① We know that

$$\Delta v = I_{sp} \cdot g_0 \cdot \ln\left(\frac{M_0}{M_f}\right)$$

Here, $\Delta v = 7.6\, kms^{-1} = 7600\, ms^{-1}$

$I_{sp} = 400\, s$

$g_0 = 9.8\, m/s^2$

$M_0 = M_{propellant} + M_{else}$

$M_f = M_{else}$

Putting the values,

$$7600 = 400 \times 9.8 \ln\left(\frac{M_0}{M_f}\right)$$

$$\Rightarrow 1.938 = \ln\left(\frac{M_0}{M_f}\right)$$

$$\Rightarrow 6.944 = M_0/M_f = \frac{M_{prop} + M_{else}}{M_{else}}$$

$$\therefore 6.944\, M_{prop} = 5.944\, M_{else}$$

$$\Rightarrow 6.944\, M_{prop} = 5.944\, [M_0 - M_{prop}]$$

$$\Rightarrow \frac{M_{prop}}{M_0} = \frac{5.944}{12.888} = \underline{\underline{0.46}} = \frac{23}{50}$$

$\therefore$ The propellation fraction should be $\frac{23}{50}$ of the initial wet mass.

(2)

$$n = \frac{F_{thrust}}{m_0 g} = \frac{+c\dot{m}}{m_0 g}$$

Let $\frac{dm}{dt} = -\dot{m}$

$$\mu = \frac{m(t)}{m_0} = \frac{m_0 - \dot{m}t}{m_0}$$

$$F_{net} = F_{thrust} + F_{inertial} = \frac{c\dot{m}}{m_0 g} - mg$$

$$\therefore m\frac{dv}{dt} = \frac{c\dot{m}}{\cancel{m_0 g}} - mg = n m_0 g - mg$$

$$m\frac{dv}{dm} \cdot \frac{dm}{dt} = n m_0 g - mg$$

$$\Rightarrow -m\frac{dv}{dm}\dot{m} = n m_0 g - mg$$

$$m\frac{dv}{dt} = c\dot{m} - mg \Rightarrow \frac{dv}{dt} = \frac{c\dot{m} - mg}{m}$$

where $m = m_0 - \dot{m}t$

$$\therefore \frac{dv}{dt} = \frac{c\dot{m} - m_0 g + \dot{m}tg}{m_0 - \dot{m}t} = c$$

$$-m\frac{dv}{dm}\dot{m} = n m_0 g - mg$$

$$\Rightarrow -dv = dm \left(\frac{n m_0 g}{m\dot{m}} - \frac{g}{\dot{m}}\right)$$

Integrating:

$$-\int_0^v dv = \frac{n m_0 g}{\dot{m}} \ln\left(\frac{m}{m_0}\right) - \frac{g}{\dot{m}}(m - m_0)$$

$$v = \frac{nm_0 g}{\dot{m}} \ln\left(\frac{M_0}{m}\right) - gt$$

$$\boxed{v = -gt - \frac{nm_0 g}{\dot{m}} \ln(\mu)}$$

(b) $$\frac{dz}{dt} = -gt - \frac{nm_0 g}{\dot{m}} \ln\left(\frac{m_0 - \dot{m}t}{m_0}\right)$$

$$\Rightarrow \quad z = -\frac{gt^2}{2} + \frac{nm_0 g}{\dot{m}} t + \frac{nm_0 g}{\dot{m}^2} (m_0 - \dot{m}t) \ln(\mu)$$

(c) Total time of flight:

First we take the burnout time t_b.

~~Let~~ Let $v(t_b) = v_b$

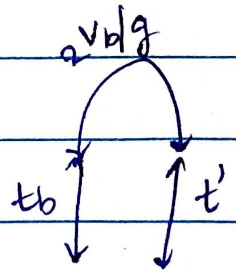
After that rocket is in gravity free fall.

further time taken $= \frac{v_b}{g}$

$\therefore$ Total time taken $= t_b + \frac{2v_b}{g} + t'$

Where t' is given by: $h_b = v_b t' + \frac{1}{2} g t'^2$

where h_b is altitude at burnout time.



(d) Physically, n signifies about the exhaust speed and the thrust, the ejecting gases are able to provide to the rocket.

(3) $\Delta v = 8 kms^{-1} = \underline{8000 ms^{-1}}$

<u>SSTO:</u> $v_e = 4.5 km/s = \underline{4500 m/s}$.

$$\Delta V = V_{ex} . \ln\left(\frac{M_f}{M_o}\right)$$

$$\Rightarrow 8000 = +4500 \ln\left(\frac{M_o}{M_f}\right)$$

$$\Rightarrow 1.78 = \ln(M_o/M_f)$$

$$\Rightarrow \underline{\underline{M_o = 5.93 M_f}}$$

$$\Rightarrow M_{prop} + M_{pay} + M_{rocket} = 5.93[M_{pay} + M_{rocket}]$$

$$\Rightarrow 4.93(M_{pay} + M_{rocket}) = M_{prop}$$

Let $M_{pay} + M_{rocket} = m$

$$\therefore 4.93 m = M_o - m$$

$$\boxed{m = \frac{M_o}{5.93}}$$

(4) $$\lambda = \frac{M_{rocket}}{M_{rocket} + M_{pay}}$$

$$\Delta V = I_{sp} \cdot g_0 \ln\left[\frac{M_{prop} + M_{pay} + M_{rocket}}{M_{pay} + M_{rocket}}\right]$$

$$\Delta v = I_{sp} \cdot g_0 \cdot \ln\left[\frac{M_0}{M_{pay} + M_{rocket}}\right]$$

Now, $(M_{pay} + M_{rocket})\,\lambda = M_{rocket}$

$$M_{rocket} = \frac{\lambda M_{pay}}{1-\lambda}$$

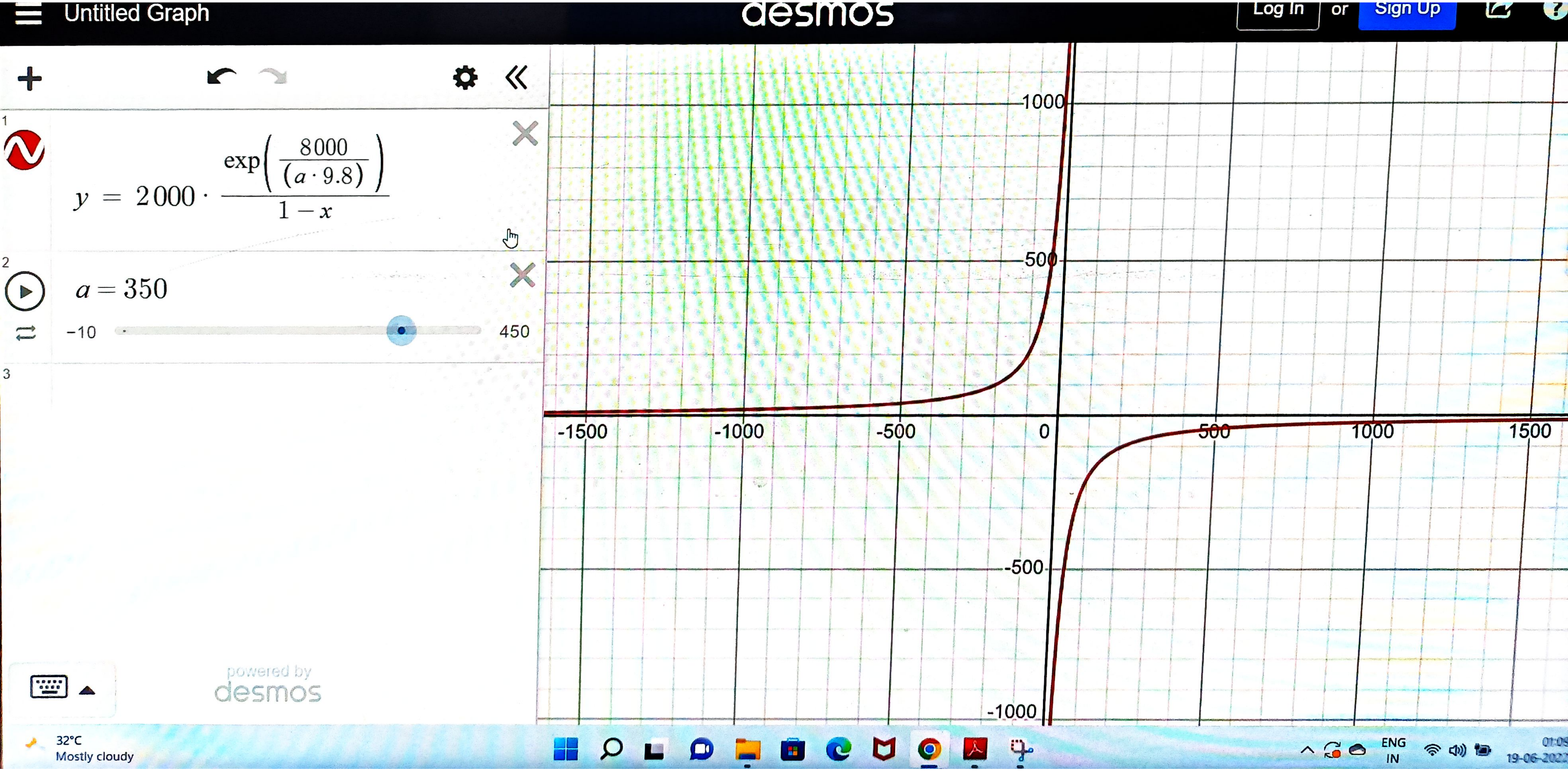
$$\therefore \quad M_{pay} + M_{rocket} = \frac{\lambda M_{pay}}{1-\lambda} + M_{pay} = \frac{M_{pay}}{1-\lambda}$$

$$\Delta V = I_{sp} \cdot g_0 \cdot \ln\left[\frac{M_0 (1-\lambda)}{M_{pay}}\right]$$

$$\Rightarrow \quad e^{\frac{\Delta V}{I_{sp} g_0}} = \frac{M_0 (1-\lambda)}{M_{pay}}$$

$$\therefore \quad \boxed{M_0 = \frac{M_{pay} \cdot e^{\frac{\Delta V}{I_{sp} g_0}}}{(1-\lambda)}}$$

(Definition of λ in slides is wrong. graphs are therefore in second and fourth quad. which is absurd)


Untitled Graph
desmos
Log In
or
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1
$y = 2000 \cdot \frac{\exp\left(\frac{8000}{(a \cdot 9.8)}\right)}{1 - x}$
2
$a = 350$
−10
450
3
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-500
-1000
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1500
powered by
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32°C
Mostly cloudy
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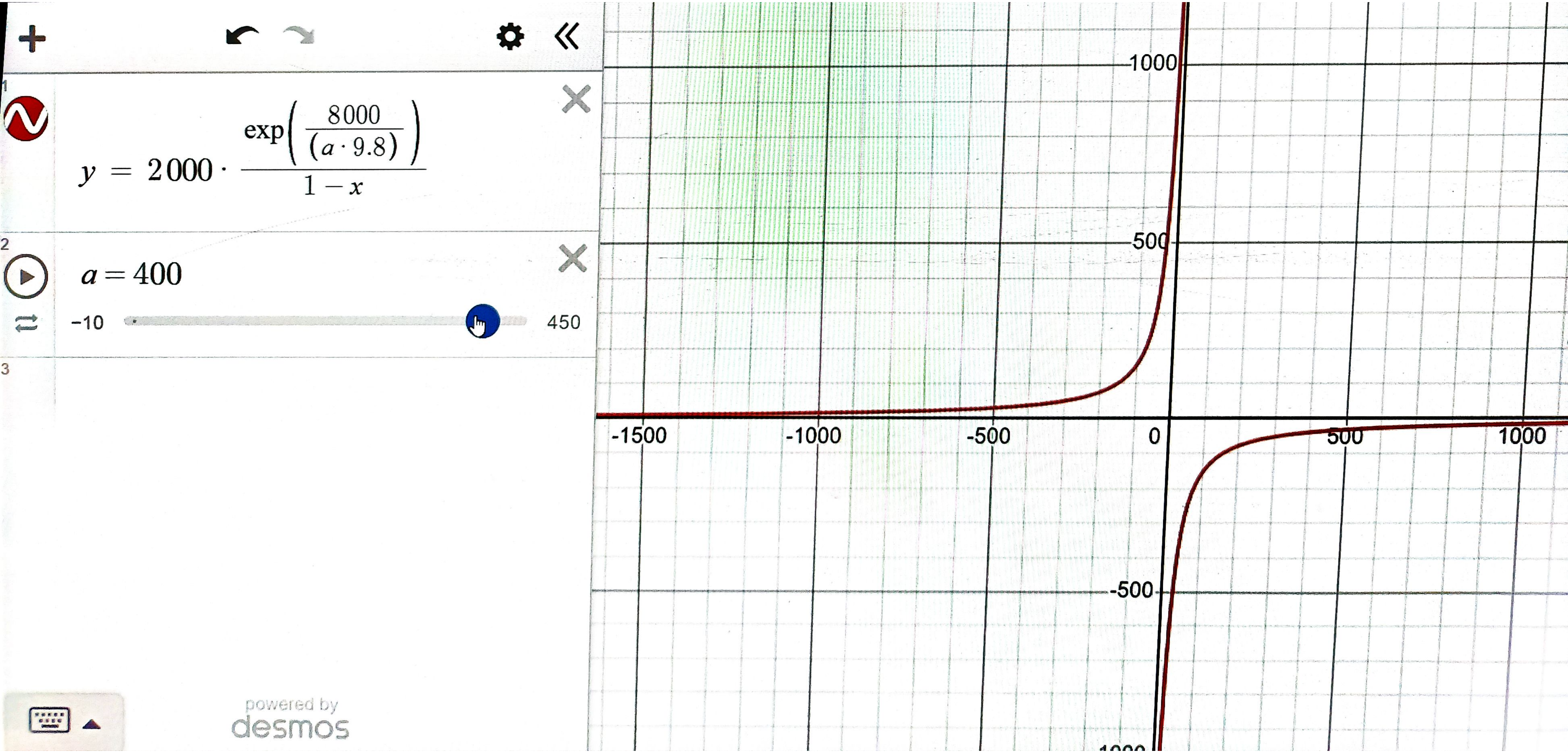
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$a = 400$
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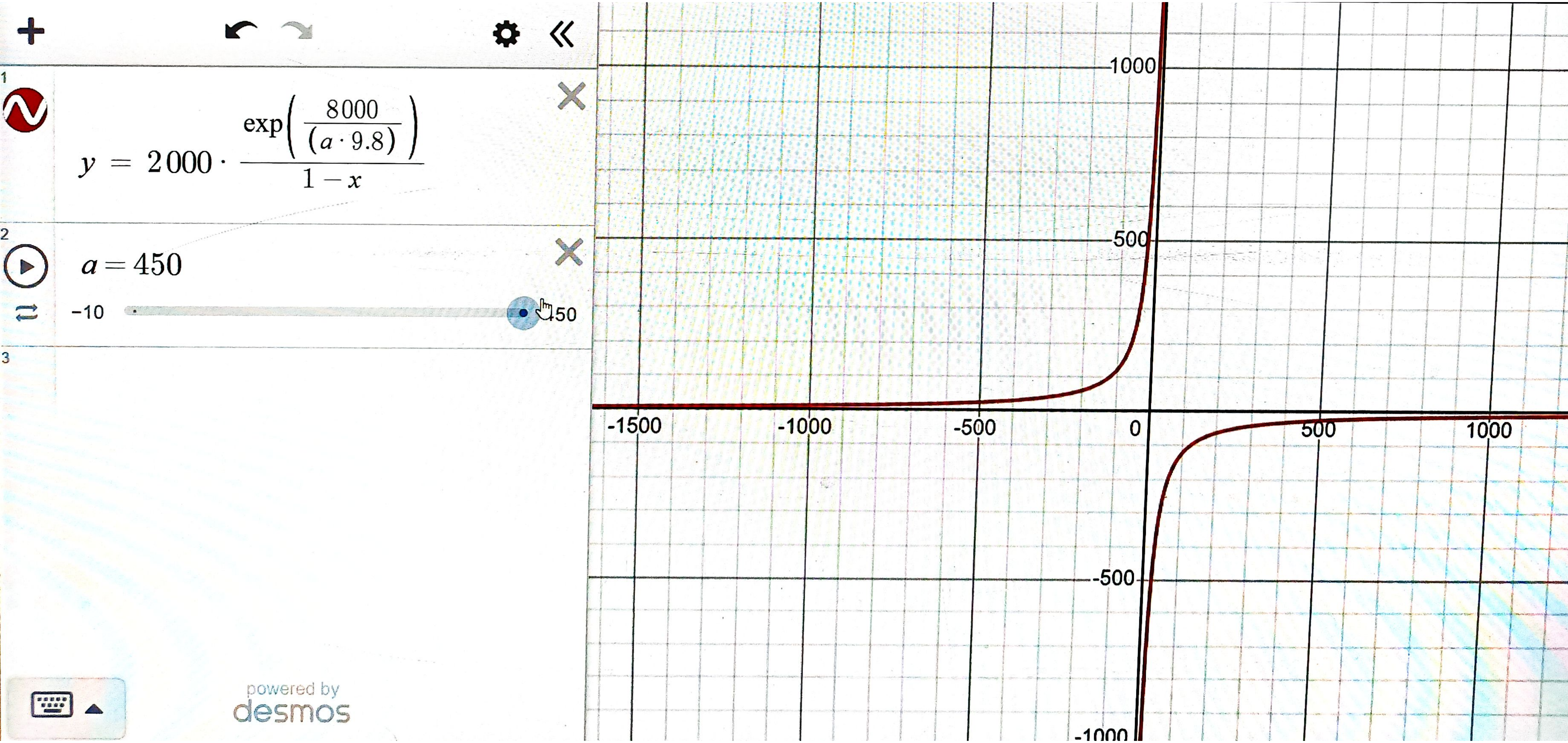

1
$y = 2000 \cdot \frac{\exp\left(\frac{8000}{(a \cdot 9.8)}\right)}{1 - x}$
2
$a = 450$
−10
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1000
500
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