



2020牛客暑期多校训练营（第一场）

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B-Suffix Array

- Let $C_i = \min_{\{j > i \text{ and } s_j = s_i\}} \{j - i\}$
- The B-Suffix Array is equivalent to the suffix array of $C_1 C_2 \dots C_n$
- Detailed proof can be found in “Parameterized Suffix Arrays for Binary Strings
- ” <http://www.stringology.org/event/2008/p08.html>





Infinite Tree

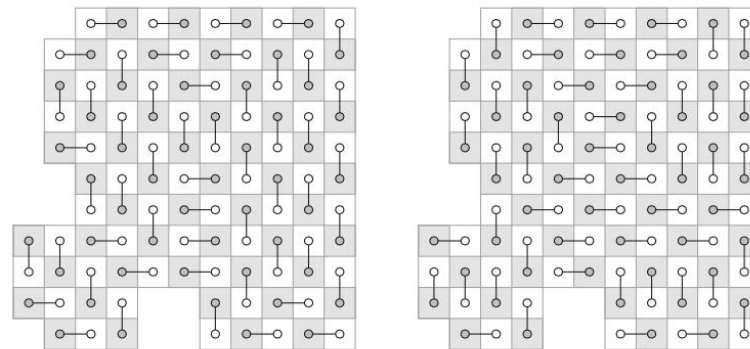
- First, compute the “virtual tree” of $\{1!, 2!, \dots, n!\}$
- Second, to compute the actual cost, use Segment Tree or Fenwick Tree.
- $O(m \log^2 m)$





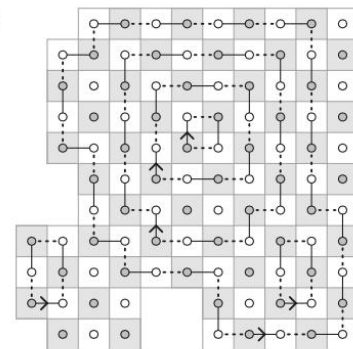
Domino

- See “Distances in Domino Flip Graphs”

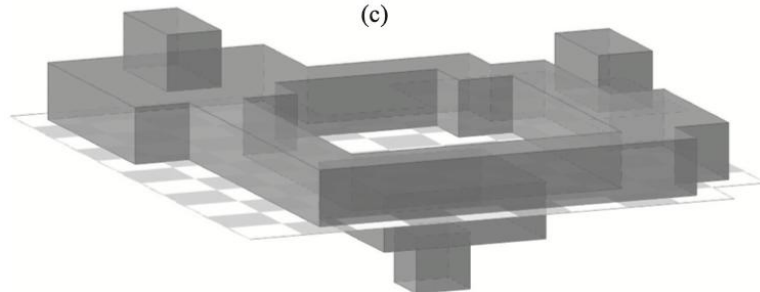


(a)

(b)



(c)



(d)



Quadratic Form

- The answer is $\mathbf{b}^T \mathbf{A}^{-1} \mathbf{b}$, which can be proved by Lagrange Duality.
- All we need is to compute the inverse matrix of the matrix \mathbf{A} .





Counting Spanning Trees

- The number of spanning trees is $\prod_{i \geq 2} \deg(x_i) \deg(y_i)$
- Detailed proof can be found in “Enumerative properties of Ferrers graphs”
<https://arxiv.org/pdf/0706.2918.pdf>





Infinite String Comparision

- Compare the string a^{∞} and b^{∞} directly
- By the Periodicity Lemma, if there is no mismatches in the first $a + b - \gcd(a, b)$ characters, the two string are identical





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- For simplicity, we denote the multiplication as $+$, and exponentiation as $*$
- Precompute $B_{\{i, j\}} = 2^{\{W * j\}} * v_i$
- To compute $\text{sum}_{\{i, j\}} (\text{sum } e'_{\{i, j\}} * 2^{\{W * j\}}) v_i$
 - $= \text{sum}_x x \text{sum}_{\{i, j\}} [e'_{\{i, j\}} = x] B_{\{i, j\}} = \text{sum}_x x Q_x$
- To compute $\text{sum}_x x Q_x$
 - $= \text{sum}_x (\text{sum}_{\{y \geq x\}} Q_y)$
- The overall complexity is $O(nm / W + 2^W)$
- Taking $W = 16$ yields a fast enough solution



Minimum-cost Flow

- We denote the cost in a network with capacity c and flow f as $\text{cost}(c, f)$.
- $\text{cost}(c, 1) = \text{cost}(c * 1/c, 1 / c) * c = \text{cost}(1, 1 / c) * c$
- For a network with unitary capacity, its cost grows linearly with the flow f , with at most $O(m)$ pieces.
- Thus, we can compute $O(m)$ pieces first, and query in $O(\log m)$ time.





1 or 2

- For an edge $e=(x, y)$ where $d_x = d_y = 2$, add the following edges:
 - $(x, e) (x', e)$
 - $(y, e') (y', e')$
 - (e, e')
- The problem is turned into finding a perfect matching in a general graph, which can be solved with Edmond's Algorithm.





Easy Integration

- The value is $(n!)^2 / (2n+1)!$
- Detailed proof can be found in “Wallis' integrals”.
https://en.wikipedia.org/wiki/Wallis%27_integrals



Thanks

