DATE: / /
PAGE NO.
B+

Assignment 2

1) Using $\Delta v = I_{sp} \cdot g_0 \cdot \ln\left[1 + \frac{M_{prop}}{M_{pay} + M_{rocket}}\right]$

Let $\frac{M_{prop}}{M_{prop} + M_{rocket} + M_{pay}} = x$

$\Rightarrow$ Given $\Delta v = 7.6\ Kms = 7600\ mps$

$I_{sp} = 400s \quad g_0 = 9.8\ ms^{-2}$

$\Rightarrow 7600 = 400 \times 9.8 \times \ln\left[1 + \frac{M_{prop}}{M_{pay} + M_{rock}}\right]$

$1.939 = \frac{190}{98} = \ln\left[1 + \frac{M_{prop}}{M_{pay} + M_{r}}\right]$

$1 + \frac{M_{prop}}{M_{pay} + M_{ro}} = e^{1.939} = 6.95 \Rightarrow \frac{M_i}{M_{pay} + M_{ro}} = 6.95$

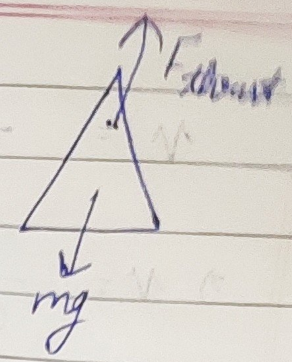
$\frac{M_{prop}}{M_{pay} + M_r} = 5.95 \Rightarrow \frac{5.95(M_{pay} + M_r)}{M_{pay} + M_r + M_{prop}} = x$

~~$\Rightarrow x = 5.95(M_{prop} + M_{ro} + M_{pay})$~~

$\Rightarrow \frac{5.95(M_{pay} + M_r)}{6.95(M_{pay} + M_r)} = x$

$\Rightarrow x = 0.856$

2.) a) $F_{thrust} = -C\frac{dM}{dt}$



$\Rightarrow$ For lift,

$$F_{thrust} - mg = F_{net} = m\frac{dv}{dt}$$

$$-C\frac{dM}{dt} - mg = m\frac{dv}{dt}$$

$$-\int C\,dM - \int mg\,dt = \int m\,dv$$

$$-\int_{m_0}^{m_f} \frac{C\,dM}{m} - \int_0^t g\,dt = \int_{v_0}^{v} dv \Rightarrow -\left[C\ln m\right]_{m_0}^{m_f} - gt = v - v_0$$

$$\Rightarrow C\ln\frac{m_0}{m_f} - gt + v_0 = v$$

$$\Rightarrow \boxed{-C\ln u - gt = v} \quad (\text{If } v_0 = 0)$$

DATE: / /
PAGE NO.

B+

b) $v = -gt - c \ln u$

$v = -gt - c \ln \frac{m_f}{m_0}$

$v = -gt - c \ln\left(\frac{m_0 - kt}{m_0}\right)$

$v = -gt - c \ln\left(1 - \frac{kt}{m_0}\right)$

$\int_0^z dz = -\int_0^t gt\,dt - \int_0^t c \ln\left(1 - \frac{kt}{m_0}\right) dt$

Let burning rate be constant then mass rate also will be constant,

$\Rightarrow -\frac{dm}{dt} = k$

$\Rightarrow -\int_{m_0}^{m_f} dm = kt$

$\Rightarrow (m_f - m_0) = -kt$

$\Rightarrow \boxed{m_f = m_0 - kt}$

$\frac{m_f}{m_0} = 1 - \frac{kt}{m_0}$

$z = -\frac{gt^2}{2} + \frac{c\,m_0}{k}\left(1 - \frac{kt}{m_0}\right)\left(\ln\left(1 - \frac{kt}{m_0}\right) - 1\right)$

$z = -\frac{gt^2}{2} + \cancel{\frac{c\,m_0}{k}}\, u \frac{c\,m_0}{k}\left(\ln u - 1\right)$

$$\boxed{z = -\frac{gt^2}{2} + \frac{u c t}{1-u}\left(\ln u - 1\right)}$$

DATE: / /
PAGE NO. B+

c) Let burnout time be t_b,

Velocity of rocket at t_b = $\vec{v_b} = -c\ln \eta - g t_b$

Thus, Extra time taken till velocity reaches 0,

$$0 = v_b - gt \Rightarrow t = \frac{v_b}{g}$$

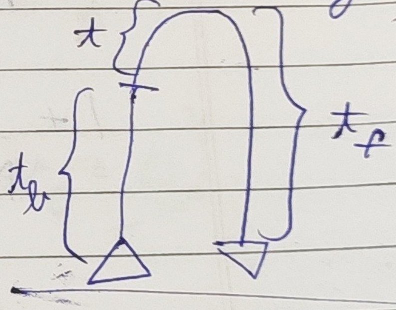
~~Thus~~, Final time to fall back to Earth, using $h = ut + \frac{1}{2}at^2$

$$z_b = \frac{g t_f^2}{2} \Rightarrow t_f = \sqrt{\frac{2z_b}{g}}$$

z_b is total distance covered in one way.

Total time of flight = $t_b + t + t_f$

$$\boxed{T = t_b + \frac{v_b}{g} + \sqrt{\frac{2z_b}{g}}}$$



d) $n = \frac{F_{thrust}}{m_0 g}$ is the ratio of the lift force generated against the gravitational force. Thus, higher the value of n, greater is the lift force and greater is the acceleration of the rocket.

DATE: / /
PAGE NO. B+

3) Given, $\Delta v = 8$ kmps
$v_e = 4.5$ kmps

Using, $\Delta v = -v_e \ln\left(\frac{m_f}{m_0}\right)$

$\Rightarrow 8 = -4.5 \ln\left(\frac{m_f}{m_0}\right)$

→ For SSTO,

$\Rightarrow \frac{\cancel{80}^{16}}{\cancel{45}_{9}} = \ln\left(\frac{m_{pay} + m_{prop} + m_{r}}{m_{pay} + m_{r}}\right)$

$\Rightarrow \frac{16}{9} = \ln\left(\frac{m_{pay} + m_{prop} + m_{r}}{m_{pay} + m_{r}}\right)$

$\Rightarrow$

$5.93 = \frac{m_{pay} + m_{prop} + m_{r}}{m_{pay} + m_{r}}$

$m_i = m_{pay} + m_{prop} + m_{r} = 5.93(m_{pay} + m_{r})$ — (1)

Also,

$5.93 = 1 + \frac{m_{prop}}{m_{pay} + m_{r}}$

$m_{prop} = 4.93(m_{pay} + m_{r})$

$\Rightarrow \frac{m_{prop}}{m_{initial}} = \frac{4.93(m_{pay} + m_{r})}{5.93(m_{pay} + m_{r})} = 0.831$

4) Structural coefficient $\lambda = \dfrac{M_{rocket}}{M_{pay} + M_{rocket}}$

Thus,

$$1 - \lambda = 1 - \frac{m_r}{m_{pay} + m_r} = \frac{m_{pay}}{m_{pay} + m_r}$$

$$\frac{\lambda}{1-\lambda} = \frac{m_{rocket}}{m_{pay}} \rightarrow m_{rocket} = m_{pay}\left[\frac{\lambda}{1-\lambda}\right]$$

Thus, Using,

$$\Delta v = I_{sp}\, g_0 \ln\left(\frac{m_i}{m_{pay} + m_r}\right) = I_{sp} \cdot g_0 \cdot \ln\left(\frac{m_i}{m_{pay}\left[\frac{\lambda}{1-\lambda} + 1\right]}\right)$$

$$= I_{sp} \cdot g_0 \cdot \ln\left(\frac{m_i(1-\lambda)}{m_{pay} \times 1}\right)$$

$$e^{\frac{\Delta v}{I_{sp} \cdot g_0}} = \frac{M_i(1-\lambda)}{m_{pay}}$$

$$\Rightarrow \boxed{m_i = \frac{m_{pay}}{(1-\lambda)} e^{\left(\frac{\Delta v}{I_{sp} \cdot g_0}\right)}}$$

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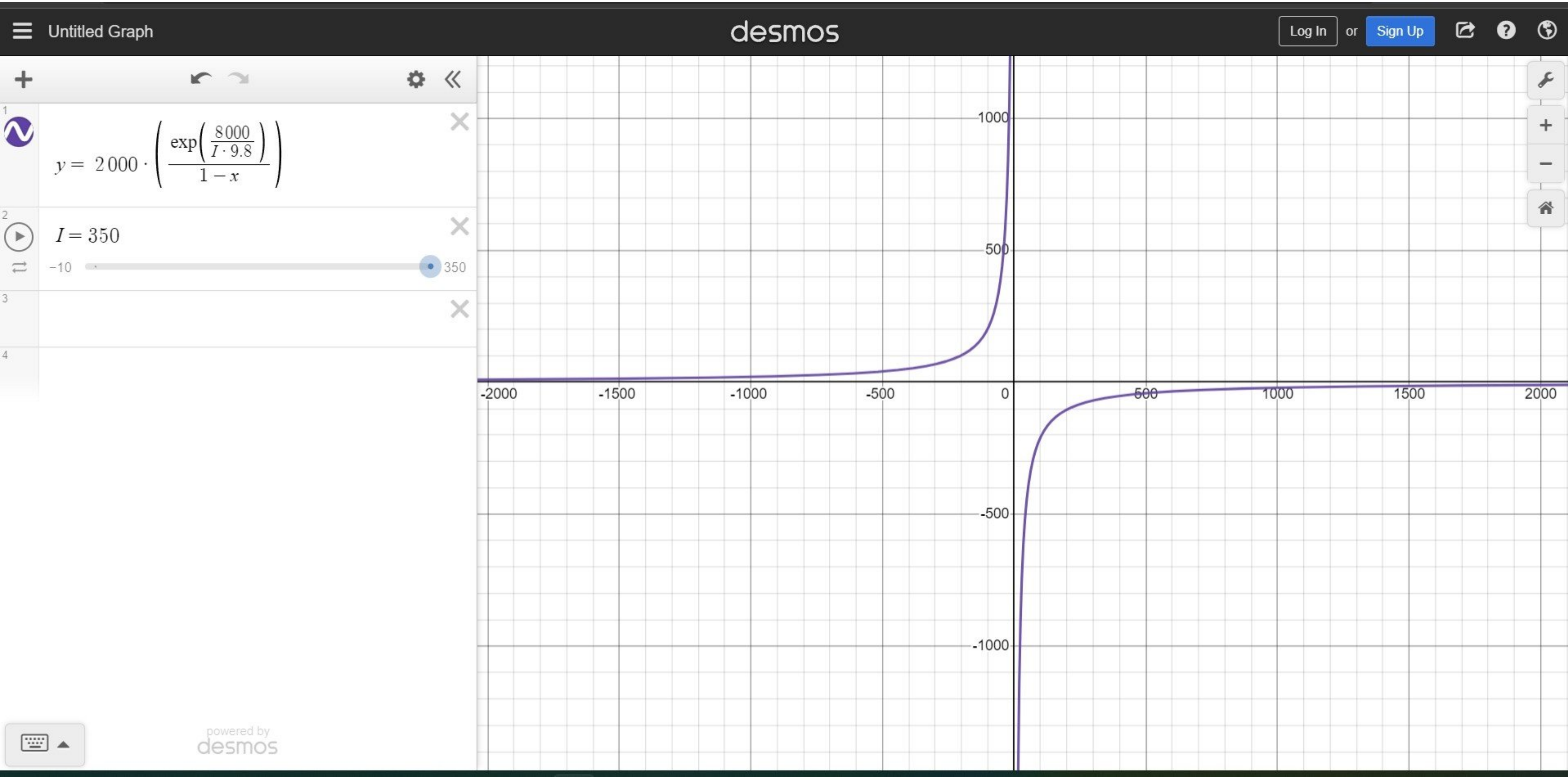

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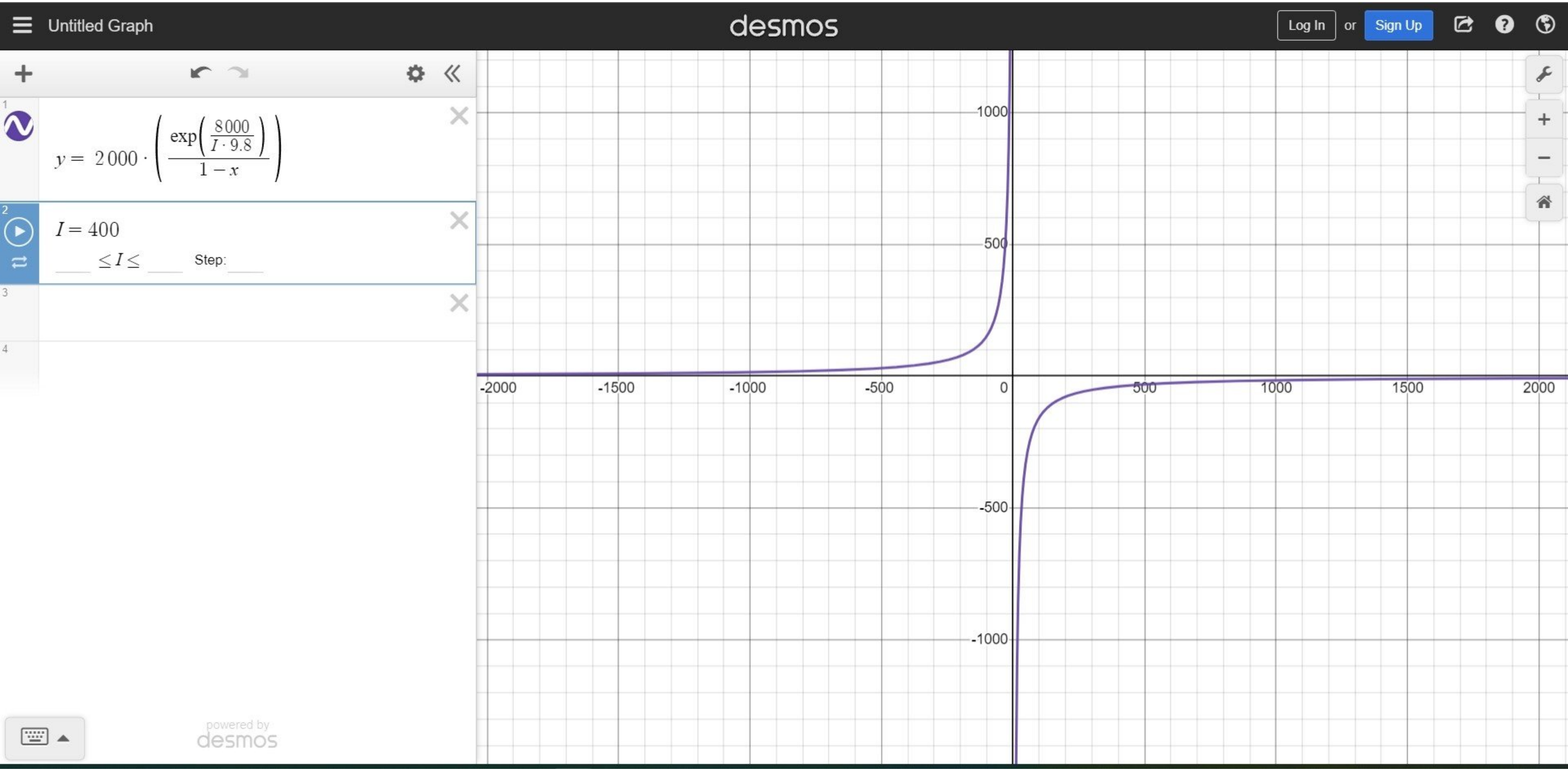
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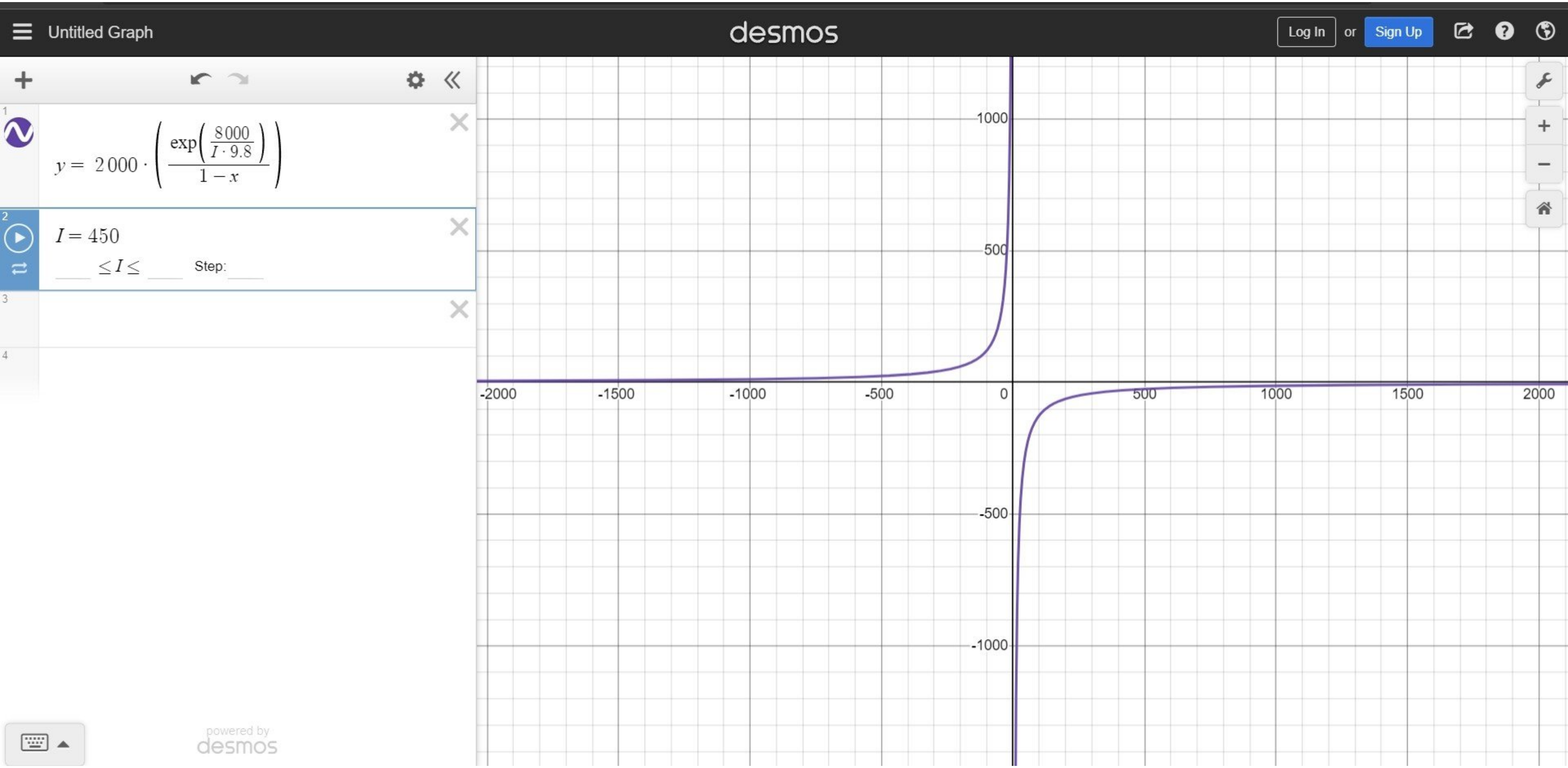
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$y = 2000 \cdot \left(\frac{\exp\left(\frac{8000}{I \cdot 9.8} \right)}{1 - x} \right)$
$I = 450$
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