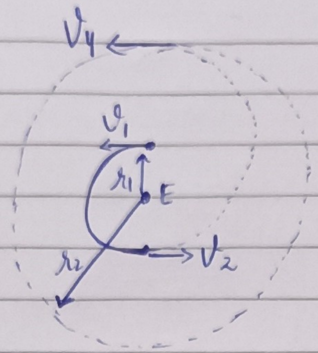
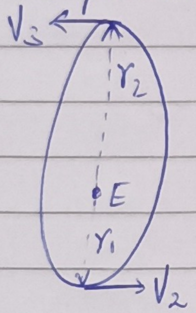
1



$v_1 = \sqrt{\frac{GM}{r_1}}$

$v_4 = \sqrt{\frac{GM}{r_2}}$

elliptical orbit,



$a = \frac{r_1 + r_2}{2}$

$-\frac{GM\cancel{m}}{2a} = \frac{1}{2}\cancel{m}v_2^2 - \frac{GM\cancel{m}}{r_1}$

$\frac{GM}{r_1} - \frac{GM}{r_1+r_2} = \frac{v_2^2}{2}$

$GM\left(\frac{\cancel{r_1} + r_2 - \cancel{r_1}}{r_1(r_1+r_2)}\right) = \frac{GMr_2}{r_1(r_1+r_2)} = \frac{v_2^2}{2}$

$v_2 = \sqrt{\frac{2GMr_2}{r_1(r_1+r_2)}}$

by consv of L,

$\cancel{m}v_2r_1 = \cancel{m}v_3r_2$

$v_3 = \frac{r_1}{r_2}\sqrt{\frac{2GMr_2}{r_1(r_1+r_2)}} = \sqrt{\frac{2GMr_1}{r_2(r_1+r_2)}}$

$\Delta v_{total} = (v_2 - v_1) + (v_4 - v_3)$

$= \sqrt{\frac{2GMr_2}{r_1(r_1+r_2)}} - \sqrt{\frac{GM}{r_1}} + \sqrt{\frac{GM}{r_2}} - \sqrt{\frac{2GMr_1}{r_2(r_1+r_2)}}$

$$\Rightarrow \sqrt{GM}\left[\sqrt{\frac{2r_2}{(r_1+r_2)r_1}} - \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}} - \sqrt{\frac{2r_1}{r_2(r_1+r_2)}}\right]$$

$$\sqrt{GM}\left(\sqrt{\frac{2}{r_1+r_2}}\left(\sqrt{\frac{r_2}{r_1}} - \sqrt{\frac{r_1}{r_2}}\right) + \frac{1}{\sqrt{r_2}} - \frac{1}{\sqrt{r_1}}\right)$$

$$\cancel{\sqrt{\frac{2}{(r_1+r_2)}}\frac{r_2-r_1}{\sqrt{r_1 r_2}} + \frac{\sqrt{r_1}-\sqrt{r_2}}{\sqrt{r_1 r_2}}}$$

$\rightarrow r_2 = 36050 \times 10^3\ m + R_e \Rightarrow 42421393\ m$

$r_1 = 35800 \times 10^3\ m + R_e \Rightarrow 42171393\ m$

putting r_2, r_1 in the abv eq^n we get

$\Delta v \Rightarrow 9.0692086\ m/s$ for transferring to the graveyard orbit

$r_2 \Rightarrow 400 \times 10^3 + R_e \Rightarrow 6771393\ m$

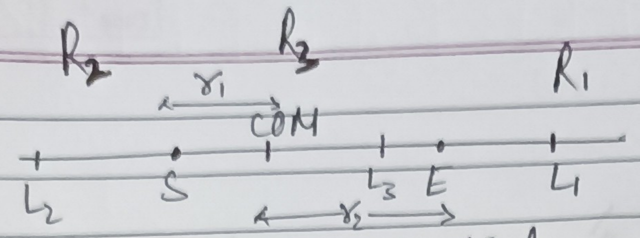
$r_1 \Rightarrow 35800 \times 10^3 + R_e = 42171393\ m$

putting r_2, r_1 in the eq^n we get

$\Delta v \Rightarrow -3856.6016\ m/s$ for transferring to LEO.

transferring to graveyard orbit will be favourable as Δv is less in that case.

(2)



for an equipotential surface,

$V = $ constt

$\frac{\partial V}{\partial r} = 0 \Rightarrow \underline{\underline{F = 0}}$

$(F_{grav.})_{s} + (F_{grav.})_{E} = F_{centrifugal}$

Let R_1: region right of earth

R_2: region left of sun

R_3: region b/w sun and earth

in R_1,

$$\frac{GM_s}{(x+r_1)^2} + \frac{GM_e}{(x-r_2)^2} = \frac{G(M_s+M_e)}{R_{sE}^3} x$$

x: dist of L_1 from COM

$$M_s(x-r_2)^2 + M_e(x+r_1)^2 \Rightarrow \frac{(M_s+M_e)x}{R_{sE}^3}\left[(x+r_1)^2(x-r_2)^2\right]$$

~~which on solving~~ gives,

$$L_1 = R\left[1 + \left(\frac{M_e}{3M_s}\right)^{1/3}\right)$$

$L_1 \Rightarrow 1.515 \times 10^{11}$ m

in R_2,

$$\frac{GM_s}{(x-r_1)^2} + \frac{GM_e}{(x+r_2)^2} \Rightarrow \frac{G(M_s+M_e)}{R^3} x$$

$$M_s(x+r_2)^2 + M_e(x-r_1)^2 = \frac{(M_s+M_e)x}{R^3}\left[(x-r_1)^2 \cdot (x+r_2)^2\right]$$

~~on solving this~~ gives, $L_2 = R\left[1 - \left(\frac{M_e}{3M_s}\right)^{1/3}\right)$

$L_2 = 1.48 \times 10^{11}\,m$

in R_3,

$$\frac{GM_{\phi}}{(x+r_1)^2} - \frac{GM_e}{(r_2-x)^2} = \frac{G(M_{\phi}+M_e)}{R_{\phi E}^3}\,x$$

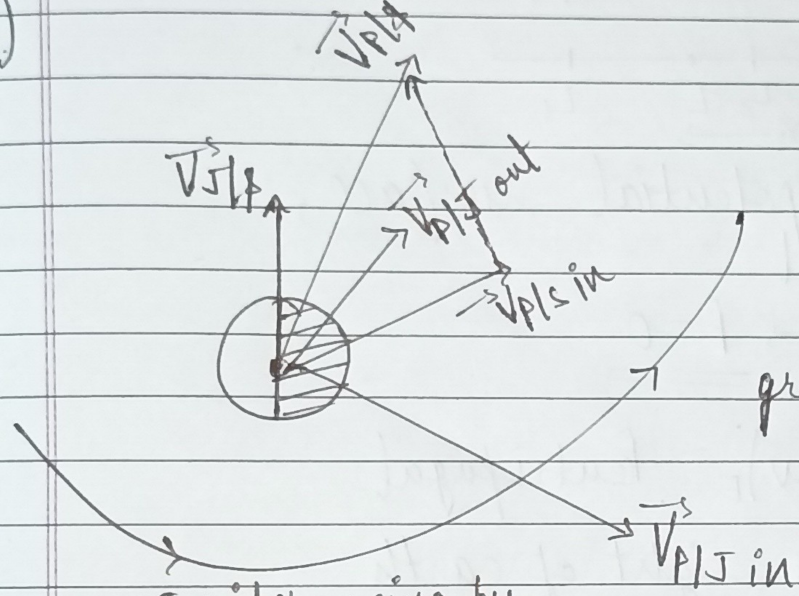
$$M_{\phi}(r_2-x)^2 - M_e(x+r_1)^2 = \frac{(M_{\phi}+M_e)x}{R_{\phi E}^3}\left[(x+r_1)^2\cdot(r_2-x)^2\right]$$

on solving this gives,

$$L_3 = -R\left(1+\frac{5}{12}\frac{M_e}{M_{\phi}}\right)$$

$L_3 \simeq -1.5 \times 10^{11}\,m$ → as $\left(\frac{M_e}{M_{\phi}}\right)$ gets very small in comparison to 1

3)



gravity assist

Jupiter swing by

when the rocket comes in the influence of Jupiter's gravity, it changes it's speed and direction from it's initial state so energy and momentum is conserved in this process.

$\therefore \vec{V}_{new} = \vec{V}_{P|S\,in} + \vec{V}_{J|S}$

$-|\vec{V}_{P|S\,in}| + |\vec{V}_{P|S\,out}| = \Delta V = 16\ km/s$