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2)a) Let initial mass of rocket be m_r, then since $\mathring{m}$ is constant fuel rate, mass of rocket after time $t = m_r - \mathring{m} t = m_t'$ — ①

Now, specific impulse $I_{sp} = \frac{F_{thrust}}{\mathring{m} g_0}$

$$\Rightarrow F_{thrust} = I_{sp} \mathring{m} g_0 \quad — ②$$

Now,

$$F_{thrust} - m_t g_0 = m_t a$$

$$F_{thrust} - m_t g_0 = m_t \frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \left(\frac{F_{thru}}{m_t} - g_0\right)$$

$$\Rightarrow \int_0^v \frac{dv}{\cancel{dt}} = \int_0^t \left(\frac{F_t}{(m_r - \mathring{m} t)} - g_0\right) dt \quad \text{(Using ①)}$$

$$\Rightarrow v = \int_0^t \frac{I_{sp} \mathring{m} g_0}{m_r - \mathring{m} t} - g_0 t \quad \text{(Using ②)}$$

$$\Rightarrow v = -\frac{I_{sp} g_0 \cancel{\mathring{m}}}{\cancel{\mathring{m}}} \left[\ln(m_r - \mathring{m} t)\right]_0^t - g_0 t$$

$$\Rightarrow v = -I_{sp} g_0 \ln\left(\frac{m_r - \mathring{m} t}{m_r}\right) - g_0 t$$

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$$\Rightarrow \boxed{v = +I_{sp}\, g_0 \ln\left(\frac{m_r}{m_r - \dot{m}t}\right) - g_0 t}$$

2) b) When $t = t_{b}$,

$$v = I_{sp}\, g_0 \ln\left(\frac{m_r}{m_r - \dot{m}t_b}\right) - g_0 t_b \quad \text{--- (1)}$$

From previous equation,

$$\Rightarrow \frac{dh}{dt} = I_{sp}\, g_0 \ln\left(\frac{m_r}{m_r - \dot{m}t}\right) - g_0 t$$

$$\Rightarrow \int_0^H dh = I_{sp} g_0 \int_0^{t_b} \ln\left(\frac{m_r}{m_r - \dot{m}t}\right) dt - \int_0^{t_b} g_0 t \, dt$$

$$\Rightarrow H = I_{sp} g_0 \int_0^{t_b} \ln(m_r)\, dt - \int_0^{t_b} \ln(m_r - \dot{m}t)\, dt - \frac{g_0 t_b^2}{2}$$

$$H = I_{sp} g_0 t_b \ln m_r - (m_r - \dot{m}t_b)(\ln(m_r - \dot{m}t_b) - 1) + m_r(\ln m_r - 1) - \frac{g_0 t_b^2}{2}$$

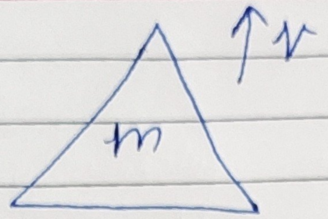
$$\boxed{H = I_{sp} g_0 t_b \ln m_r - (m_r - \dot{m}t)(\ln(m_r - \dot{m}t) - 1) + m_r \ln m_r - m_r - \frac{g_0 t_b^2}{2}}$$

(where, H is height at burnout time t_b) — (2)

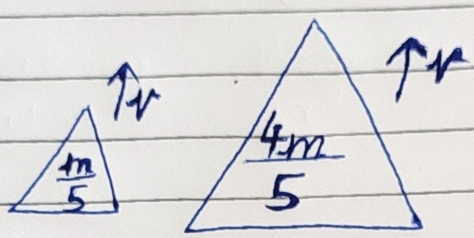
So, final maximum height achieved,

$$= H + \frac{v^2}{2g_0}$$, Solving by putting eqn (1) & (2),

3) Initially →



Finally →
when quad is detached



Since, debris start flying after t_0 sec, to escape them, at any time t, final distance of the quad should be greater than debris,
Thus,
at any time t,

→ $$d_{quadrot} > d_{debris}$$

, d_{debris} can be easily calculated as

→ $$d_{debris} = v t_0 + \frac{v}{2}(t - t_0)$$

→ Speed of ~~spe~~ quadrant at time t,

$$v_f = v + u_e \ln\left(\frac{m_f}{m_i}\right) = v + u_e \ln\left(\frac{\frac{m}{5} - \dot{m}t}{\frac{m}{5}}\right)$$

$$= v + u_e \ln\left(\frac{m - 5\dot{m}t}{m}\right)$$

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distance of ship after time t,

$$\frac{dh}{dt} = v + u_e \ln\left(\frac{m - 5\dot{m}t}{m}\right)$$

$$\Rightarrow h = vt + u_e (m - 5\dot{m}t)\left(\ln(m - 5\dot{m}t) - 1\right) - t\ln m - m\ln m - m$$

$$\Rightarrow h > d_{debris},$$

$$\Rightarrow vt + u_e (m - 5\dot{m}t)\left(\ln(m - 5\dot{m}t) - 1\right) - \ln m(t+m) - m > vt_0 + \frac{vt}{2} - \frac{vt_0}{2}$$

$$\Rightarrow u_e (m - 5\dot{m}t)\left[\ln(m - 5\dot{m}t) - 1\right] > \frac{vt_0}{2} - \frac{vt}{2} + m + \ln m(t+m)$$