Q1.

r_1 = 35800 + 6400 = 42200 Km = 4.22*10^7 m

r_2 = 6400 + r

a = (r_1 + r_2)/2

For circular trajectory,

$$\frac{mv_0^2}{r_1} = \frac{GMm}{r_1^2}$$

So, Initial speed $v_0 = (GM/r_1)^{1/2}$

For elliptical trajectory,

$$\frac{1}{2}mv_f^2 - \frac{GMm}{r_2} = -\frac{GMm}{2a}$$

So, Final speed $v_f = (GM\,(2/r_2 - 1/a)\,)^{1/2}$

Change required = $\Delta v = |v_f - v_0|$

Case I:

For first case, r = 400Km => r_2 = 6800 Km = 6.8*10^6 m

a = 24500 Km = 2.45*10^7 m

v_0 = 3079.51494919 ms^{-1}

v_f = 10068.3260016 ms^{-1}

Δv = 6,988.81105241 ms^{-1}

Case II:

For second case, r = 36050Km => r_2 = 42450 Km = 4.245*10^7 m

a = 42325 Km = 4.2325*10^7 m

v_0 = 3079.51494919 ms^{-1}

$v_f = 3065.896128\ ms^{-1}$

$\Delta v = 13.61882119\ ms^{-1}$

The second transfer is preferable because the change required for the second transfer is less as compared to the first.

Q2.

The location of L_1 is the solution to the following equation, gravitation providing the centripetal force:

$$\frac{M_1}{(R-r)^2} - \frac{M_2}{r^2} = \left(\frac{M_1 R}{M_1 + M_2} - r\right) \frac{M_1 + M_2}{R^3}$$

Numerically solving the equation,

We get r = $1.4954*10^9$ m

Also, $R(M_2 / 3M_1)^{1/3}$ = 1.500418854×10^9 m

The location of L_2 is the solution to the following equation, gravitation providing the centripetal force:

$$\frac{M_1}{(R+r)^2} + \frac{M_2}{r^2} = \left(\frac{M_1}{M_1 + M_2}(R+r)\right) \frac{M_1 + M_2}{R^3}$$

Numerically solving the equation,

We get r = $1.5054*10^9$ m

Also, $R(M_2 / 3M_1)^{1/3}$ = 1.500418854×10^9 m

The location of L_3 is the solution to the following equation, gravitation providing the centripetal force:

$$\frac{M_1}{(R-r)^2} + \frac{M_2}{(2R-r)^2} = \left(\frac{M_2}{M_1 + M_2}R + R - r\right) \frac{M_1 + M_2}{R^3}$$

Numerically solving the equation,

We get r = 262719.171 m

Also, $R(7M_2 / 12M_1)$ = 262719.959779 m

Thus, our approximations are quite close to the actual values.

Q3.

The diagram illustrates how a spacecraft can achieve an extraordinary Δv using gravity-assist. It changes the direction of the probe's motion in such a way that the velocity components (due to the relative motion with respect to the Sun and Jupiter) add up at an obtuse angle as the probe recedes from the planet. This manuouvre would require an enormous amount of propellant in the absence of the planet.