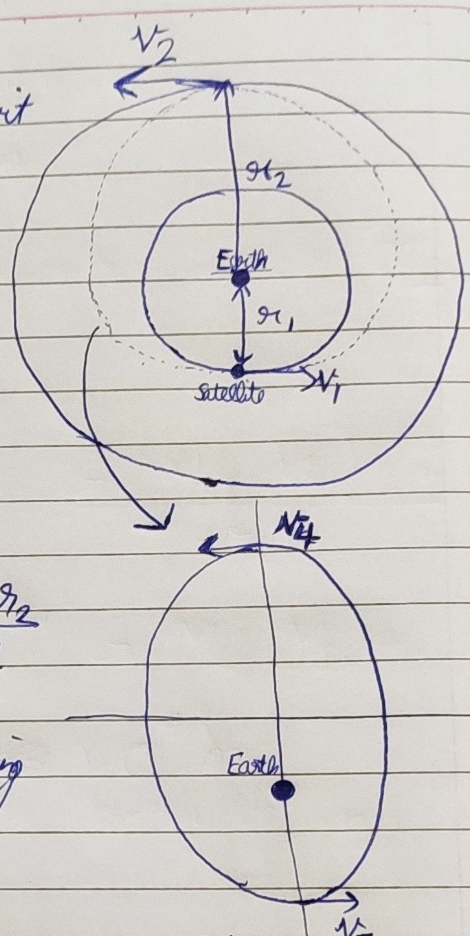
ASSIGNMENT - 4



Elliptical Orbit for Hohmann transfer

(1) → Velocity of satellite when in shorter orbit

$$v_1 = \sqrt{\frac{GM}{r_1}}$$

, where M is mass of Earth.
Also, let mass of satellite be m,

→ Velocity of satellite when in larger orbit $v_2 = \sqrt{\frac{GM}{r_2}}$

→ For elliptical trajectory $a = \frac{r_1 + r_2}{2}$

→ Now let speed of object in elliptical orbit at point r_1 be v_3, then, using conservation of energy,

$$\frac{mv_3^2}{2} - \frac{GMm}{r_1} = -\frac{GMm}{2a}$$

$$\Rightarrow v_3^2 = GM\left(\frac{2}{r_1} - \frac{1}{a}\right)$$

$$= GM\left(\frac{2}{r_1} - \frac{2}{r_1+r_2}\right) = \frac{GM}{r_1}\left(\frac{2r_2}{r_1+r_2}\right)$$

Thus, $v_3 = \sqrt{\frac{GM \times 2r_2}{r_1(r_1+r_2)}}$

Therefore, change in ~~step~~ speed $\Delta v_1 = v_3 - v_1$

$$= \sqrt{\frac{GM}{r_1}}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right]$$

Similarly, let speed of object in elliptical orbit at point r_2 be v_4, then using conservation of Energy,

$$\frac{m v_4^2}{2} - \frac{GMm}{r_2} = -\frac{GMm}{2a}$$

$$\Rightarrow v_4 = \sqrt{\frac{GM}{r_2} \times \frac{2r_1}{(r_1+r_2)}}$$

Therefore, change in speed $\Delta v_2 = -[v_4 - v_2] = \sqrt{\frac{GM}{r_2}}\left[1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right]$

Now, total $\Delta v = \Delta v_1 + \Delta v_2$

$$= \sqrt{\frac{GM}{r_1}}\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right] + \sqrt{\frac{GM}{r_2}}\left[1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right]$$

CASE-1 : Geostationary (35800 km) to LEO (400 km)

$r_1 = 35800 + 6400 = 42200$ km

$r_2 = 400 + 6400 = 6800$ km

$\Delta v_{LEO} = 3856$ m/s

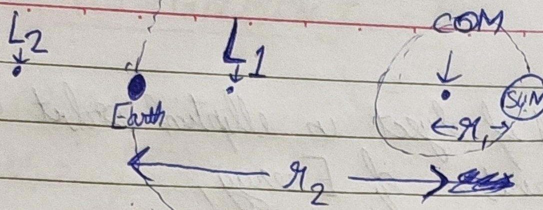
CASE-2 : Geostationary (35800 km) to Graveyard Orbit (36050 km)

$r_1 = 42200$ km $\qquad r_2 = 36050 + 6400 = 42450$ km

$\Delta v_{GR} = 9.07$ m/s

→ Since, $\Delta v_{LEO} > \Delta v_{GR}$, therefore transfer to graveyard orbit is more preferred for disposal of the satellite.

(2)



Let the distance of L_1 from the centre of mass be x_1,

Then, equating forces of gravitation,

$$\frac{GM_S m}{(x_1+r_1)^2} - \frac{GM_E m}{(-x_1+r_2)^2} = \frac{G(M_S+M_E)x m}{(r_1+r_2)^3}$$

Solving and putting ~~the~~ the values of G, M_E, M_S, r_1 & r_2, we get,

$L_1 = +1.5 \times 10^9$ m towards the Sun from Earth

Similarly for L_2,

$$\frac{GM_S m}{(x_2+r_1)^2} + \frac{GM_E m}{(x_2-r_2)^2} = \frac{G(M_S+M_E)m x_2}{R_{SE}^3}$$

Solving,

$L_2 = 1.5 \times 10^9$ m away from Sun from Earth

For L_3,

$$\frac{GM_E m}{(x_3+r_2)^2} - \frac{GM_S m}{(x_3-r_1)^2} = \frac{G(M_S+M_E)x_3 m}{R_{ES}^3}$$

Solving, ~~$L_2 = 1.48 \times 10^{11}$ m~~

$L_3 = 1.48 \times 10^{11}$ m from Sun, opposite to Earth.

3) The illustration is trying to show how gravity assist can be soo useful in moving objects around the space. The diagrams on the right comparatively show how much fuel would be required to make an 5kg object take an equal obtuse turn, in the absence of gravity assist. When the object comes into the influence of the Jupiter, the direction and speed of the object change greatly. Momentum conservation is applicable on the system and the total energy of planet-object system remains the same. The object borrows its increase in kinetic energy from planet's total energy. This leaves negligible effect on planet's total orbital speed.