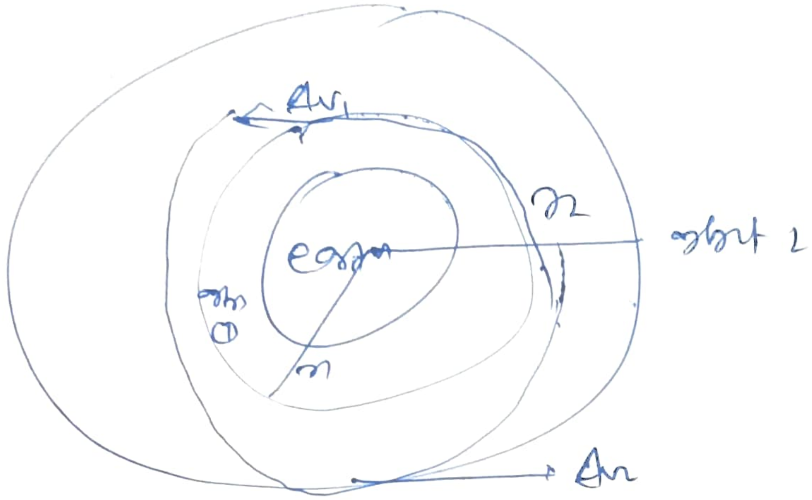
(Q1)



orbit 1 to intermediate orbit:

$$-\frac{Gm_em}{2r_1} + \frac{1}{2}mv^2 = -\frac{Gm_em}{r_1+r_2}$$

$$2\left(\frac{Gm_e}{2r_1} - \frac{Gm_e}{r_1+r_2}\right) = v^2$$

$$\sqrt{\frac{Gm_e}{r_1}}\sqrt{1-\frac{2r_1}{r_1+r_2}}$$

$$\Delta v_1 = \sqrt{\frac{GM}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)$$

$r = r_1$ from the circular orbit

Similarly

$$\Delta v_2 = \sqrt{\frac{GM}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right)$$

to leave the elliptical orbit at $r = r_2$ to the r_2 circular orbit

$\Delta V = \Delta V_1 + \Delta V_2$

$$= \sqrt{\frac{GM}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right) + \sqrt{\frac{GM}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right)$$

i) $r_1 = \cancel{400\,km}\ 35800$

$r_2 = 400$

$$\Delta V = \sqrt{GM}\left[\left(\frac{\sqrt{\frac{2r_2}{r_1+r_2}} - 1}{\sqrt{r_1}}\right) + \left(\frac{1 - \sqrt{\frac{2r_1}{r_1+r_2}}}{\sqrt{r_2}}\right)\right]$$

$$= \cancel{\sqrt{GM}\left(\frac{-0.97}{\sqrt{r_1}} + 1 - \right.}$$

$$= \cancel{\sqrt{GM}\left(\frac{-0.85}{\sqrt{35800 \times 10^3}}\right.} + \left(1 - \right.$$

$$= \sqrt{GM}\left(\frac{-0.85}{\sqrt{35800}} - \frac{0.40}{\sqrt{400}}\right)$$

$$= \sqrt{GM}\left(\frac{-0.85}{189.20} - 0.02\right)$$

$$= \sqrt{GM}\left(-0.00449 - 0.02\right)$$

$$\Delta V_{LEO} = -\sqrt{GM} \times 0.02449$$

ii) $r_1 = 35800$

$r_2 = 36050$

$$\Delta V = \sqrt{GM}\left[\left(\frac{\sqrt{\frac{2r_2}{r_1+r_2}} - 1}{\sqrt{r_1}}\right) + \left(\frac{1 - \sqrt{\frac{2r_1}{r_1+r_2}}}{\sqrt{r_2}}\right)\right]$$

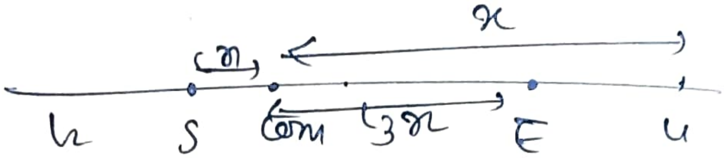
$$\Delta V = \sqrt{GM}\,(2.98) \times 10^{-6}$$

so

$\Delta v|_{\text{graveyard}} < \Delta v_{LEO}$

we prefer hohmann transfer to graveyard orbit

(a)



$r_1 = \frac{M_E R_{ES}}{M_E + M_S}$

$r_2 = \frac{M_S R_{ES}}{M_E + M_S}$

for an equipotential surface $\frac{dV}{dr} = 0 \Rightarrow F = 0$

$(F_{grav})_S + (F_g)_E = F_{centrifugal}$

for L1 region

$$\frac{GM_s}{(x+r_1)^2} + \frac{Gm_e}{(x-r_2)^2} = \frac{G(M_E+M_S)}{R^3}x$$

$$GM_s(x-r_2)^2 + Gm_e(x+r_1)^2 = \frac{G(m_e+m_s)x}{R^3}$$

$$M_s(x-r_2)^2R^3 + m_e(x+r_1)^2R^3 = (m_e+m_s)x$$

$\Rightarrow$ this will give x

for L_1 we get

$$L_1 = R\left(1 + \left(\frac{M_E/M_S}{3}\right)^{1/3}\right]$$

$\cancel{L_2 = R(1 -}$ n

Checking for $L_1 = 1.515 \times 10^{11}$ km

L_1 satisfies eqn (1)

Now for L_2

$$\frac{GM_s}{(x-x_1)^2} + \frac{GM_E}{(x+x_2)^2} = \frac{G(M_s+M_E)x}{R^3}$$

$$M_s(x+x_2)^2R^3 + M_E(x-x_1)^2R^3 = (M_s+M_E)x$$

on solving we get

$$x_{L_2} = \left(1 - \left(\frac{M_E/M_s}{3}\right)^{1/3}\right)R \quad L_2 = 1.49 \times 10^{11} \text{ km}$$

L_2 satisfies eqn (2).

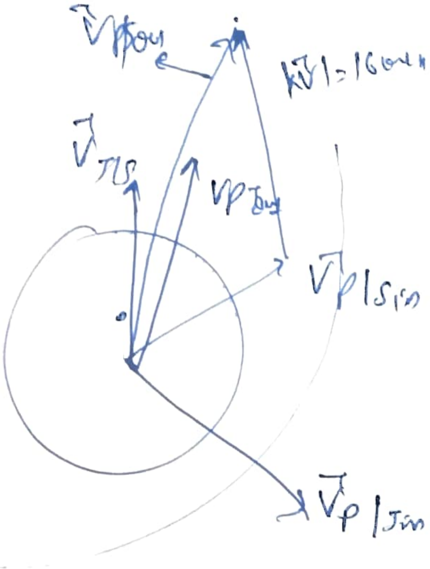
Similarly for L_3

$$\frac{GM_s}{(x+x_1)^2} - \frac{GM_E}{(x-x_2)^2} = \frac{G(M_s+M_E)x}{(x+x_1)^2} \quad - (3)$$

$L_3 = -1.50001807 \times 10^{11}$

$$L_3 = \left(-R\left[1 + \frac{5M_2}{12M_1}\right], 0\right)$$

Q3



Initially $V_{P/J}$ points in another direction but ~~watta~~ when it comes in range of Jupiter its gravity changes its speed as well as direction.

So here we can apply momentum conservation b/w satellite and Jupiter and also energy conservation

$$\vec{V}_{new} = \vec{V}_{P/J,final} + \vec{V}_{J/S}$$

$$|\vec{V}_{new}| - |\vec{V}_{old}| = 16\,km/s$$