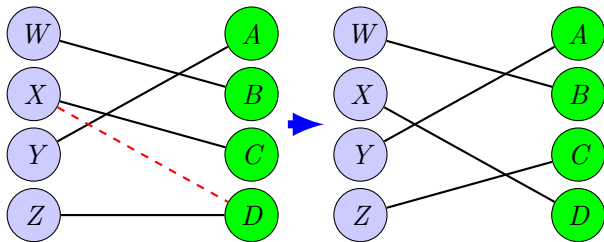
Stable 12



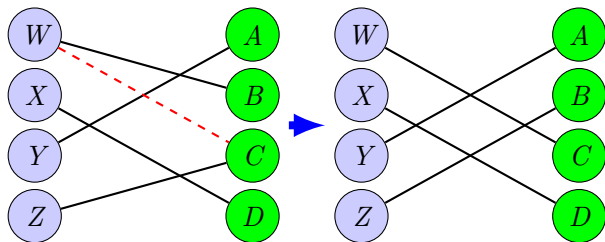
Stable 13



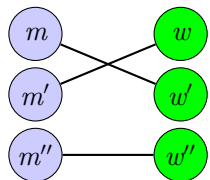
Stable 14



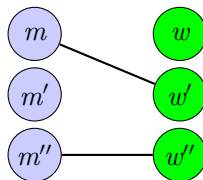
Stable 15



Stable Proof 1

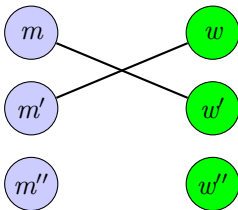


complete solution

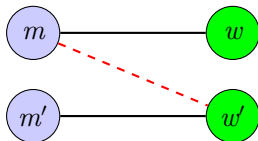


partial solution

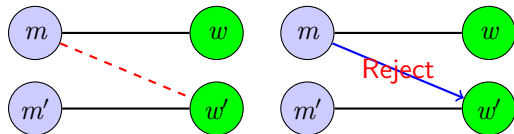
Stable Proof 2



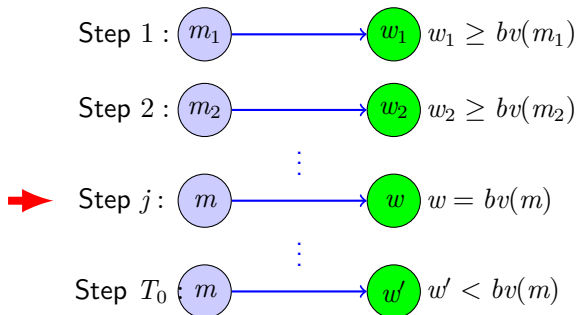
Stable Proof 3



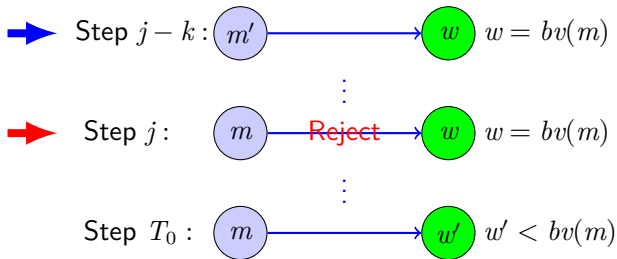
Stable Proof 4



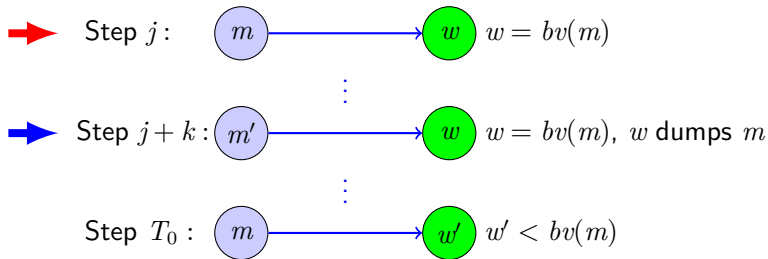
Stable Proof 5



Stable Proof 6

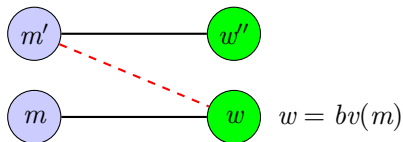


Stable Proof 7

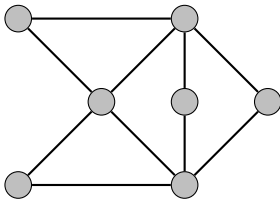


Stable Proof 8

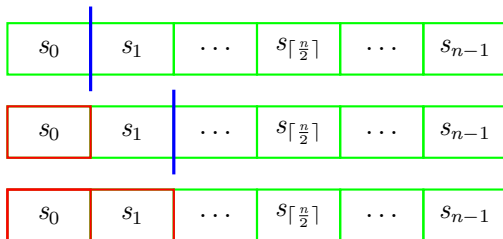
Stable match S'

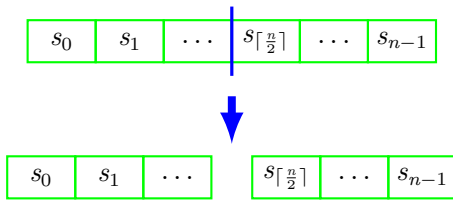


network example

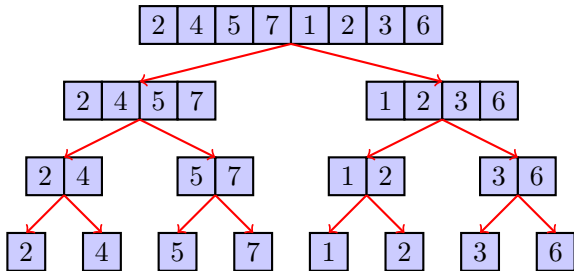


Lec5 1

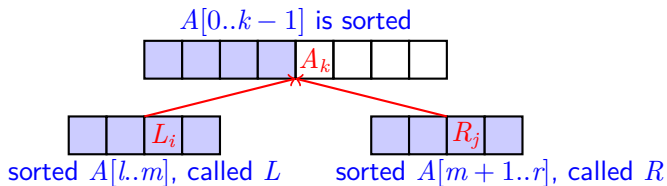




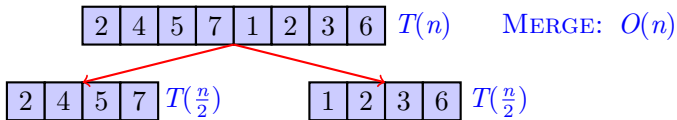
Lec5 how to divide?



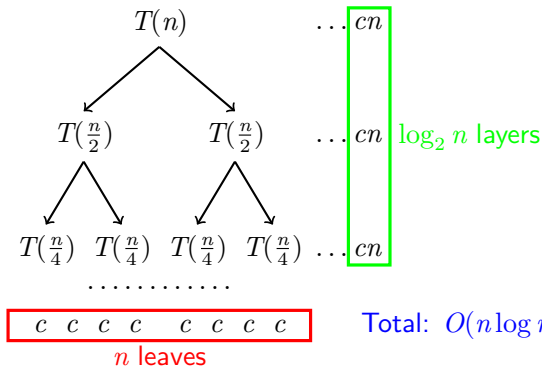
Lec5 how to combine?



Lec5 time complexity?



Lec5 tree



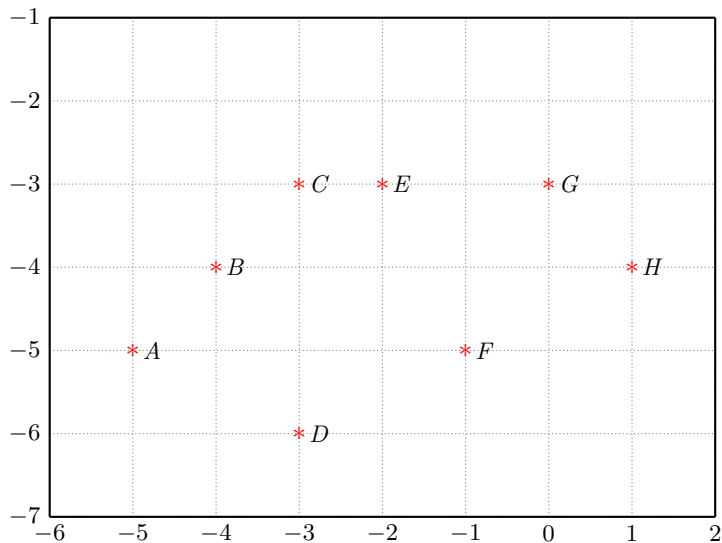
Lec5 Splitter

best splitter

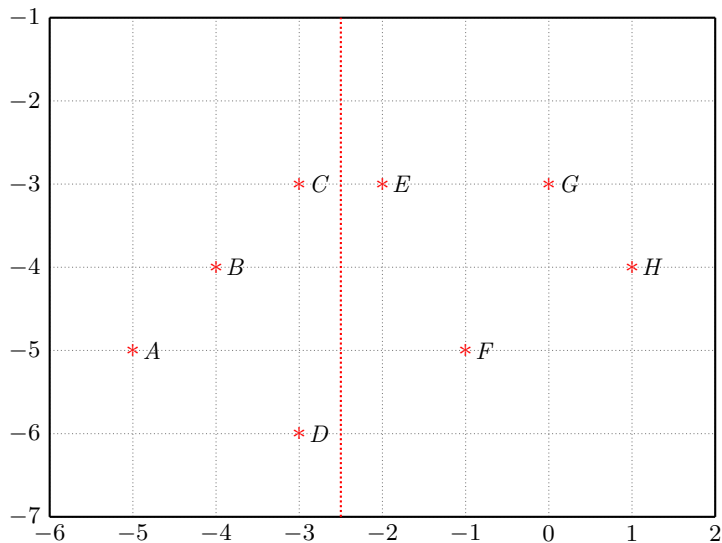


good splitters

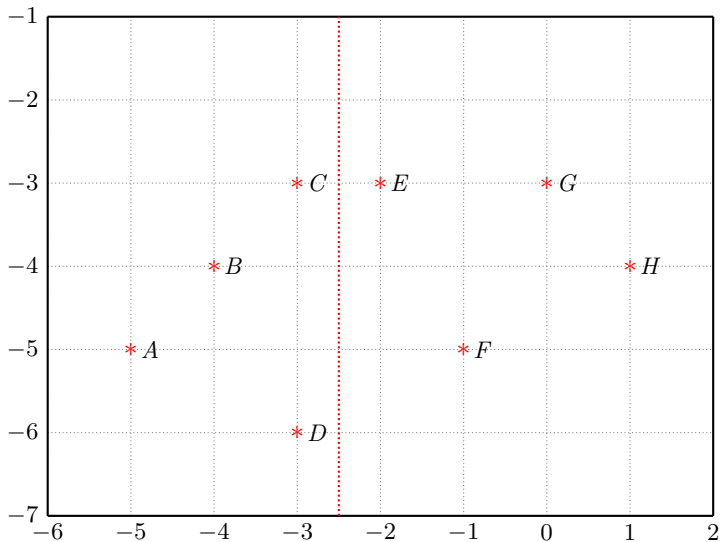
Lec5 Closest Pairs



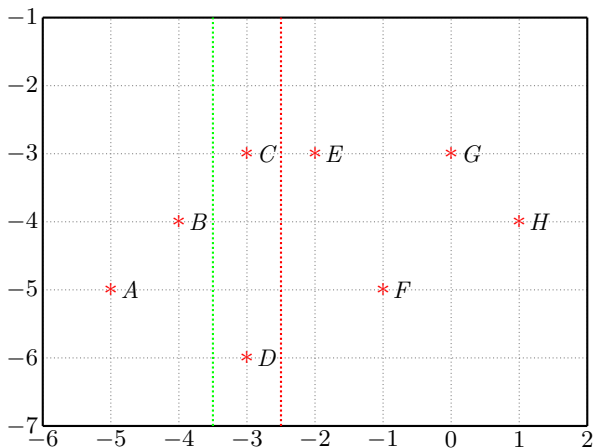
Lec5 Closest Pairs



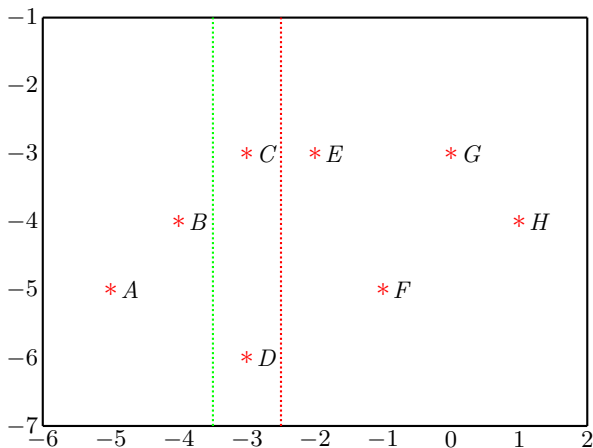
Lec5 Closest Pairs Middle Red Line



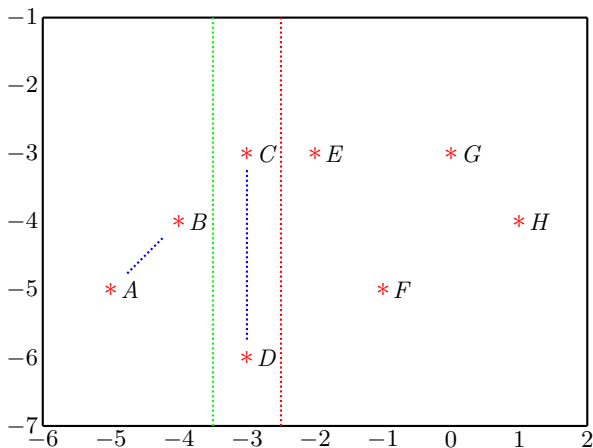
Leech Closest Pairs Left



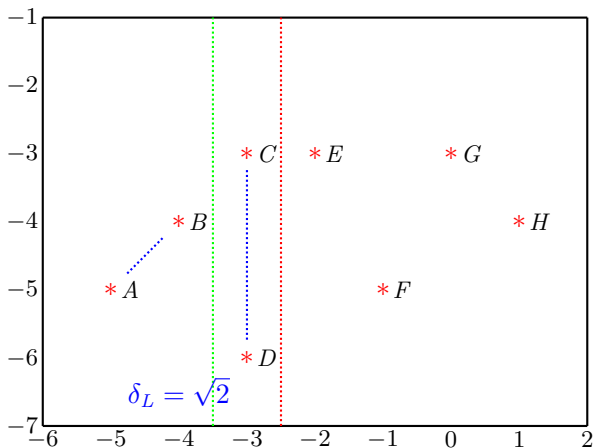
Lec5 Closest Pairs Left Grids



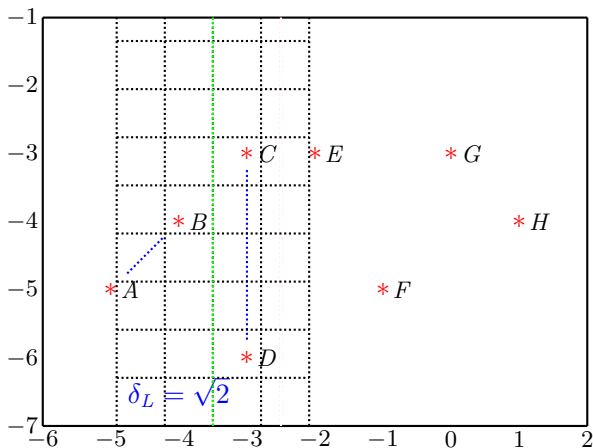
Lec5 Closest Pairs Left Grids



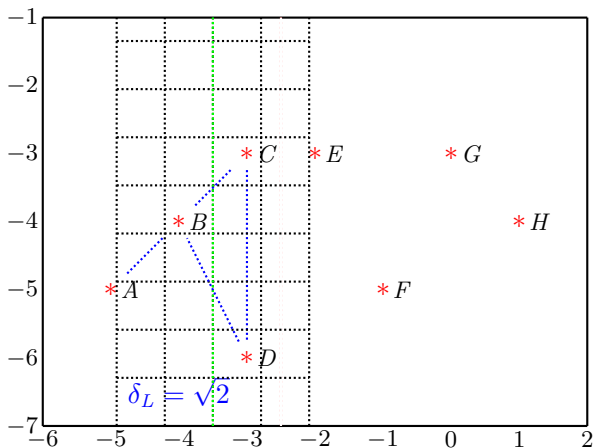
Lec5 Closest Pairs Left Grids



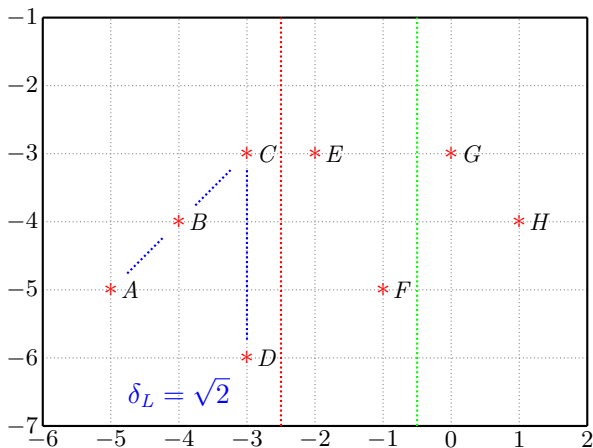
Lec5 Closest Pairs Left Grids



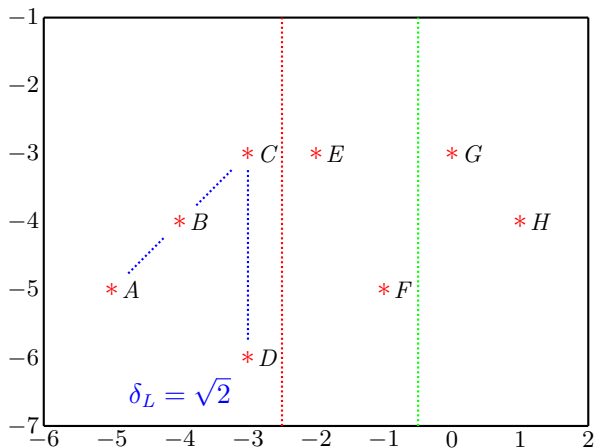
Lec5 Closest Pairs Left Grids



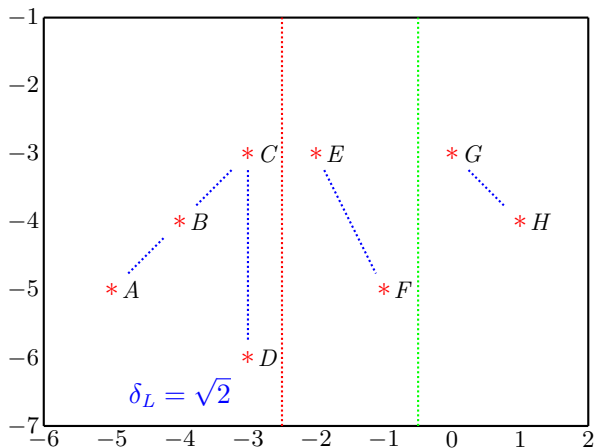
Lec5 Closest Pair Right



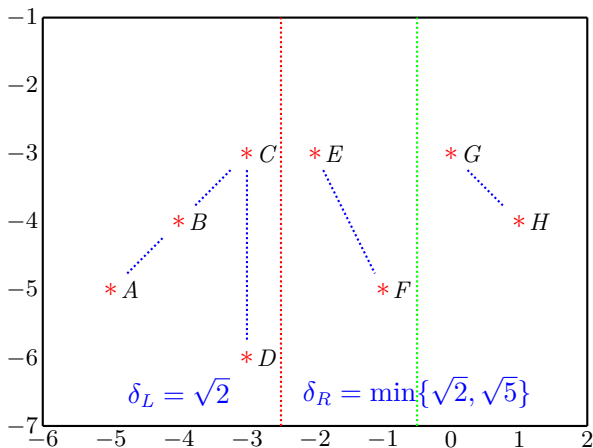
Lec5 Closest Pairs Right Grids



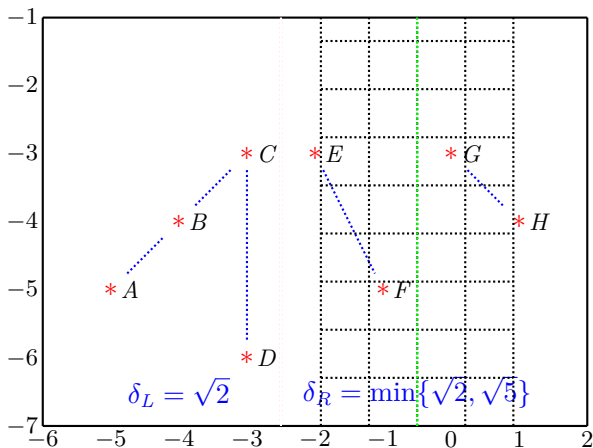
Lec5 Closest Pairs Right Grids



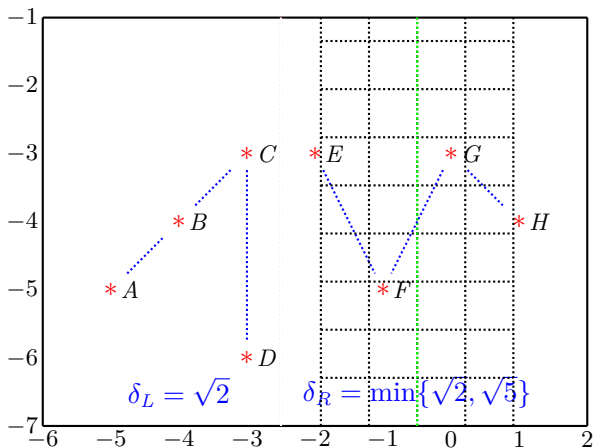
Lec5 Closest Pairs Right Grids



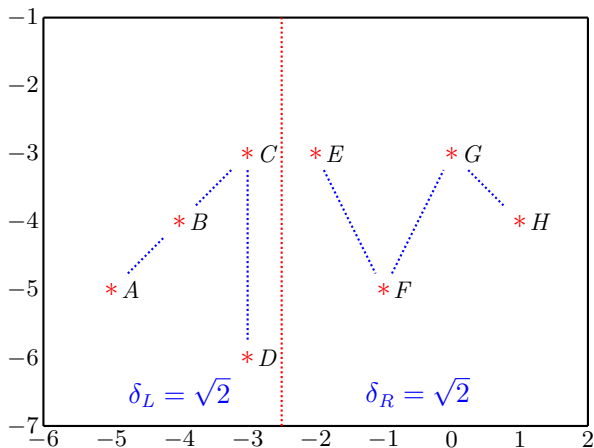
Lec5 Closest Pairs Right Grids



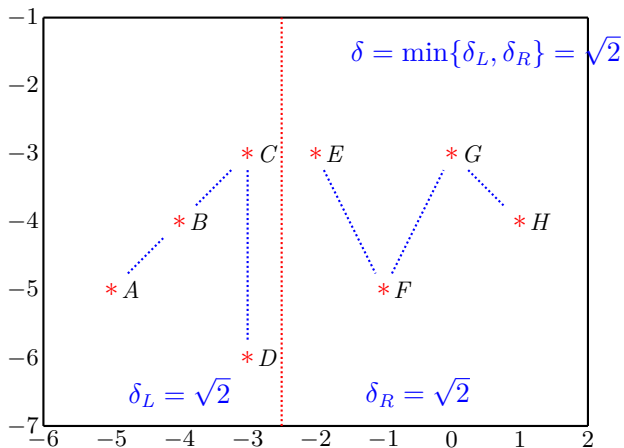
Lec5 Closest Pairs Right Grids



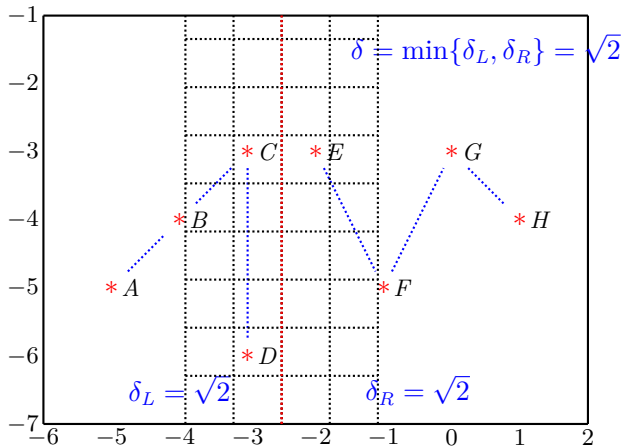
Lec5 Closest Pairs Final



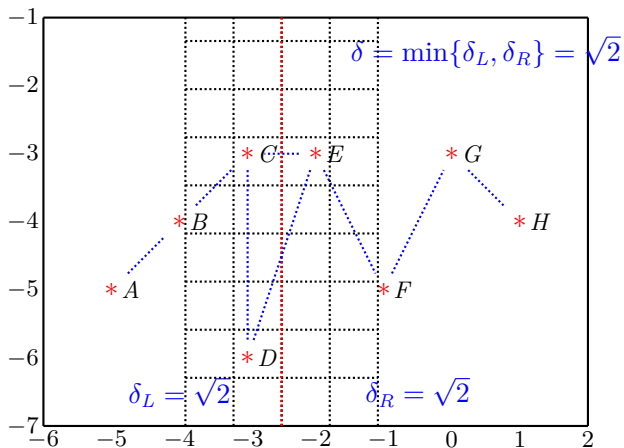
Lec5 Closest Pairs Final



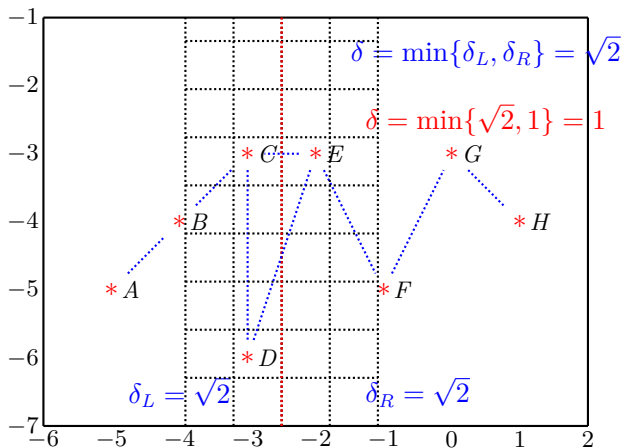
Lec5 Closest Pairs Final

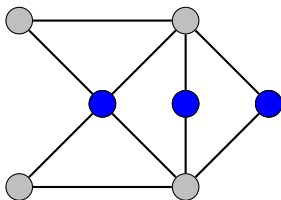


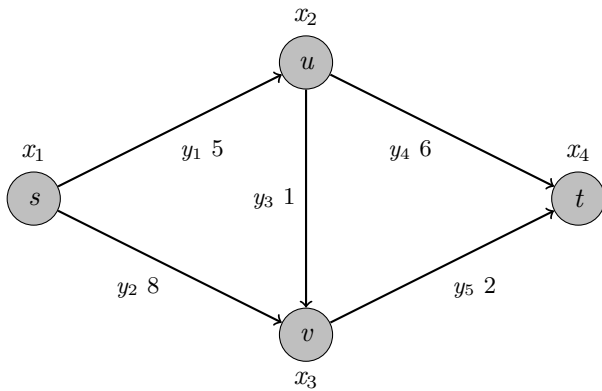
Lec5 Closest Pairs Final



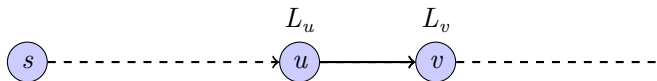
Lec5 Closest Pairs Final



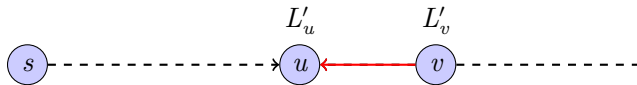




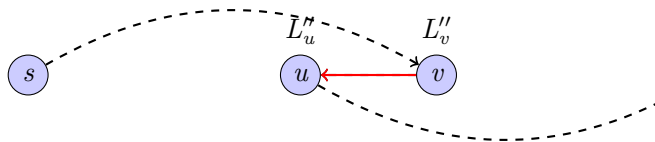
Step k :



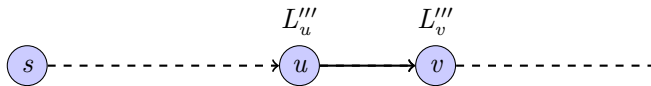
Step $k + 1$:

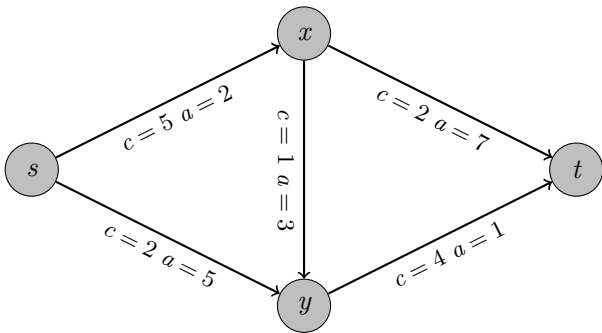


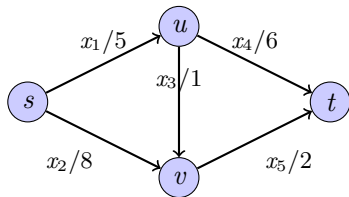
Step k'' :

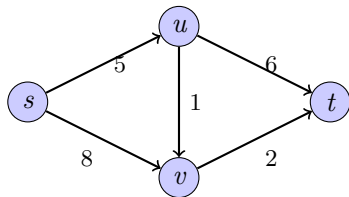


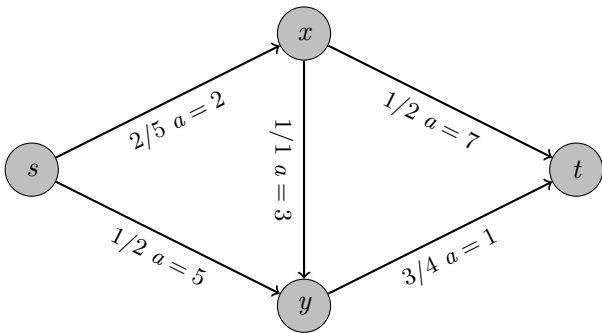
Step $k''' :$

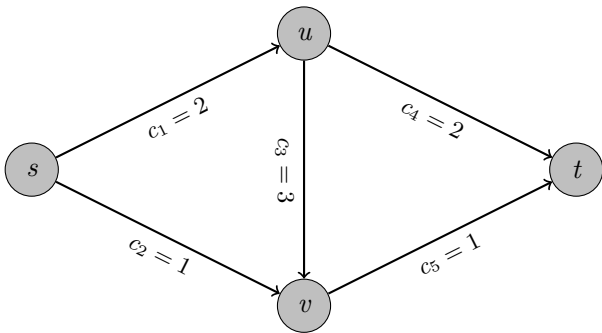


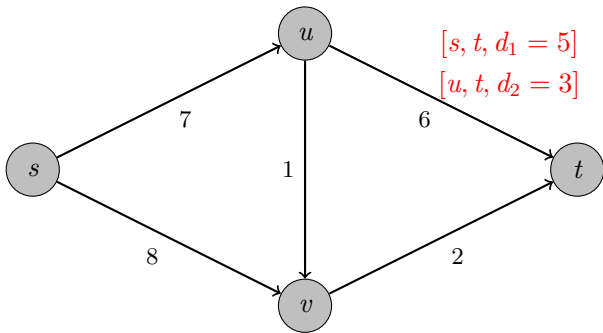


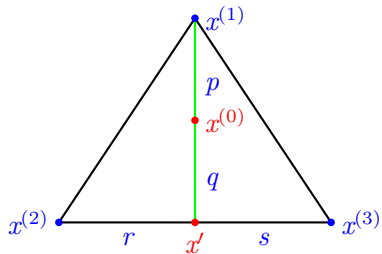




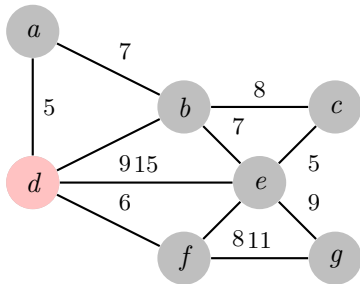




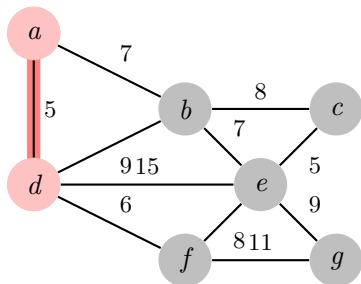




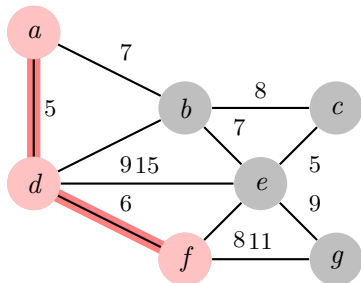
Prim's algorithm



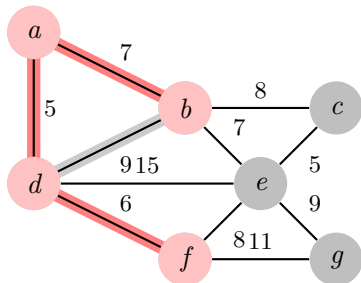
Prim's algorithm



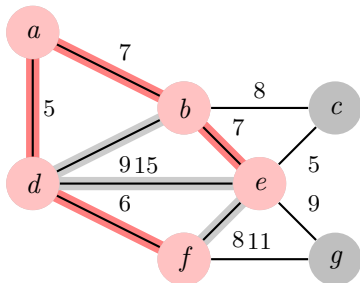
Prim's algorithm



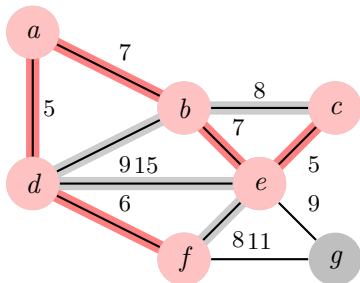
Prim's algorithm



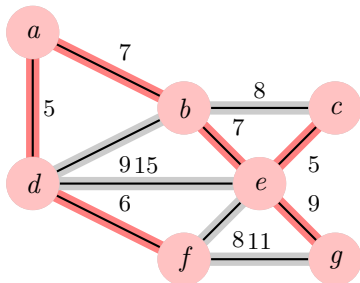
Prim's algorithm



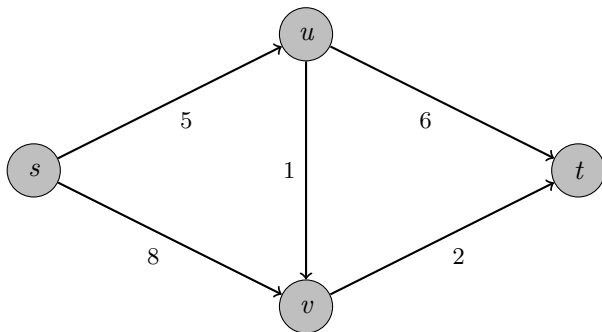
Prim's algorithm



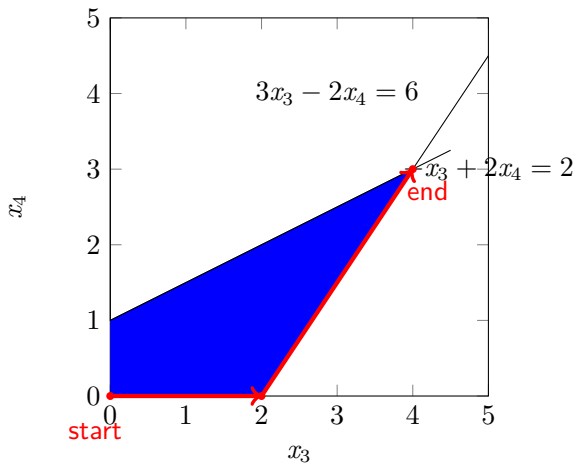
Prim's algorithm

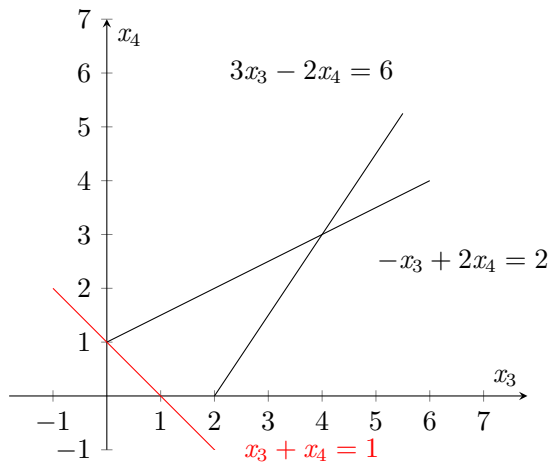


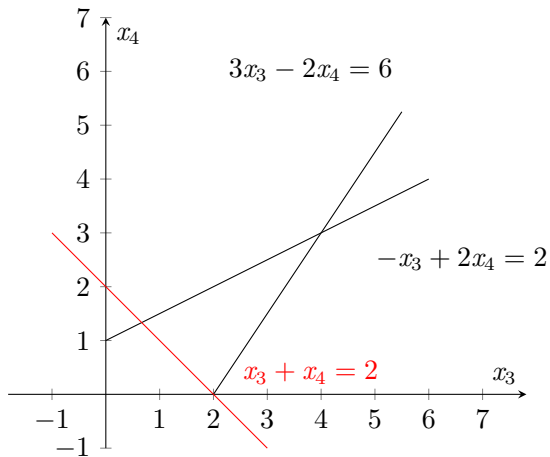
Max Flow

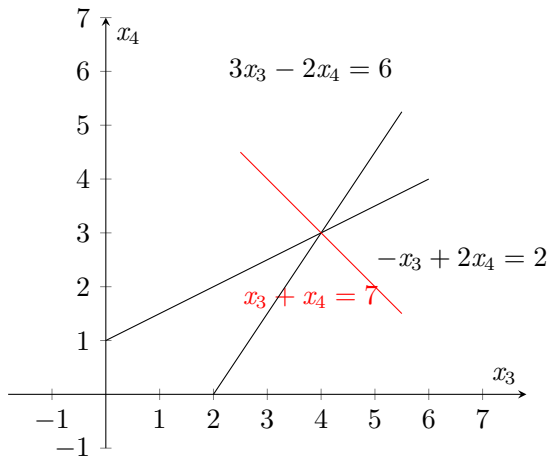


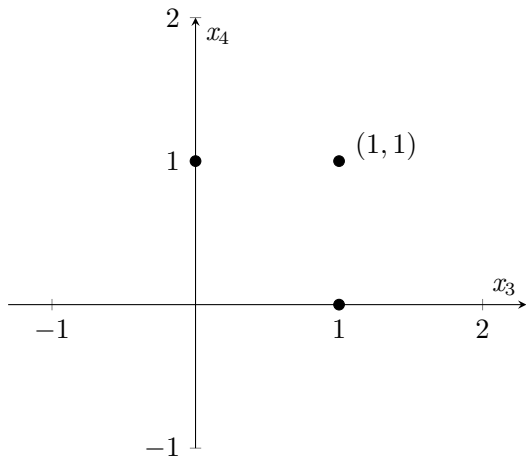
LP example

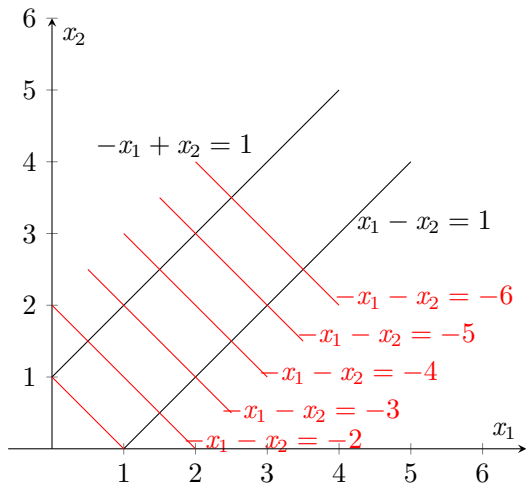


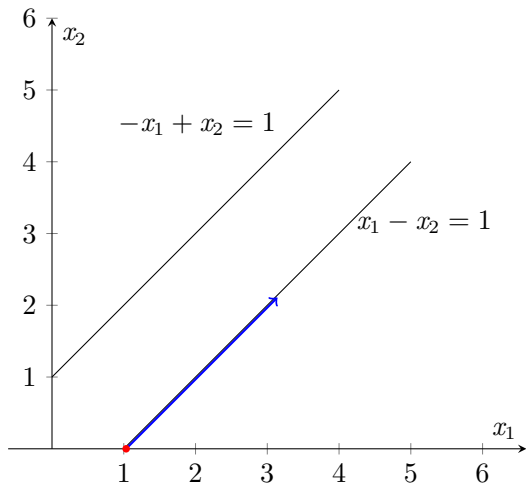


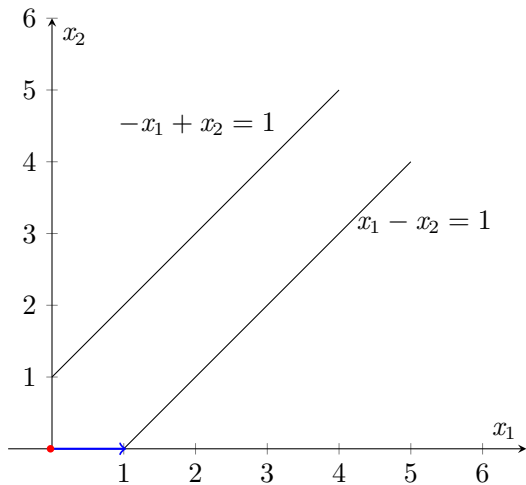












$$\begin{array}{cccccc}
 c_1 & c_2 & \dots & c_n & & \\
 a_{11} & a_{12} & \dots & a_{1n} & b_1 & \\
 a_{21} & a_{22} & \dots & a_{2n} & b_2 & \\
 & & \dots & & & \\
 a_{m1} & a_{m2} & \dots & a_{mn} & b_m &
 \end{array}$$

$$\min \quad c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \geq b_1$$

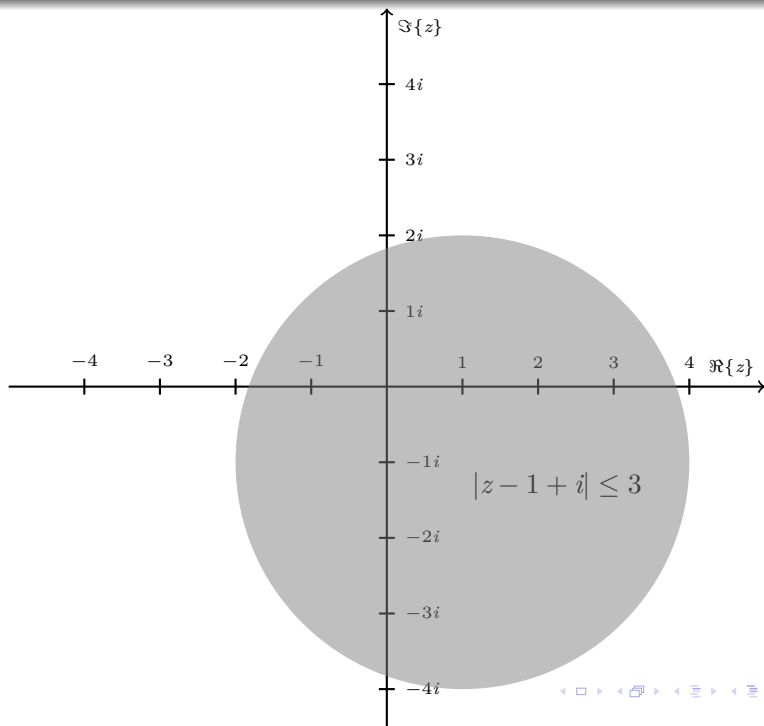
$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n \geq b_2$$

...

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n \geq b_m$$

$$x_i \geq 0 \quad \text{for each } i$$

$$\begin{array}{cccccc}
c_1 & c_2 & \dots & c_n & & \\
\vee & \vee & & \vee & \max & \\
y_1 a_{11} & y_1 a_{12} & \dots & y_1 a_{1n} & y_1 b_1 & \\
+ & + & & + & + & \\
y_2 a_{21} & y_2 a_{22} & \dots & y_2 a_{2n} & y_2 b_2 & \\
+ & + & & + & + & \\
& & \dots & & & \\
+ & + & & + & + & \\
y_m a_{m1} & y_m a_{m2} & \dots & y_m a_{mn} & y_m b_m & \\
& & & y_j \geq 0 & \text{for each } j &
\end{array}$$



$$\begin{pmatrix} 0 & c_1 & c_2 & \cdots & c_m & \cdots & c_n \\ b_1 & a_{11} & a_{12} & \cdots & a_{1m} & \cdots & a_{1n} \\ b_2 & a_{21} & a_{22} & \cdots & a_{2m} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_m & a_{m1} & a_{m2} & \cdots & a_{mm} & \cdots & a_{mn} \end{pmatrix}$$

$$\Rightarrow \times \mathbf{B}^{-1}$$

$$\begin{pmatrix} \mathbf{c}_B^T \mathbf{B}^{-1} \mathbf{b} & 0 & 0 & \cdots & 0 & \mathbf{c}_N^T - \mathbf{c}_B^T \mathbf{B}^{-1} \mathbf{N} \\ & 1 & 0 & \cdots & 0 & \\ \mathbf{B}^{-1} \mathbf{b} & 0 & 1 & \cdots & 0 & \mathbf{B}^{-1} \mathbf{N} \\ & \vdots & \vdots & \ddots & \vdots & \\ & 0 & 0 & \cdots & 1 & \end{pmatrix}$$

$$\begin{pmatrix} 0 & \dots & 0 & \dots & c_e & \dots \\ b_1 & \dots & 0 & \dots & a_{1e} & \dots \\ b_2 & \dots & 0 & \dots & a_{2e} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_l & \dots & 1 & \dots & a_{le} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_m & \dots & 0 & \dots & a_{me} & \dots \end{pmatrix}$$

\Rightarrow

$$\begin{pmatrix} -\frac{a_{me}}{a_{le}} b_l & \dots & -\frac{c_e}{a_{le}} & \dots & 0 & \dots \\ b_1 - \frac{a_{1e}}{a_{le}} b_l & \dots & -\frac{a_{1e}}{a_{le}} & \dots & 0 & \dots \\ b_2 - \frac{a_{2e}}{a_{le}} b_l & \dots & -\frac{a_{2e}}{a_{le}} & \dots & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{1}{a_{le}} b_l & \dots & \frac{1}{a_{le}} & \dots & 1 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_m - \frac{a_{me}}{a_{le}} b_l & \dots & -\frac{a_{me}}{a_{le}} & \dots & 0 & \dots \end{pmatrix}$$

LP x1x2-1

