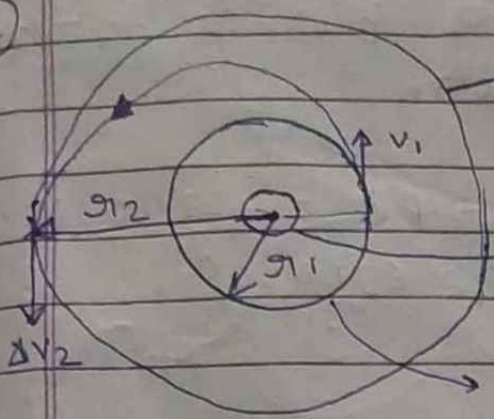
Assignment - 4

(1)



$$v = \sqrt{\frac{GM}{r_1}} \qquad \mu = GM$$

$$\frac{mv'^2}{2} - \frac{GMm}{r_1} = -\frac{GMm}{2a}$$

$$a = \frac{r_1 + r_2}{2} \qquad (v')^2 = \mu\left(\frac{2}{r_1} - \frac{1}{a}\right)$$

$$r = r_1 \qquad \Delta V_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)$$

, to enter the elliptical orbit at $r = r_1$ from r_1 circular orbit.

$$v' = \sqrt{\frac{\mu}{r_2}}$$

$$\frac{mv^2}{2} - \frac{GMm}{r_2} = -\frac{GMm}{2a}$$

$$v^2 = \mu\left(\frac{2}{r_2} - \frac{1}{a}\right)$$

$$\Delta V_2 = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right).$$

$$\Delta V_{Total} = \Delta V_1 + \Delta V_2.$$
$$= \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right) + \sqrt{\frac{GM}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right)$$

On putting value for transfer b/w geostationary orbit and low earth orbit.

$$r_1 = 35800 + R_e = 42200 \text{ km}$$
$$r_2 = 400 + R_e = 6800 \text{ Km}$$

$$|\Delta v_{total}| = \Delta v_1 + \Delta v_2$$
$$= +3856 \text{ m/s}.$$

similarly
On putting value for transfer b/w geostationary orbit & graveyard orbit.

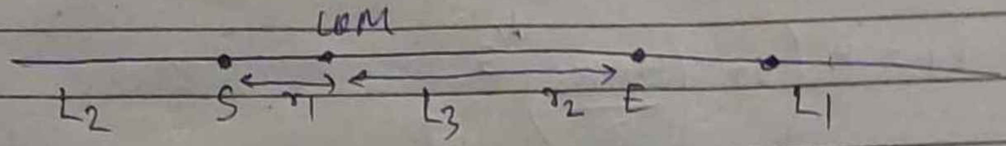
$$r_1 = 35800 + R_e = 42200 \text{ km}$$
$$r_2 = 6050 + R_e = 42450 \text{ km}.$$

$$\Delta v_{total} = \Delta v_1 + \Delta v_2$$
$$= 9.07 \text{ m/s}.$$

$$(\Delta v_{Total})_{grav} < (\Delta v_{total})_{low\ earth}.$$

$\Rightarrow$ we would prefer transfer to graveyard orbit.

Que. 2



$\because$ for an equipotential region
$V =$ constant.

$$\frac{\partial U}{\partial r} = 0 \implies F = 0.$$

$$(F_G)_S + (F_G)_E = F_{centrifugal\ force} = F_2$$

for L1 region.

$$\frac{GM_S}{(x+r_1)^2} + \frac{GM_E}{(x-r_2)^2} = \frac{G(M_S+M_E)x}{R_{SE}^3}$$

$r_1 =$ Distance of sun from COM $= \left(\frac{M_e}{M_e+M_S}\right) R_{SE}$

$r_2 =$ Distance of earth from COM $= \left(\frac{M_S}{M_S+M_e}\right) R_{SE}$

$$GM_S(x-r_2)^2 R_{SE}^3 + GM_e(x+r_1)^2 R_{SE}^3 = G(M_S+M_e)x\,(x+r_1)^2(x-r_1)^2$$

(2).

$$L_1 = R\left(1 + \left(\frac{M_e/M_s}{3}\right)^{1/3}\right)$$

Putting M_E ; M_S , R_{SE} in eq^n - L_1 and checking

L_1 satisfy eq^n or not.

$$L_1 = 1.515 \times 10^{11} \text{ m.}$$

Similarly for L_2.

$$\frac{GM_S}{(x-r_1)^2} + \frac{GM_E}{(x+r_2)^2} = \frac{G(M_S+M_E)x}{R_{SE}^3}$$

$$R_{SE}^3 M_S (x+r_2)^2 + M_E (x-r_1)^2 R_{SE}^3 = (M_S+M_E)\, x\, (x-r_1)^2 (x+r_2)^2$$

$$L_2 = \left(1 - \left(\frac{M_e}{3M_s}\right)^{1/3}\right) R.$$

for M_e, M_s we get $\rightarrow$ $L_2 = 1.48 \times 10^{11}$ m.

L_2 satisfy eq.

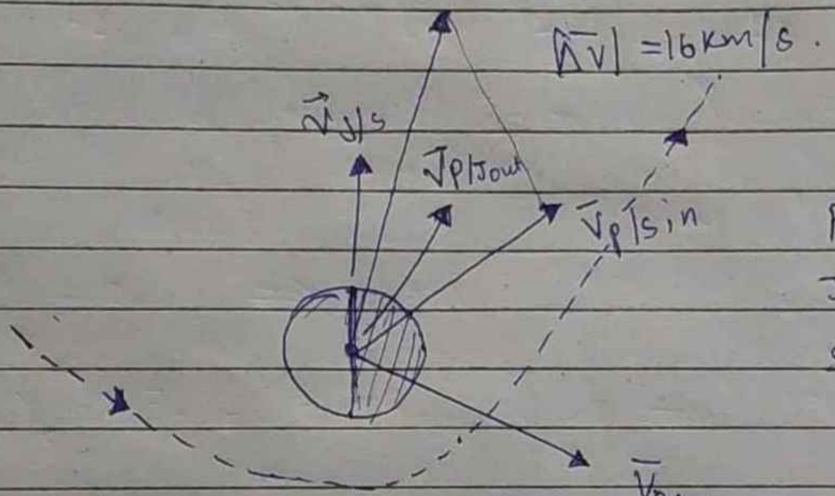
for L_3 :-

$$\frac{GM_s}{(x+r_1)^2} - \frac{GM_e}{(x-r_2)^2} = \frac{G(M_s+M_e)x}{(r_1+r_2)^3}$$

$L_3 = -1.5000187 \times 10^{11}$ m.

$$L_3 = -R\left[1 + \frac{5}{12}\frac{M_e}{M_s}\right]$$

Que 3



P = Probe
J = Jupiter
S = Sun.

$\vec{V}_{J/s}$ = Velocity of Jupiter Relative to sun.

Initially $V_{p/J}$ was in another direction but when it comes to range of Jupiter's gravity it changes its speed as well as dirn.

so here we can apply momentum conservation b/w satellite & Jupiter and also energy conservation since only gravity is acting here.

Here $\vec{V}_{new} = \vec{V}_{p/s in} + \vec{V}_{J/s}$

So $|\vec{V}_{new}| - |\vec{V}_{p/s in}| = 16$ km/s

$\vec{V}_{new} = \vec{V}_{p/s out}$.