Assignment - 4

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1. We know that

$$R_{LEO} = \underset{(\times 10^3)}{400} + R_E$$

$$R_{GO} = 36050 \times 10^3 + R_E$$

$$R_{GSO} = 35,800 \times 10^3 + R_E$$

Thus

$$\Delta v_{LEO} = \left| \sqrt{GM_E} \left(\frac{1}{\sqrt{R_{LEO}}} - \frac{1}{\sqrt{R_{GSO}}} \right) \right|$$

$$= \sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24}} \left(\frac{1}{\sqrt{6.77 \times 10^6}} - \frac{1}{\sqrt{42.17 \times 10^6}} \right)$$

$$= 2 \times 10^7 \times (0.38 - 0.15) \times 10^{-3}$$

$$= 4.6 \times 10^3 = \boxed{4.6 \text{ km/s}}$$

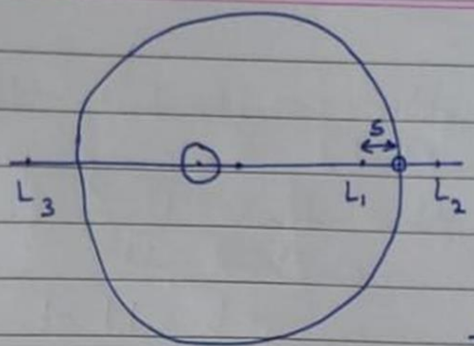
and $$\Delta v_{GO} = \left| \sqrt{GM_E} \left(\frac{1}{\sqrt{R_{GO}}} - \frac{1}{\sqrt{R_{GSO}}} \right) \right|$$

$$= \left| 2 \times 10^7 \times (0.1536 - 0.154) \times 10^{-3} \right|$$

$$= \boxed{0.0078 \text{ km/s}}$$

Thus a transfer to Graveyard Orbit would be preferable.

2.



Equating ~~potentials~~ forces due to gravity and centripetal acc. we get (for L_1), assuming dist. betn the center of the Sun and CoM of Earth-Sun system to be very small —

$$\frac{GM_S}{(R_{SE}-s)^2} = \frac{GM_E}{s^2} + \omega^2 (R_{SE}-s)$$

$$\therefore \quad GM_S s^2 = GM_E (R_{SE}-s)^2 + \omega^2 (R_{SE}-s)^3 s^2$$

$$= GM_E (R_{SE}-s)^2 + \cancel{(R_{SE}-s)^3} \cdot \frac{G\mu_{SE} s^2}{\cancel{(R_{SE}-s)^3}}$$

Since $M_S >> M_E$, ~~$\mu_{SE} \approx M_S$~~

$$\mu_{SE} = \frac{M_S M_E}{M_S + M_E} \approx M_E$$

$$\therefore \quad \left(\frac{M_S}{M_E} - 1\right) s^2 = (R_{SE}-s)^2 = R_{SE}^2 - 2R_{SE}s + s^2$$

$$\therefore \quad \left(\frac{M_S}{M_E} - 2\right) s^2 + 2R_{SE}s - R_{SE}^2 = 0.$$

Now $M_S >> M_E$, so we neglect '-2'. Our final quadratic equation is

$$\frac{10^6}{3} \cdot s^2 + 3\times 10^{11} \cdot s - 2.25\times 10^{22} = 0.$$

$$\therefore \quad s = \frac{-3\times 10^{11} + \sqrt{9\times 10^{22} + 9\times \frac{1}{3}\times 10^{22+6}}}{2\times \frac{10^6}{3}}$$

$$= 2593\times 10^5 \text{ m}$$

$$= 2.593\times 10^8 \text{ m}$$

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Using the given formula, and values of R, M_2 and M_1, we get

$$s = R_{SE} - (x \text{ co-ordinate of } L_1)$$

$$= R - \left(R - R\left(\frac{M_2/M_1}{3}\right)^{1/3}\right)$$

$$= R\left(\frac{M_2/M_1}{3}\right)^{1/3}$$

$$= R\left(\frac{3\times10^{-6}}{3}\right)^{1/3}$$

$$= 1.5\times10^{11}\times10^{-2} = \cancel{1.5}\ 15\times10^{8}\ m.$$

Now there seems to be a large error (which is due to the approximations made However the ~~logarithmic~~ difference in the decimal logarithms of both is ~~less than 1~~ <1, thus it would be a fairly good estimate taking the size of the Sun-Earth system into account.

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3. The figure illustrates how ~~a~~ a spacecraft can achieve an extra-ordinary Δv using a gravity assist. It ~~also~~ changes the direction of the probe's motion, in such a way that the velocity components (due to the relative motion w.r.t. the Sun and Jupiter) add up at an obtuse angle as the probe recedes from the planet. This manouvre would require an enormous amount of ~~fuel~~ propellant in the planet's 'absence'. ~~We would need to launch it along with the~~