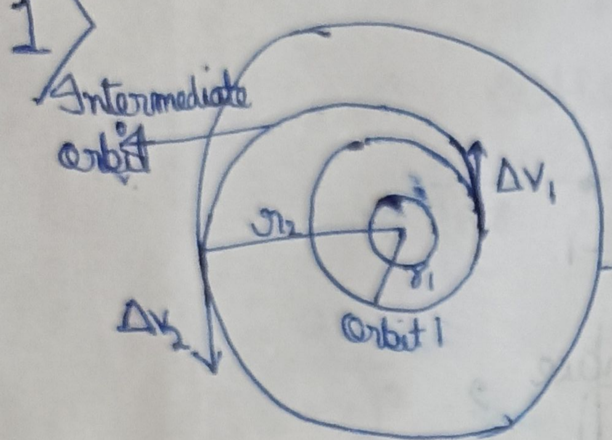
Assignment - 4

1) 

Transferring from orbit-1 intermediate orbit

$$V = \sqrt{\frac{GM}{r_1}} \qquad \mu = GM$$

$$\frac{mv_1'^2}{2} - \frac{GMm}{r_1} = -\frac{GMm}{2a} \qquad a = \frac{r_1+r_2}{2}$$

$$\Rightarrow (v_1')^2 = \mu\left(\frac{2}{r_1} - \frac{1}{a}\right)$$

$$\Rightarrow \Delta V_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)$$

From intermediate orbit to orbit-2.

$$v' = \sqrt{\frac{\mu}{r_2}}$$

$$\frac{mv^2}{2} - \frac{GMm}{r_2} = -\frac{GMm}{2a}$$

$$\Rightarrow v_2^2 = \mu\left(\frac{2}{r_2} - \frac{1}{a}\right)$$

$$\Rightarrow \Delta V_2 = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right)$$

$$\Delta V_{total} = \Delta V_1 + \Delta V_2$$

1) Geostationary orbit — low earth orbit

$r_1 = 35800 + R_e = 42200\,km$

$r_2 = 2400 + R_e = 6800\,km$

$\therefore (\Delta v_{total})|_{LEO} = |\Delta v_1 + \Delta v_2| = 3856\ m/s$

- from geostationary orbit $\longrightarrow$ graveyard orbit

$r_1 = 42200$ km

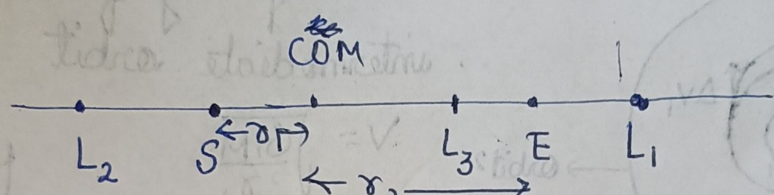
$r_2 = 42450$ km

$(\Delta v_{total}) = \Delta v_1 + \Delta v_2 = 9.07\ m/s$

$(\Delta v_{total})_{gv} < (\Delta v_{total})_{LEO}$

$\therefore$ We would prefer to transfer to graveyard orbit.

2)



$\because$ For an equipotential surface,

$V =$ constant

$$\frac{\partial V}{\partial r} = 0 \Rightarrow F = 0$$

$$\Rightarrow (F_G)_S + (F_G)_E = F_{centrifugal\ force} = F_2$$

For L1 region, (x = distance of L_1 from COM)

$$\frac{GM_S}{(x+r_1)^2} + \frac{GM_e}{(x-r_2)^2} = \frac{G(M_S+M_e)\,x}{R_{SE}^3}$$

$$\Rightarrow \cancel{G} M_S R^3 (x-r_2)^2 + \cancel{G} M_e (x+r_1)^2 R^3 = \cancel{G}(M_S+M_e)\,x\,(x+r_1)^2 (x-r_2)^2$$

which on solving gives

$$L_1 = R\left(1 + \left(\frac{M_e}{3M_S}\right)^{1/3}\right)$$

Putting M_e, M_S, R in eq, we get

$L_1 = 1.515 \times 10^{11}$ km

Similarly for L_2

$$\frac{-GM_s}{(x-r_1)^2} + \frac{GM_e}{(x+r_2)^2} = \frac{-G(M_s+M_e)x}{R^3}$$

$$\Rightarrow -M_s(x+r_1)^2R^3 + M_e(x-r_2)^2R^3 = (M_s+M_e)x(x-r_1)^2(x+r_2)^2$$

Solving this eqn. we get

$$L_2 = \left(1-\left(\frac{M_e}{3M_s}\right)^{1/3}\right)R$$

For M_e, M_s and R we get $L_2 = 1.48\times10^{11}$ which satisfies the eqn.

Now, for L_3

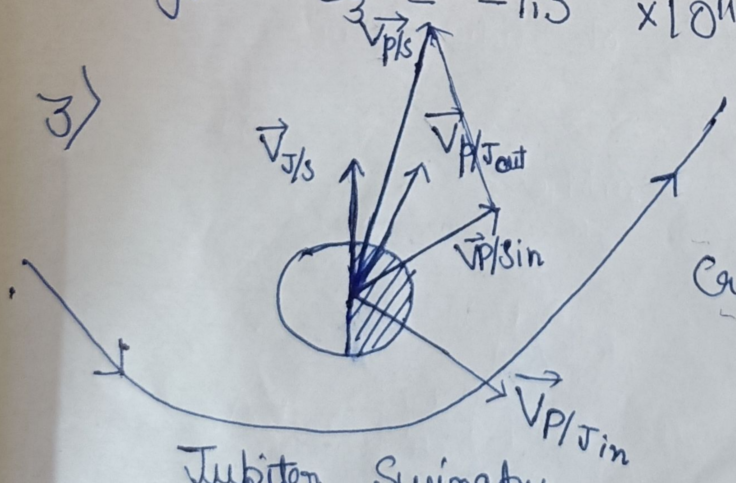
$$\frac{GM_s}{(x+r_1)^2} - \frac{GM_e}{(x-r_2)^2} = \frac{G(M_s+M_e)}{(r_1+r_2)^3}x$$

Solving for this we get $L_3 = \left(-R\left[1+\frac{5}{12}\frac{M_E}{M_s}\right], 0\right)$

Putting the values for M_e, M_s an R, we get

$$L_3 = -1.5\times10^{11}\ m$$

3)



Gravity assist

Jupiter Swingby

When the rocket comes in the influence of Jupiter's gravity, it changes its speed and direction from its initial state.

So, energy and momentum is conserved in this process.

Thus $\vec{V}_{new} = \vec{V}_{p/s\,in} + \vec{V}_{J/s}$

$|\vec{V}_{p/s\,out}| - |\vec{V}_{p/s\,in}| = \Delta v = 16\ km/s$